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COSMOLOGICAL TESTS OF THE HOYLE-NARLIKAR CONFORMAL GRAVITY

V. M. CANUTO[1]
NASA Goddard Institute for Space Studies, Goddard Space Flight Center
AND
J. V. NARLIKAR
Tata Institute for Fundamental Research, Bombay, India
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ABSTRACT

For the first time we subject the Hoyle-Narlikar theory with creation of matter and a variable gravitational constant G, to the following cosmological tests: the magnitude versus z relation, the $N(m)$ versus m relation, the metric angular diameters versus z relation, the isophotal angles versus z relation, the log N–log S radio source count, and finally the 3 K radiation.

It is shown that the theory passes all these tests just as well as the standard cosmology, with the additional advantage that the geometry of the universe is uniquely determined, with $k = 0$.

It is also interesting to note that the variability of G affects the log N–log S curve in a way similar to the density evolution introduced in standard cosmologies. The agreement with the data is therefore achieved without recourse to an ad hoc density evolution.

Subject headings: cosmology — gravitation

I. INTRODUCTION

In 1964 Hoyle and Narlikar (HN hereafter) proposed a gravitation theory that tried to implement the Machian view that the inertia of a particle is due to the rest of the particles in the universe (Hoyle and Narlikar 1964). Subsequently, the theory was widened to include the possibility of positive and negative inertial coupling constants (HN 1972*a*) as well as the possibility of creation of matter (HN 1972*b*). Applications of the basic features of the theory to spacetime singularities, to anomalous redshifts, and to the cosmic microwave background have been discussed elsewhere (Kembhavi 1979; Narlikar 1977; Hoyle 1975).

From the observational point of view, the theory presents novel features. For example, the prediction of a variable gravitational constant G has many observable implications, especially in the field of geophysics (Hoyle 1972; Wesson 1978).

In this paper we shall, however, be concerned with the performance of the HN theory (HN 1972*b*) in the cosmological framework. We shall in fact subject the predictions of the HN theory to the following tests: the magnitude versus z relation, the number count $N(m)$ versus m for QSOs, the metric angular diameter versus z relation, the isophotal angles versus z relation, the log N versus log S radio source count, and finally the 3 K radiation. The last five tests had never been performed before on this cosmology. The m versus z test was attempted by Barnothy and Tinsley (1973), but, as will be shown later, their analysis is invalid.

The conclusion of this paper is that in all the above observational tests the HN theory fares equally well as the standard cosmology, with the following additional advantages.

First, the HN cosmology is uniquely specified by the curvature parameter $k = 0$ and the deceleration parameter $q_0 = 1$, and therefore makes clear parameter-free predictions about the large-scale structure of the universe. Second, the variability of G affects the log N–log S curve of radio sources in a way similar to the density evolution introduced in standard cosmologies. The agreement with the data is therefore achieved in this cosmology without recourse to an ad hoc density evolution.

It cannot be inferred from this analysis, however, that the gravitational constant must necessarily vary with epoch. We shall limit ourselves to concluding that a time-varying G, in a cosmological framework that implements Mach's principle, is perfectly compatible with the present observational data.

The first part of the following discussion is concerned with the basic features of the HN cosmology. The second part deals with their applications to observable tests, and makes use of the recent techniques developed for handling the observational tests of G-varying cosmologies (Canuto *et al.* 1977; Canuto, Hsieh, and Adams 1977; Canuto and Hsieh 1978, 1979; Canuto, Hsieh, and Owen 1979; Canuto and Owen 1979).

[1] Also with the Department of Physics, CCNY, New York.

II. THEORETICAL BACKGROUND

a) *The Machian Basis of HN Cosmology*

Let $a, b, c, \ldots$ label the particles in the universe, which is described by a Riemannian spacetime manifold with the metric tensor g_{ik}. Let da denote the element of proper time on the world line of particle a, so that if da^i are the coordinate differentials along the world line of a,

$$da^2 = g_{ik} da^i da^k . \tag{2.1}$$

We shall use $A, B, C, \ldots$ as typical world points on the world lines of $a, b, c, \ldots$ respectively, and X as a typical point in the spacetime manifold. In macroscopic problems it is often convenient to replace summations over discrete sets of particles by integrations over continuum distributions of matter. This can be done by introducing particle number densities at spacetime points.

The basic equations of the HN theory are derived from an action principle $\delta S = 0$, with the action

$$S = -\sum_{a \neq b}\sum \int\int \epsilon_A \epsilon_B \tilde{G}(A, B) da db , \tag{2.2}$$

where $\tilde{G}$ is the symmetric Green's functions of the wave operator $\Box + \frac{1}{6}R$, and R is the scalar curvature of the Riemannian spacetime manifold. The quantities $\epsilon_A, \epsilon_B, \ldots$ are coupling constants. This is an action-at-a-distance theory, but it can be rewritten in a field form under the prescription given by Narlikar (1968). Define a Machian mass function

$$m(X) = \sum_b \int \tilde{G}(X, B)\epsilon_B db = \int \tilde{G}(X, B)\epsilon_B n(B) d^4 b , \tag{2.3}$$

where $n(B)$ is the particle density at the point B.

The relation (2.2) shows how the mass function arises from the rest of the particles in the universe. The mass of a typical particle at A is given by $\epsilon_A m(A)$. The coupling constants $\epsilon_A, \epsilon_B, \ldots$ can be positive or negative according to the rule described by Hoyle and Narlikar (1972*a*). For the present discussion, it is sufficient to assume that, for a particle at X, the net contribution to $m(X)$ is restricted to particles lying in the particle horizon of X.

It is convenient to define

$$F = \tfrac{1}{2} m^2(X) , \qquad \Phi_{ik} = \tfrac{1}{2} g_{ik} m^l m_l - m_{;i} m_{;k} . \tag{2.4}$$

The gravitational equations are given by $\delta S / \delta g^{ik} = 0$, and take the form

$$R_{ik} - \tfrac{1}{2} g_{ik} R = -\frac{3}{F}[T_{ik}{}^m + \Phi_{ik} + \tfrac{1}{3}\{g_{ik}\Box F - F_{;ik}\}] , \tag{2.5}$$

where (we suppress the index m for the moment)

$$T^{ik}(X) = \sum \int \delta_4(X, A)[-g(A)]^{-1/2} \epsilon_A m(A) \frac{da^i}{da}\frac{da^k}{da} da . \tag{2.6}$$

The theory is conformally invariant. Thus, if g_{ik} and $m(X)$ describe the metric tensor and mass function in a given cosmological solution of the equations (2.5), then $\Omega^2 g_{ik}$, $\Omega^{-1} m(X)$ are solutions of the same equations, for a well behaved Ω. (One requires Ω to be $C^{(2)}$ and to satisfy $0 < \Omega < \infty$.) In the next section we discuss the specific conformal transformations of relevance to this work.

b) *Different "Gauges" of the HN Cosmology*

The primary motive of the Hoyle-Narlikar cosmology was to give a quantitative expression to Dirac's large number hypothesis. To see how the theory deals with the problem, it is instructive to consider the solution of the basic equations in the simplest form.

i) *Minkowski Gauge*

In this case we obtain the solution of the equations in a flat background space:

$$ds^2 = d\tau^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) . \tag{2.7}$$

The cosmological fluid is homogeneous and isotropic and the solution is given by

$$\epsilon_A n(A) = \epsilon_B n(B) = \lambda n = \text{constant} , \tag{2.8}$$

where λ and n are functions of cosmic epoch τ; $\lambda(\tau)$ denotes the coupling constant and $n(\tau)$ the particle number density. We then have

$$m(\tau) = \tfrac{1}{2}\lambda\tau^2 n = m_0(\tau^2/\tau_0^2)\,, \tag{2.9}$$

$$\lambda(\tau) = \lambda_0\tau_0/\tau\,, \tag{2.10}$$

$$n(\tau) = n_0\tau/\tau_0\,, \tag{2.11}$$

$$G(\tau) = G_0\tau_0^4/\tau^4\,. \tag{2.12}$$

The suffix zero refers to the present epoch of observation. The mass of a typical particle, say a proton, is given by

$$m_p = \lambda m = m_p{}^0 H_0\tau\,. \tag{2.13}$$

Here H_0 is the present value of Hubble's constant. Notice that the "large number" is explained by the fact that the dimensionless number

$$\lambda^2(\tau^3 n)^{1/2} \equiv \text{constant} = \lambda_0{}^2(\tau_0{}^3 n_0)^{1/2} \tag{2.14}$$

is of the order unity. To see this in our usual units, we have to restore $\hbar$ and c into the relation (2.14).

ii) *Einstein Gauge*

The above mode has $n(\tau) \neq$ constant, and thus permits creation of matter. It is, however, possible to look at it in a conformal frame in which it resembles the simplest solution of Einstein's cosmological equations, viz., the Einstein–de Sitter cosmology. With a conformal function of the type

$$\Omega_E \propto \tau^2 \tag{2.15}$$

we can transform (2.7) to the standard Einstein–de Sitter line element, i.e.,

$$ds_E{}^2 \equiv \Omega_E{}^2 ds^2 = dt^2 - \alpha t^{4/3}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2)\,, \tag{2.16}$$

where α is a constant and $t \propto \tau^3$. Notice that in this frame the gravitational constant

$$G_E = \text{constant}\,. \tag{2.17}$$

However, the proton mass m_p is *not* a constant. The nonconstancy of n and m_p conspire to produce the same dependence of nm_p on epoch as in standard cosmology. Thus, while the *macroscopic gravitational results* can be taken over from the usual theory, the interpretation of *microscopic atomic physics* needs to be suitably changed.

iii) *The Atomic Gauge*

This gauge is particularly suitable for the discussion of atomic physics, astrophysics etc. For this we choose as a conformal factor

$$\Omega_A = H_0\tau\,, \tag{2.18}$$

so that

$$m_p = \text{constant} = m_p{}^0\,. \tag{2.19}$$

However, the gravitational constant G_A is epoch-dependent, namely,

$$G_A = G_0(H_0\tau)^{-2}\,, \tag{2.20}$$

where G_0 is the present value of the gravitational constant. The transformation

$$2H_0 t = (H_0\tau)^2 \tag{2.21}$$

changes the original line element to the form

$$ds_A{}^2 = dt^2 - 2H_0 t[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)]\,. \tag{2.22}$$

In our discussion of cosmological tests we shall use this line element since we have to interpret observations conducted in the atomic gauge. Thus in this gauge we have

$$G_A = G_0(H_0 t)^{-1}\,. \tag{2.23}$$

Henceforth we will drop the suffix A and restrict our attention to the line element (2.22).

c) HN Cosmology with Electromagnetic Radiation

The action (2.2) describes the inertial and gravitational interaction of uncharged massive particles. It has to be modified to include the effects of electromagnetic interaction. This is done as follows. Let a typical particle a have charge e_a ($e_a = 0$ for a neutral particle). Then the modified action is given by

$$S = -\sum_{a \neq b}\sum \int\int \epsilon_A \epsilon_B \tilde{G}(A, B) da db - \sum_{a \neq b}\sum \int\int e_a e_b \bar{G}_{i_A i_B} da^{i_A} db^{i_B} , \tag{2.24}$$

where $\bar{G}_{i_A i_B}$ is the symmetric bivector Green's function of the vector wave operator $\Box \delta_k^i + R_k^i$, R_k^i being the Ricci tensor of the spacetime manifold (HN 1961). The variation of the metric gives the field equations in the form

$$R_{ik} - \tfrac{1}{2} g_{ik} R = -\frac{3}{F}[\{T_{ik}{}^m + J_{ik}\} + T_{ik}{}^{em}] . \tag{2.25}$$

In (2.25), $T_{ik}{}^m$ represents the energy momentum tensor of matter as given by (2.6) and J_{ik} stands for the additional terms in parentheses on the right-hand side of (2.5). Both $T_{ik}{}^m$ and J_{ik} arise from the first term of the modified action (2.24) whereas $T_{ik}{}^{em}$, the electromagnetic energy momentum tensor, arises from the second term of (2.24). It should be noted that $T_{ik}{}^m$ and J_{ik} go together and are essentially decoupled from $T_{ik}{}^{em}$. Thus if we take the divergence of (2.25), we should get, separately,

$$[F^{-1}\{T_m{}^{ik} + J^{ik}\}]_{;k} = 0 , \tag{2.26}$$

$$[F^{-1} T_{em}{}^{ik}]_{;k} = 0 . \tag{2.27}$$

It can be verified by direct calculation that these conservation laws hold provided that the matter and the electromagnetic tensors are decoupled and provided that the electromagnetic energy tensor does not make a significant contribution to the right-hand side of (2.25). These conditions apply to the present epoch of the universe as well as to the past epochs over which the various cosmological tests of discrete objects apply. These conditions are violated in the very early stages of the big bang universe when the dynamics of the universe was determined largely by radiation and when there was a significant exchange of energy and momentum between matter and the electromagnetic radiation. In that case, instead of (2.25) and (2.27) holding separately, only their sum, viz.,

$$[F^{-1}\{T_m{}^{ik} + J^{ik} + T_{em}{}^{ik}\}]_{;k} = 0 , \tag{2.28}$$

will hold as the joint conservation law for matter and radiation.

It is necessary now to clarify the status of the photon in this theory. It was shown by Hoyle and Narlikar (1974) that *all* the observable phenomena of quantum electrodynamics can be described by the above action-at-a-distance picture. Although electromagnetic fields do not have an independent existence in this theory and consequently photons do not arise as the result of field quantization, all the effects associated with photons can be reproduced in this theory. Thus spontaneous transition, scattering phenomena, radiation pressure, spectral lines, etc., can be described in this framework without having to postulate the existence of photon as a massless particle of specified energy and momentum. Nevertheless, the exchanges of energy and momentum which take place between interacting charged particles can be described in this theory in terms of discrete packets of energy and momentum, very similar to the photons in quantum *field* theory. In the rest of this work we will use the word "photon" for such packets, since for our purpose here they are indistinguishable from the photons of field theory. There is one important difference, however.

The photons in the HN theory do not take part in the inertial interaction given by (2.2). They neither acquire inertia from other particles nor contribute to it. Thus the electromagnetic radiation in this theory can be looked upon as made of photons which do not interact with particles via the inertial interaction (2.2). Their only interaction with spacetime geometry is via the gravitational equations (2.25).

d) The Variation of Observable Parameters

A look at equations (2.5) or (2.25) shows that these are similar to the Einstein field equations if we identify

$$8\pi G = 3/F . \tag{2.29}$$

However, there are two differences from Einstein's framework: (i) the gravitational constant G, defined by (2.29) varies with space and time, and (ii) there are inertial terms J_{ik} in the energy momentum tensor. We now consider the consequences of a variable G on the parameters which are involved in the various cosmological tests.

We first note that for this cosmology the curvature parameter $k = 0$. However, unlike the Friedmann model with $k = 0$, the deceleration parameter is given by

$$q_0 = 1 \tag{2.30}$$

at all epochs. This difference is because of the above mentioned two points.

Next we write the field equations in the form

$$R_{ik} - \tfrac{1}{2}g_{ik}R = -8\pi G[(T_{ik}{}^{m} + J_{ik}) + T_{ik}{}^{em}] , \qquad (2.31)$$

where $T_{ik}{}^{em}$ denotes the energy momentum tensor for electromagnetic radiation. Since we are discussing a homogeneous and isotropic universe, G will depend on time only.

From (2.26) we now have

$$[G(T_m{}^{ik} + J^{ik})]_{;k} \equiv 0 \qquad (2.32)$$

as the modified conservation law for matter. In our present cosmological framework, this law, which can alternatively be written in the form (suppressing the index m for clarity)

$$m^i[\Box m + \tfrac{1}{6}Rm] = T^{ik}{}_{;k} , \qquad (2.33)$$

gives for the matter density ρ_m ($p_m = c_s{}^2\rho_m$, $c_s{}^2 = 0$)

$$\rho_m \propto \frac{1}{G^{1/2}R^3} \propto t^{-1} \qquad (2.34)$$

We would have obtained an incorrect result if we had "decoupled" $T_m{}^{ik}$ from J^{ik} and written separately

$$(GT_m{}^{ik})_{;k} = 0 . \qquad (2.35)$$

In fact, in the atomic gauge (2.35) yields (R is here the scale factor of the Robertson-Walker metric)

$$\rho_{m\,(\mathrm{decoupled})} \propto \frac{1}{GR^3} \propto t^{-1/2} , \qquad (2.36)$$

However, as pointed out in § II*c*, equation (2.27), the photons *are* indeed decoupled from J_{ik}. For them, we have from (2.27):

$$[GT_{em}{}^{ik}]_{;k} = 0 \qquad (2.37)$$

if this radiation is essentially decoupled from matter. Writing

$$T_1{}^1 = T_2{}^2 = T_3{}^3 = -\tfrac{1}{3}\rho_\gamma \quad \text{and} \quad T_4{}^4 = \rho_\gamma ,$$

we easily check that

$$\rho_\gamma \propto \frac{1}{GR^4} \cdot \qquad (2.38)$$

For a constant G, we get the usual dependence of the radiation energy density on epoch. In the present case, the extra factor G must be present (Canuto *et al.* 1977; Canuto and Hsieh 1979; Canuto, Hsieh, and Owen 1979; Canuto and Owen 1979).

If we consider ρ_γ as made of n_γ photons of energy ϵ_γ,

$$\rho_\gamma = n_\gamma\epsilon_\gamma \equiv \frac{N_\gamma}{V}\epsilon_\gamma , \qquad (2.39)$$

we can use (2.38) to recover the usual cosmological redshift formula. In fact, if we demand that n_γ should depend on V as n(matter) does, i.e., equation (2.36),

$$n_\gamma \propto \frac{1}{GR^3}; \qquad N_\gamma \propto \frac{1}{G} , \qquad (2.40)$$

then we deduce

$$\epsilon_\gamma \propto 1/R . \qquad (2.41)$$

Since ϵ_γ is proportional to the frequency ν of the photon, the frequency is reduced in the ratio R^{-1}. Thus a photon emitted at epoch t and received at epoch t_0 suffers a redshift z given by

$$1 + z = \frac{R(t_0)}{R(t)} \equiv \frac{R_0}{R} \cdot \qquad (2.42)$$

Finally, in the present cosmological model, G varies in the atomic gauge as (see (2.23) and (2.22))

$$G = G_0(1 + z)^2 . \qquad (2.43)$$

III. THE COSMOLOGICAL TESTS

a) The m versus z Relation

Because of the historical importance of the m versus z relation as a discriminating test for or against G-varying cosmology, we shall first present a detailed derivation based on the exact relation (2.38) just derived.

The apparent bolometric luminosity l is clearly defined as

$$l = \frac{h\nu_0 N_{\gamma 0}}{\Delta t_0} \frac{1}{4\pi r^2 R_0^2}, \tag{3.1}$$

where $4\pi r^2 R_0^2$ is the area of the pseudo-sphere surrounding the observer at a distance rR_0 from the emitter. The index zero on ν, N_γ, and Δt refers to the present epoch. Introducing the absolute luminosity at the time of emission t,

$$L(t) = h\nu N_\gamma / \Delta t\,, \qquad N_\gamma \equiv N_\gamma(t)\,, \tag{3.2}$$

we have

$$l = \frac{L(t)}{4\pi r^2 R_0^2} \left(\frac{\Delta t}{\Delta t_0}\right) \left(\frac{\nu_0}{\nu}\right) \left(\frac{N_{\gamma 0}}{N_\gamma}\right). \tag{3.3}$$

The ratio $\Delta t / \Delta t_0$ is easily seen to be equal to $R(t)/R_0$ from the general definition of a geodesic motion for photons $ds^2 = 0$. Because of (2.40) and (2.42), (3.3) reduces to

$$l = \frac{L(t)}{4\pi r^2 R_0^2} \frac{1}{(1+z)^2} \frac{G(t)}{G_0}. \tag{3.4}$$

Since the space curvature is zero, and

$$R(t) = R_0(t/t_0)^{1/2}\,, \qquad H_0 t_0 = \tfrac{1}{2}\,, \tag{3.5}$$

the coordinate rR_0 is simply given by

$$rR_0 = \frac{c}{H_0}\left[1 - \left(\frac{t}{t_0}\right)^{1/2}\right] = \frac{c}{H_0}\frac{z}{1+z}, \tag{3.6}$$

yielding

$$l = \frac{1}{4\pi}\left(\frac{H_0}{c}\right)^2 \frac{L(t)}{z^2} \frac{G(t)}{G_0}. \tag{3.7}$$

Even though the last term in (3.7) can be written using (2.43), we shall use a more general approach that will allow us to follow the effect of $G(t)$ at any point of the derivation. We shall therefore write

$$G(t)/G_0 = (t/t_0)^{-g} = (1+z)^{2g}\,, \tag{3.8}$$

and analogously

$$L(t)/L_0 = (t/t_0)^{-e} = (1+z)^{2e}\,; \tag{3.9}$$

so that finally

$$l = \frac{1}{4\pi}\left(\frac{H_0}{c}\right)^2 L_0 \frac{(1+z)^{2(e+g)}}{z^2}, \tag{3.10}$$

or

$$m = m_0 + 5\log cz - 5(e+g)\log(1+z)\,. \tag{3.11}$$

The factor g is missing in the Barnothy and Tinsley (1973) analysis. The observed plot and the theoretical m versus z curve are shown in Figures 1 and 2, but we defer a discussion of these till § IV.

b) The Metric Angular Diameters versus z

Consider two events occurring at the time t at the points $A(r, \theta, \phi)$ and $B(r + dr, \theta + d\theta, \phi)$. The observer is located at $r = 0$ at the time t_0. The two emission events are separated by a local distance y. The metric angular diameter θ_m is then given by

$$\theta_m = \frac{y}{rR(t)}; \tag{3.12}$$

or, using (3.6),

$$\theta_m = \theta_0(1 + z)^2/z \,, \tag{3.13}$$

with

$$\theta_0 = \frac{H_0}{c}\, y = 8.6\left(\frac{H_0}{50}\right)\left(\frac{y}{250\ \mathrm{kpc}}\right) \text{arcsec.} \tag{3.14}$$

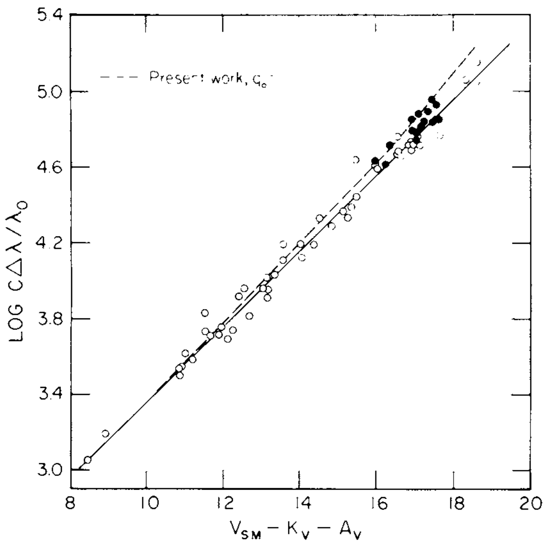

FIG. 1.—The magnitude versus redshift diagram predicted by the present theory (*dashed curve*). The normalization m_0 is -6.78. The full curve has the equation $V_c = 5 \log cz - 6.803$ and corresponds to the standard cosmology with $q_0 = 1$ and no evolutionary effects. The data are from the work of Sandage *et al.* (1976).

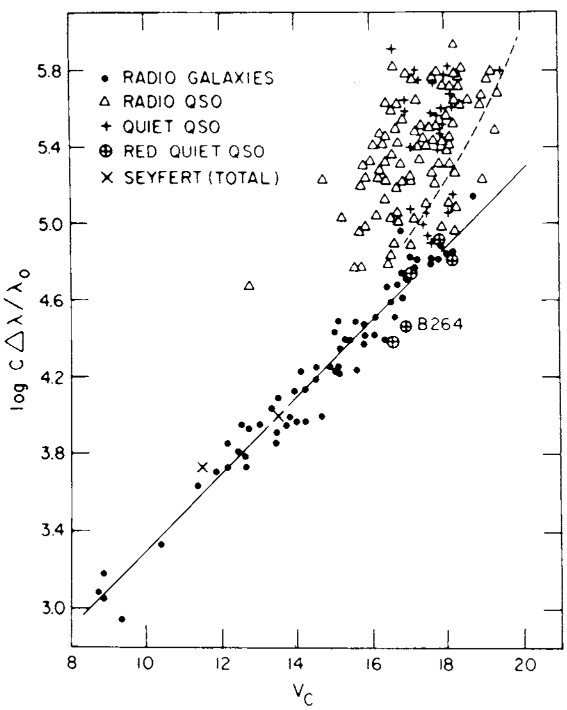

FIG. 2.—The magnitude versus redshift diagram for QSOs predicted by the present theory (*dashed curve*). The observational points are taken from Sandage (1972*a*).

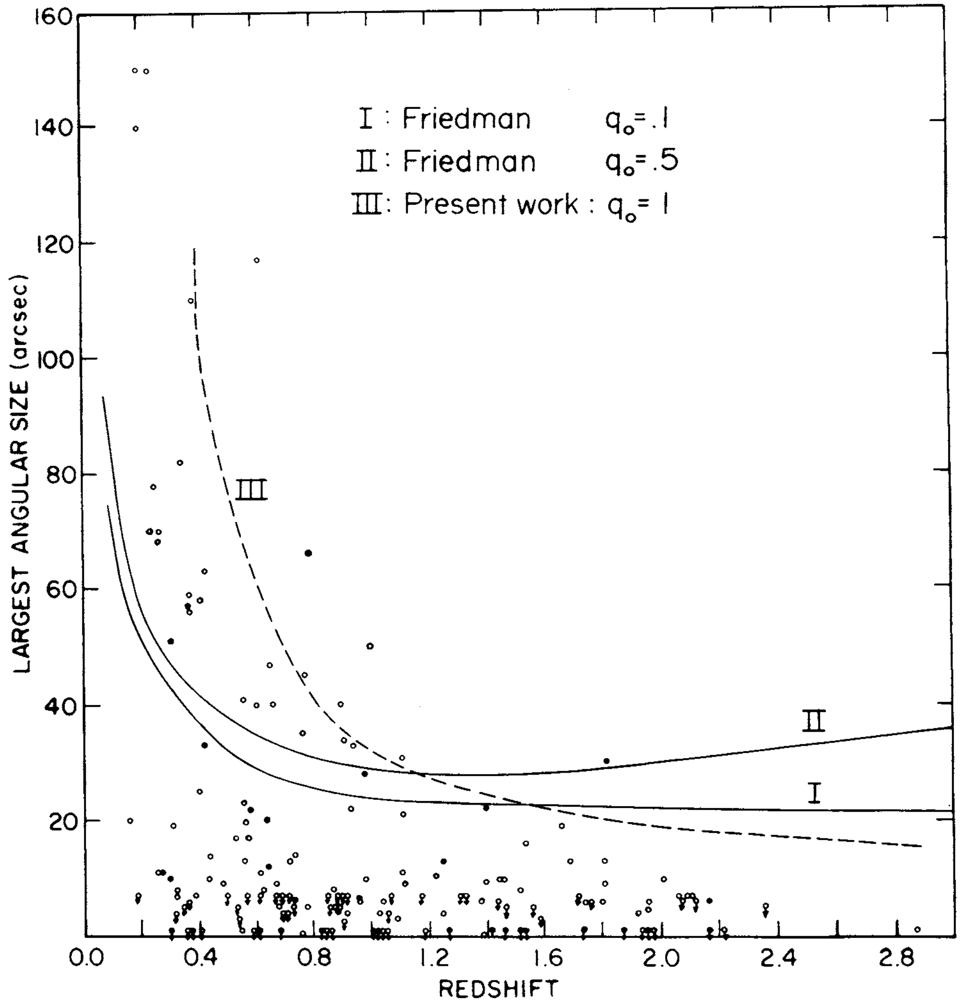

FIG. 3.—Metric angular diameters versus redshift from the present theory (*dashed curve*). The data are from Wardle and Miley (1974).

The θ_m versus z has a minimum at $z = 1$. Before plotting θ_m versus z and comparing the results with observations, we must warn the reader against taking equation (3.13) too literally. In fact, the quantity y can depend on z due to the expansion of the radio source. Most likely the variation of G is too slow to have a sizable effect on time scales of the order of 10^8 yr corresponding to the generally estimated lifetime of expanding radio sources. One can therefore take for $y = y(z)$ the standard G-constant expression. However, since no generally accepted model exists as yet, we decided not to commit ourselves to any particular model. For this reason we have plotted (3.13) with y constant. Once a model for $y = y(z)$ is chosen, Figure 3 can easily be rescaled.

c) *The Isophotal Angles θ_i versus z*

Another test often used in cosmology is the isophotal angles versus redshift. We shall now derive the θ_i versus z relation within the present cosmology. To that end, we shall first define the surface brightness B as

$$B = l/\theta_m^2 \, , \tag{3.15}$$

where the symbols have already been defined. Using (3.4) and (3.12), we obtain

$$B \propto \frac{L(t)}{y^2(1+z)^4}\left[\frac{G(t)}{G_0}\right] \tag{3.16}$$

independent of r. The determination of θ_i is usually made using an empirical formula due to Hubble, namely,

$$\theta_i/\theta_m \propto B^{1/p} \quad (p \sim 2) \, . \tag{3.17}$$

We derive

$$\frac{\theta_i}{\theta_m} \propto (1 + z)^{-4/p}\left[\frac{L(t)}{y^2}\frac{G(t)}{G_0}\right]^{1/p} ; \tag{3.18}$$

or, using the expression (3.12) for θ_m,

$$\theta_i \propto \left[\frac{L(t)}{r^2(1 + z)^2}\frac{G(t)}{G_0}\frac{y^{p-2}}{r^{p-2}}(1 + z)^{p-2}\right]^{1/p} ; \tag{3.19}$$

or, using (3.4),

$$\theta_i \propto \left[l\frac{y^{p-2}}{r^{p-2}}(1 + z)^{p-2}\right]^{1/p} ,$$

so that

$$\log \theta_i = -\frac{2}{5p}m + \frac{p-2}{p}\log\left(\frac{1+z}{r}\right) + \frac{p-2}{p}\log y + \text{const} . \tag{3.20}$$

For the value $p = 2$ suggested by Hubble,

$$\log \theta_i = -\tfrac{1}{5}m + \text{const} . \tag{3.21}$$

The results are presented in Figure 4.

d) The N(m) versus m Relation

A cosmological test originally proposed by Hubble that has become increasingly important since the discovery of QSOs is the count of sources with a magnitude m. The relation is usually written as

$$\log N = a + bm , \tag{3.22}$$

and the discussion concentrates on the value of the parameter b. Sandage and Luyten (1969) found $b = 0.75$, Braccesi and Formiggini (1969) reported the value $b = 0.72$, and more recently Green and Schmidt (1978) have published the value $b = 0.93$, the largest value so far published. Without evolutionary corrections, the maximum value of b allowed by standard Friedmann cosmology is 0.6, corresponding to a Euclidean universe.

In the present $k = 0$ cosmology, the total number of sources per square degree is given by (Q is the total number of square degrees in the sky)

$$N = \frac{4\pi}{3}\frac{n_0}{Q}R_0^3 r^3 ,$$

or using (3.6)

$$N = N_0\left(\frac{z}{1+z}\right)^3 , \tag{3.23}$$

where

$$N_0 \equiv \frac{4\pi n_0^3}{3Q}\left(\frac{c}{H_0}\right)^3 . \tag{3.24}$$

Defining $m_0^* \equiv m_0 + 5 \log c$, and rewriting (3.11) in the form

$$\text{dex}\,[0.2(m - m_0^*)] = \frac{z}{(1 + z)^{e+g}} ,$$

we obtain

$$\log N(m) = 0.6m + 3(e + g - 1)\log(1 + z) + \text{const.} , \tag{3.25}$$

whose slope can easily be evaluated as

$$\text{slope} = \frac{\partial \log N}{\partial m} = \frac{0.6}{1 - (e + g - 1)z} . \tag{3.26}$$

Calling α the slope and eliminating the quantity $(e + g)z$ between (3.26) and (3.11), we obtain the m versus z relation

$$m = 5 \log cz - 5\left[1 + \frac{\alpha - 0.6}{\alpha z}\right]\log(1 + z) + m_0 , \tag{3.27}$$

a relation which is now written in terms of observable quantities only.

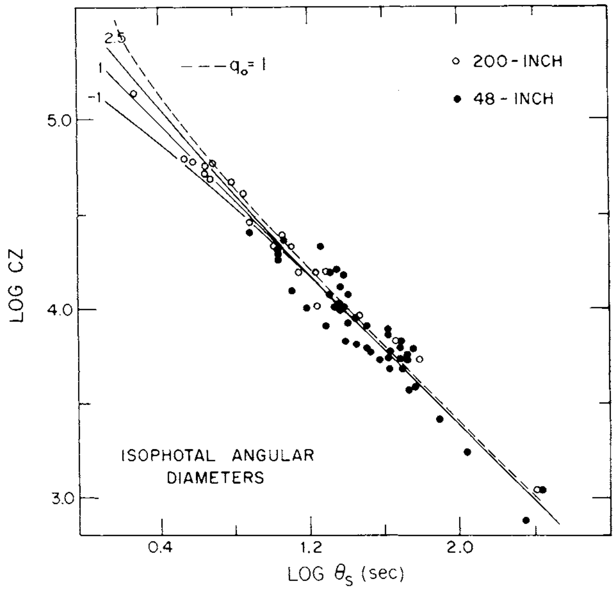

FIG. 4.—The isophotal angles versus redshift from the present theory (*dashed curve*). The full curves (characterized by the values of q_0) are from standard cosmology. The observational data are from Sandage (1972*b*).

e) The log *N versus* log *S Relation*

One of the tests that have played a fundamental role in cosmology is the log N–log S test for radio sources. Unfortunately, most workers in this field have felt it necessary to invoke evolutionary effects in order to get an acceptable fit to the data, with the result that the log N–log S test seems to be more useful as an indicator of evolutionary effects than as a geometrical tool. The evolution to be posited ad hoc is usually a strong function of z, of the form $(1 + z)^p$ where $p \approx 4$–5. Several possible evolutionary effects have been analyzed—e.g., luminosity, density, etc. We suggest a paper by von Hoerner (1973) for more details on this subject. The aim of our computation here is twofold: We want to show (*a*) that a G-varying cosmology yields a perfectly acceptable log N–log S relation, and (*b*) that a good fraction of the ad hoc evolutionary correction $(1 + z)^p$ is actually accounted for by the variation of G itself and need not be posited as a free parameter.

In order to proceed to the derivation, let us first of all change (3.10) to make it suitable for a source with a synchrotron-type spectrum of the form

$$P(\nu)d\nu \propto \nu^{-\alpha}d\nu\,, \tag{3.28}$$

so that, with S as the flux, (3.10) becomes

$$L_0/S = 4\pi(c/H_0)^2 z^2(1 + z)^{-2r}\,, \tag{3.29a}$$

$$L_0/S \equiv 4\pi(c/H_0)^2 J_L{}^2(z) \tag{3.29b}$$

with

$$2r \equiv 1 - \alpha + 2(e + g) \equiv 2\pi + 2g\,. \tag{3.30}$$

Equation (3.29b) is the first relation needed. The second relation gives the number of sources (per steradian) with absolute luminosity between L and $L + dL (k = 0)$

$$n(r, L)drdL = R^3(t)r^2dr\Phi(t, L)dL = R_0{}^3(t)r^2dr\Phi(L)dL\,, \tag{3.31}$$

where

$$\Phi(t_0, L) \equiv \Phi(L) \tag{3.32}$$

is the radio luminosity function. Changing variables from dr to dz, and from dL to dS using (3.6) and (3.29b), we now have

$$n(r, L)drdL = 4\pi\left(\frac{c}{H_0}\right)^5 \frac{z^2}{(1 + z)^4} J_L{}^2(z)\Phi(L)dzdS \tag{3.33a}$$

$$= 4\pi\left(\frac{c}{H_0}\right)^5 J_n{}^4\Phi(L)dzdS \tag{3.33b}$$

$$= n(S, z)dzdS\,, \tag{3.33c}$$

where we have introduced the symbols J_L^2 and J_n^4:

$$J_L^2(z) \equiv \frac{z^2}{(1+z)^{2r}} = \frac{z^2}{(1+z)^{1-\alpha+2e}} \frac{1}{(1+z)^{2g}}, \tag{3.34}$$

$$J_n^4(z) \equiv \frac{z^2}{(1+z)^4} J_L^2 = \frac{z^4}{(1+z)^{5-\alpha+2e}} \frac{1}{(1+z)^{2g}}, \tag{3.35}$$

so as to make the comparison with the standard cosmology easier. (Compare Table 1 of von Hoerner 1973.)

As for the luminosity function $\Phi(L)$, we shall adopt as in the standard case a function of the type (von Hoerner 1973)

$$\Phi(L) = \Phi_0(L/L_*)^\gamma \quad (L_* = 10^{26} \text{ watts Hz}^{-1}) \tag{3.36}$$

for the interval

$$L_{\min} \equiv 10^{-3}L_* \leq L \leq 10^3 L_* \equiv L_{\max}. \tag{3.37}$$

Integrating $n(S, z)$ over z and dividing the result by $n_E(S) \propto S^{-5/2}$, the result corresponding to the Euclidean case, we obtain the so-called normalized number count, n_n (A is a constant),

$$n_n = \frac{n(S)}{n_E(S)} = AS^{5/2+\gamma} \int_{z_m}^{z_M} \frac{z^2}{(1+z)^4} J_L^{2+2\gamma}(z) dz. \tag{3.38}$$

Here z_M and z_m are the maximum and minimum redshifts of the sources counted. To exhibit more clearly the G-dependence, we shall use (3.30) and the definition of J_L^2 as from equation (3.29) in order to write (3.38) as

$$n_n = AS^{5/2+\gamma} \int_{z_m}^{z_M} \frac{z^{4+2\gamma}}{(1+z)^{4+2\pi(1+\gamma)}} (1+z)^{2g(|\gamma|-1)} dz. \tag{3.39}$$

In fact, the exponent γ is always negative, $\gamma = -|\gamma|$: -2 (flat), -2.5 (critical), -3 (steep).

If we write the last term in the integrand as

$$(1+z)^{2g(|\gamma|-1)} \equiv (1+z)^p, \tag{3.40}$$

the index p turns out to be ($g = 1$, eqs. [2.43] and [3.8])

$$\text{steep}: p = 4; \quad \text{critical}: p = 3; \quad \text{flat}: p = 2, \tag{3.41}$$

which simulates the ad hoc density evolution in standard cosmology (see von Hoerner 1973, eq. [37a]).

To solve (3.38), we need z_m and z_M as a function of S. For that, let us call

$$\Lambda \equiv \frac{L}{4\pi S}\left(\frac{H_0}{c}\right)^2 = J_L^2(z) = \frac{z^2}{(1+z)^{2r}}. \tag{3.42}$$

The quantities z_m and z_M can be obtained as a function of S by inserting $L = L_{\min}$ and $L = L_{\max}$.

In the case $r = 1$, we can show that, for any value of γ, n_n is constant, i.e., the differential count is Euclidean. In this case we have

$$r = 1: \quad J_L = \frac{z}{1+z}; \quad dJ_L = \frac{dz}{(1+z)^2}.$$

Equation (3.38) now becomes

$$\begin{aligned} n_n &= AS^{5/2+\gamma} \int_{z_m}^{z_M} \left[\frac{z}{(1+z)}\right]^{4+2\gamma} \frac{dz}{(1+z)^2} \\ &= AS^{5/2+\gamma} \int_{J_m}^{J_M} J_L^{4+2\gamma} dJ_L \\ &= \frac{A}{5+2\gamma}(J_M^{5+2\gamma} - J_m^{5+2\gamma}) S^{5/2+\gamma}. \end{aligned} \tag{3.43}$$

Since by definition, (3.42),

$$J_L \sim S^{-1/2},$$

it follows that

$$n_n \sim \text{const.} \quad (r = 1, \text{ any } \gamma). \tag{3.44}$$

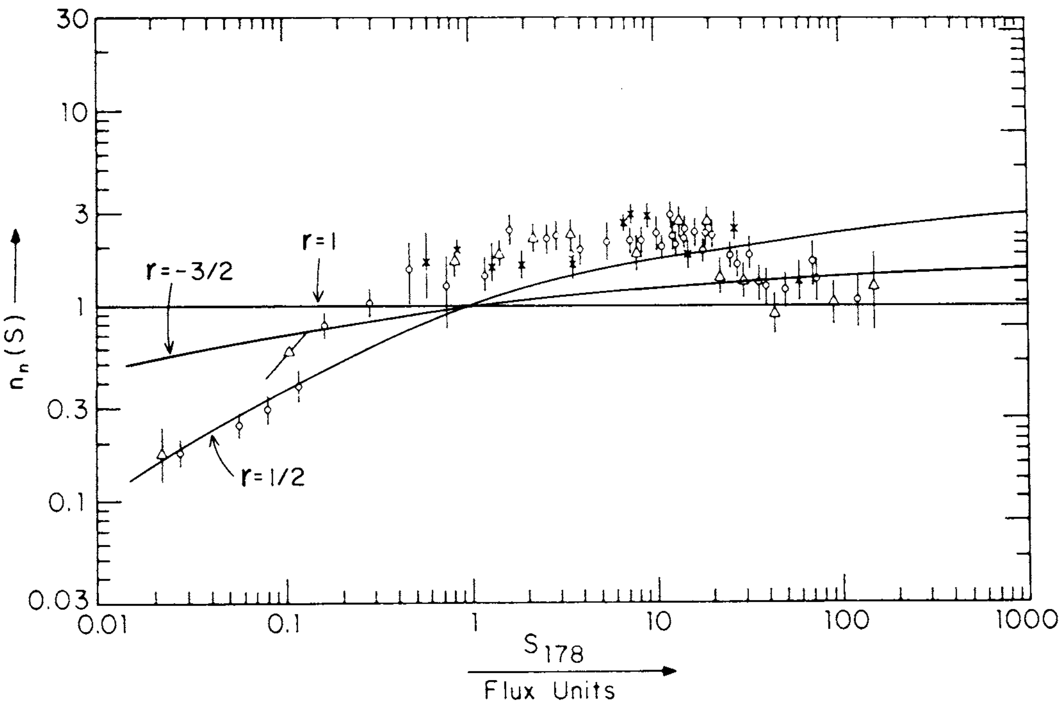

FIG. 5.—The normalized n_n versus S from the present theory (*full lines*) for $r = -3/2$, $1/2$, and 1 and for $\gamma = -2.5$, -2; the curve $r = 1$ is valid for any γ. To a given r there correspond more than one combination of α and e; see eq. (3.30). The data are from the work of von Hoerner (1973). In general, as the value of γ becomes more negative, the theoretical curves tend to flatten.

The cases $r = 1/2$ and $-3/2$ for $\gamma = -2$ and -2.5 were computed numerically, and the results are presented in Figure 5.

At this point it is necessary to make two comments. In Figure 5 we have shown the comparison of the theory with observations made at 178 MHz. It is, however, well known (Wall 1977) that at higher frequencies the n_n versus S curve flattens appreciably and becomes even more Euclidean than shown here. Thus the agreement of the present theory for $e = 0$ with observations might even be improved when these higher frequency surveys are included. As noted previously, in the parameter-fitting exercise undertaken to fit standard cosmology to the radio data, it is usual to assume a density evolution of the form $(1 + z)^p$, with p as large as 5 (Longair 1971; von Hoerner 1973), or other analytic forms which show increased density in the past. In the present theory the presence of a term dependent on a power of G in equations (3.39)–(3.40) serves the same purpose of density evolution by boosting the value of the integrand at high z. However, this term arises naturally in our present discussion rather than as postulated in the usual theory on an ad hoc basis.

IV. THE EVOLUTIONARY PARAMETER e

Before we can meaningfully compare any of the previous relations with observational data, we clearly must have an estimate of the parameter e introduced in (3.9). Since we shall be dealing with optical galaxies as well as with radio galaxies and QSOs, we shall present two different derivations of e, one valid for galaxies made of stars and one valid for QSOs and radio galaxies.

a) The Evolution of a Single Star

Suppose we consider a single star and ask ourselves how its luminosity depends on G and M as time progresses. Standard stellar evolution equations yield

$$L \propto G^7 M^5 \tag{4.1}$$

if Kramers's opacity and the p-p cycle are used. The strong dependence of L on G has been emphasized over the years by many authors—for instance, in connection with the thermal history of our Sun.

Equation (4.1) is usually employed under the approximation $M =$ const., so that

$$L \propto G^7 = L_0(t_0/t)^7 , \tag{4.1a}$$

a relation often applied to galaxies too.

Several comments are in order.

i) First of all, the use of (4.1) with the imposition of mass conservation while keeping G varying, *contradicts Newtonian physics.* In fact, within Newtonian mechanics it is easy to show that if for whatever reason G is to vary with time, then *energy cannot be conserved.* This point, discussed in detail by Bishop and Landsberg (1976), can easily be seen by considering that if $G \to G(t)$, then

$$F = -G(m_1 m_2/r^2) = -\nabla V(r) \to -\nabla V(r, t) ,$$

i.e., the energy becomes time dependent, and one cannot have consistency by requiring $G = G(t)$ and at the same time M or $E =$ const. as is done in arriving at (4.1a). The correlation between G and M can easily be seen from the Einstein equations which, when applied to an isolated spherical object (like a star), imply that

$$GM = \text{const} . \qquad (4.2)$$

This relation is clearly valid even at the Newtonian level at which we work. If one decides to use (4.1), then one would have *instead* of (4.1a), the expression

$$L \propto G^2 \propto (t/t_0)^{-2} , \qquad (4.3)$$

i.e., $e = 2$ and not $e = 7$. With such a value of e, equation (3.11) now becomes ($g = 1$)

$$m = 5 \log cz - 15 \log (1 + z) + m_0 . \qquad (4.4)$$

For example, at $\log cz = 5$, $z = \frac{1}{3}$, we obtain

$$m = 25 - 1.87 + m_0 , \qquad (4.5)$$

instead of the value of Barnothy and Tinsley (1973)

$$m = 25 - 4.3 + m_0 . \qquad (4.6)$$

ii) Can we really believe (4.3)? Legitimate doubts may arise when one considers how (4.1) has actually been derived. In fact, the ingredients that have gone into the standard derivation of (4.1) are: (*a*) the radiative transfer equation that uses $\rho_\gamma \propto T^4$ instead of

$$\rho_\gamma \propto \frac{1}{G} T^4 \qquad (4.7)$$

(see [2.38]) and (*b*) the Boltzmann distribution for particles (to obtain the opacity and the nuclear reaction energy output) that conserves the total number of particles. This again contradicts (2.34) which demands that the particle number should scale like

$$N \sim G^{-1/2} . \qquad (4.8)$$

In other words, *the very ingredients used to construct* (4.1) *conserve energy: if so, the constraint* (4.2) *implies that G is constant and* (4.1) *is therefore useless.*

iii) The correct way to derive $L = L(G, M)$ when G varies is as follows. The hydrostatic equilibrium equation can be shown (Canuto *et al.* 1977) to hold unaltered even if G varies; i.e., we still have

$$\frac{dp}{dr} = -G \frac{m(r)\rho}{r^2} . \qquad (4.9)$$

For the radiative transfer equation, we shall write

$$L(r) = -\frac{4\pi r^2 c}{3k\rho} \frac{d\rho_\gamma}{dr} , \qquad (4.10)$$

where k is the opacity. The other definition $L = \epsilon M$, where ϵ is the nuclear energy output, is clearly unchanged. Using the same dimensional arguments employed to derive (4.1), but using now (4.7), we obtain

$$L \propto \frac{R}{k\rho} \frac{T^4}{G} \propto \frac{R^4 T^4}{kGM} \propto \frac{R^4 T^4}{kg} \propto g^3 \frac{1}{k} , \qquad (4.11)$$

where we have defined

$$g \equiv GM .$$

Since $p = nkT = (\rho/m_{\rm H})kT$, it follows from (4.9) that

$$p \propto GM\rho/R \propto \rho g/R$$

and

$$TR \sim g\,. \tag{4.12}$$

Since now

$$L = \epsilon M = \epsilon GM/G = \epsilon g/G\,, \tag{4.13}$$

we finally obtain

$$k\epsilon \propto Gg^2\,. \tag{4.14}$$

Let us now use for ϵ and k the standard expressions but let us remember that in both cases we must integrate over the Boltzmann distribution function, which must now contain a factor $G^{-1/2}$ in the normalization; see (4.8). The Rosseland mean does not introduce an extra factor of G: being a mean value, the normalization of (4.7) like $1/G$ cancels out. We shall write ($f_1^2 = f_2^2 = G^{-1}$)

$$k \propto f_2 \rho^r T^n\,, \qquad \epsilon \propto f_1 \rho^s T^m \tag{4.15}$$

($r = s = 1, n = -3.5, m = 4.5$ for the Kramers's opacity and the p-p cycle). We now have from (4.14) ($f \equiv f_1 f_2$)

$$T \propto \left(\frac{G^{1+t}}{f}\, g^{2+2t}\right)^{1/q} \tag{4.16}$$

with

$$q \equiv n + m + 3t\,, \qquad t \equiv r + s\,;$$

and so

$$R \propto g\left(\frac{f}{G^{1+t} g^{2+2t}}\right)^{1/q}\,.$$

Finally, using (4.15), (4.16), and (4.17), we obtain from (4.13)

$$L \propto (GM)^a G^b (f_1^{3r+n}/f_2^{3s+m})^{1/q}\,, \tag{4.17}$$

where

$$a = \frac{9s + 3m + 2mr + 3r + n - 2sn}{n + m + 3s + 3r}\,, \qquad b = \frac{mr - n(1 + s) - 3r}{n + m + 3s + 3r}\,. \tag{4.18}$$

with $f_1^2 = f_2^2 = G^{-1}$, we get

$$L \propto (GM)^a G^{b+c}\,, \qquad 2c \equiv \frac{3s - 3r + m - n}{n + m + 3s + 3r}\,, \tag{4.19}$$

where $r = s = 1$. In fact, if r or s were different from unity, the density dependence of ϵ and k would not be linear and the correction factors f_1^2 and f_2^2 would not be simply G^{-1}.

a) When we are dealing with the evolution of a single star, the constraint (4.8), i.e., $g = GM \propto G^{1/2}$ applies, so that

$$L \propto G^{b+c+a/2}\,,$$

with

$$2(b + c) = \frac{m(2r + 1) - 3n - 9r + 3s - 2sn}{n + m + 3s + 3r}\,, \tag{4.20}$$

where again $r = s = 1$.

For Kramers's opacity, $n = -3.5$, $m = 4.5$, so

$$L \propto G^{4.5} \tag{4.21}$$

instead of

$$L \propto G^7\,. \tag{4.21a}$$

Equation (4.21) and not (4.21a) should be used to study the time evolution of the Sun's luminosity and its possible effects on the Earth's early temperature.

b) There might be cases, however, when the constraint (4.8) does not apply. In that case we must distinguish at least two cases depending on the opacity in use.

For stars with Kramers's opacity, we obtain for (4.19)

$$L \propto M^{\alpha} G^{\delta} \tag{4.22}$$

with

$$\alpha \equiv \frac{9s + 3m + 2mr + 3r + n - 2ns}{n + m + 3s + 3r}\,, \qquad 2\delta \equiv \frac{7m + 6mr + 21s - 3r - n - 6sn}{n + m + 3s + 3r}\,,$$

valid again for $r = s = 1$. With $n = -3.5$ and $m = 4.5$, we obtain

$$\alpha = 5.42\,, \qquad \delta = 7.21\,. \tag{4.22a}$$

For stars with constant opacity, we must go back to (4.17) and put $f_2 = 1$, $r = n = 0$, thereby getting

$$L \propto M^{\alpha} G^{\delta}\,, \qquad \alpha = \delta = 3\,, \tag{4.23}$$

independently of s and m. The interesting thing here is that α and δ are the same. In standard cosmology, $\alpha = 3$, $\delta = 4$. The lowering of δ is due to the extra power of G in (4.7). As (4.10) indicates, the luminosity L is in fact proportional to ρ_{γ}.

b) *Stellar Populations*

Let us now perform a full treatment of the contribution to the total luminosity of a galaxy as coming separately from dwarfs and giants. We shall generalize a treatment for constant G (Tinsley 1976; Maeder 1977; also Canuto and Hsieh 1979; Canuto, Hsieh, and Owen 1979; for a general treatment within scale covariant cosmology).

For L_d, the contribution by dwarfs, we shall write

$$L_d = \int_{M_L}^{M_f} \frac{dN}{dM} l_M dM\,, \tag{4.24}$$

where M_f is the turnoff mass at the time t. The luminosity

$$l_M \equiv l_M(t)$$

of a star of mass $M(t)$ will be written in general using (4.22). The Salpeter initial mass function (IMF)

$$\frac{dN}{dM} \propto M^{-(1+x)}\,, \tag{4.25}$$

must be generalized to include an $M(t)$. Since $GM \propto G^{1/2}$, we have

$$\frac{dN(t)}{dM(t)} \propto M^{-(1+x)}(t) G^{-x/2}\,. \tag{4.26}$$

The main-sequence lifetime τ scales like

$$\tau \propto \frac{M(t)}{l(t)} \propto M^{1-\alpha} G^{-\delta}\,, \tag{4.27}$$

where $\tau = t - t_1$ and t_1 is the time at which the galaxy was born. We have

$$M_f(t) \propto t^{1/(1-\alpha)}(1 - t_1/t)^{1/(1-\alpha)} G^{\delta/(1-\alpha)}\,. \tag{4.28}$$

Using (4.26) and (4.22) in (4.24), we obtain

$$L_d \propto G^{\delta - x/2}(t) M^{\alpha - x}(t)\big|_{M_L}^{M_f} \tag{4.29}$$

Since the major contribution comes from the upper limit, we obtain using (4.28)

$$L_d \propto G^{\delta - x/2 - \delta(\alpha - x)/(\alpha - 1)} t^{-(\alpha - x)/(\alpha - 1)} (1 - t_1/t)^{-(\alpha - x)/(\alpha - 1)}\,.$$

By putting $G = $ const., one recovers Tinsley's expression (1976). Neglecting t_1/t with respect to unity and using $G \sim t^{-1}$, we finally obtain

$$L_d \propto t^{-e_d}\,, \tag{4.30}$$

where

$$e_d \equiv \delta - \frac{x}{2} + (1 - \delta)\frac{\alpha - x}{\alpha - 1}\,. \tag{4.31}$$

The interesting thing about this expression is that for the value $x = 1$ that is usually used, e_d becomes independent of α and δ; in fact,

$$e_d = \tfrac{1}{2}\,. \tag{4.32}$$

For G = const. and $x = 1$ the corresponding e_d is unity.

Let us now study the contribution due to giants (post–main-sequence stars). This, following Tinsley (1976), will be written as

$$L_g \propto \tau_g l_g \left|\frac{dN(t)}{dM(t)}\right|\left|\frac{dM(t)}{dt}\right|\,. \tag{4.33}$$

Using (4.26) and (4.28), we now have

$$L_g \propto \tau_g l_g t^{-1-x/(1-\alpha)} G^{-x[1/2+\delta/(1-\alpha)]}\,, \tag{4.34}$$

where as usual we have assumed $G \sim t^{-1}$ and $t_1 \ll t$. For G = const., (4.34) reduces to equation (9) of Tinsley (1976), where the symbol γ stands for $\alpha/(\alpha - 1)$. Since

$$l_g \tau_g \propto M(t) \sim G^{-1/2}\,, \tag{4.35}$$

we finally have

$$L_g \propto t^{-e_g}\,, \tag{4.36}$$

$$e_g \equiv \frac{1 - x}{2} + \frac{x}{1 - \alpha}(1 - \delta)\,. \tag{4.37}$$

This expression is a particular case of a more general expression (A20) in Canuto *et al.* (1979).

The total luminosity will now be written as

$$L \propto L_d + aL_g \propto t^{-e_d}[1 + at^{-e_g + e_d}]\,. \tag{4.38}$$

It is interesting to note that $e_g - e_d$ is independent of x. In fact,

$$e_g - e_d = \frac{\delta - \alpha}{\alpha - 1} + \frac{1}{2}\,. \tag{4.39}$$

Defining

$$\frac{d \ln L}{d \ln t} \equiv -e\,, \tag{4.40}$$

we derive the general expression

$$e = e_d + \frac{R}{1 + R} e_1\,, \tag{4.41}$$

where

$$e_1 \equiv \frac{\delta - \alpha}{\alpha - 1} + \frac{1}{2}\,. \tag{4.42}$$

With the values $\alpha = \delta$, derived before, equation (4.23), e_1 is $\frac{1}{2}$. For α and δ given by (4.22a), $e_1 = 0.90$. Since for $x = 1$, $e_d = \frac{1}{2}$, the total e is therefore not larger than unity, quite different from $e = 7$ used previously (Barnothy and Tinsley 1973).

Equation (4.41) can now be used to compute the m versus z relation (3.11) with $g = 1$. The results shown in Figure 1 (*dashed curve*) correspond to $x = 1$.

For QSOs we cannot compute e from any reliable theoretical model. We have therefore followed another route. We eliminate $e + g$ between (3.26) and (3.11), thus getting (3.27) which now contains only observable quantities.

Alternatively we can determine the value of e from (3.26) by demanding that the slope be, say, 0.75, an average value between the ones quoted after equation (3.22).

In the case $g = 1$, we would then get from (3.26)

$$e_{\rm QSO} = \frac{1}{5z}\,, \tag{4.43}$$

which we shall then use for the QSO part of the m versus z diagram. Since, for QSOs, $z \geq \frac{1}{3}$ (log $cz \geq 5$), equation (4.43) implies that

$$e_{QSO} \leq \tfrac{3}{5} ,$$

a small value not very different from the one computed for optical galaxies. Putting (4.43) into (3.11) with $g = 1$ gives rise to the dashed curve of Figure 2 that begins at log $cz = 5$.

V. THE 3 K BLACKBODY RADIATION

The prediction, discovery, and confirmation of the existence of a universal background radiation with a temperature of about 3 K has been justly acclaimed as a major indication that a hot, dense phase existed in the early universe, today's radiation being the vestige of that period. To formulate a consistent treatment, we must make sure that the integrated distribution satisfies (2.38) at any time. The standard thermodynamic derivation must therefore be modified to take account of the variable gravitational constant.

We first write

$$\rho_\gamma(\nu) = \rho_\nu(V)\rho(G) , \tag{5.1}$$

where $\rho_\nu(V)$ depends on the volume V of the system while $\rho(G)$ depends only on G. From thermodynamic arguments and using the Doppler shift formula, we arrive at the well known relation

$$\frac{\partial}{\partial V}[\rho_\nu(V)V] = \frac{\nu}{3}\frac{\partial}{\partial \nu}[\rho_\nu(V)] , \tag{5.2}$$

which integrates to

$$\rho_\nu(V) \propto \nu^3\Phi(\nu V^{1/3}) . \tag{5.3}$$

Here Φ is an arbitrary function whose form is determined to be Planckian by the usual arguments of quantum statistics. Thus we have

$$\rho_\nu(V) = 8\pi\nu^3 F(\nu/T) . \tag{5.4}$$

However, in order to satisfy (2.38), we have to choose the function $\rho(G)$ to have the form $\rho(G) \propto G^{-1}$. Thus finally we get

$$\rho_\gamma(\nu) = \frac{G(t_*)}{G(t)} 8\pi\nu^3 F(\nu/T) . \tag{5.5}$$

In (5.5), $G(t)$ is the gravitational constant at the epoch t at which $\rho_\gamma(\nu)$ is measured, while $G(t_*)$ is the gravitational constant at an arbitrary but fixed epoch t_*. Equation (5.5) represents the equilibrium distribution function of radiation, and it is seen to preserve its form during the expansion of the universe even if the gravitational constant varies with time. (For a more general thermodynamic treatment leading to [5.5] as a particular case, see Canuto and Hsieh 1979.)

It is the equilibrium distribution, not the Planckian distribution, that is endowed with physical significance. In standard cosmology, the two *happen* to coincide and one therefore talks about the preservation of the Planckian form during expansion. In the present, more general, framework the two do not coincide and it is therefore unphysical to demand anything at the level of the Planckian form. What we ought to make sure of is that the equilibrium distribution (whatever its form might be) is unchanged after matter and radiation have decoupled: this does happen in the HN theory.

The confusion between the physically significant "equilibrium distribution" and the more accidental form represented by a "Planckian" led Steigman (1977) to conclude incorrectly that G cannot vary (see Canuto and Hsieh 1979).

It is evident from (5.5) that the equilibrium distribution at the present epoch agrees with the observed microwave background distribution if we set $T = T_0 \approx 3$ K and $G(t_*) \approx G(t_0)$. The most recent observations by Woody and Richards (1979) indicate deviations from the Planckian curve, and it may be possible to have a better fit of (5.5) with these observations. Such a fit will determine the parameters T_0 and $G(t_*)$ more accurately.

If it becomes possible in the future to measure the equilibrium distribution of the microwave background in remote parts of the universe (i.e., at earlier epochs), such measurements should reveal the time-varying factor $G(t)/G(t_*)$ in (5.5). This is therefore a new test of the HN cosmology.

VI. CONCLUDING REMARKS

The differences between "G-constant" and "G-varying" cosmologies are in general of two kinds: geometrical and physical. In principle, it should be possible to test these differences to see which (if either) cosmology is right. In practice, however, the situation is more complicated because these differences get mixed up. Our discussion of the various cosmological tests has emphasized this point.

Of all the tests, the oldest and the most discussed *m-z* test makes this amply clear. Although in principle one is hoping to measure the large-scale geometry of the universe, the actual distance measurement involves magnitudes which in turn involve the luminosities of the so-called standard candles. Even in standard cosmologies, the evolution of luminosity has made the *m-z* test and the measurement of the "true q_0" very uncertain. In the *G*-varying cosmologies the effect of *G*-variation on the luminosities can be very sensitive, and great care needs to be exercised in the comparison of theory with the existing data. So far as galaxies are concerned, we have shown here that the *m-z* relation of this HN cosmology is consistent with observations. For the QSOs, because of the highly uncertain nature of the luminosity evolution, we have used the $N(m)$ test to eliminate the evolutionary parameter. When this is done, the theory is in satisfactory agreement with the *m-z* plot of QSOs.

For the log *N*–log *S* test, it is interesting to note that a decreasing *G* can play a role analogous to the evolutionary parameters usually introduced ad hoc to make standard cosmologies agree with the source count data.

Finally, although the equilibrium-background radiation in this cosmology differs from a blackbody curve, the differences are not detectable by observations at a single epoch (i.e., the present epoch). It may well be possible to devise tests which could compare the cosmic microwave background at earlier epochs with the present background. Only in this way can a distinction be made between the standard cosmology and the *G*-varying Hoyle-Narlikar cosmology.

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V. M. Canuto: NASA Goddard Institute for Space Studies, 2880 Broadway, New York, NY 10025

J. V. Narlikar: Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay, India 400005