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Existence of the Gravitomagnetic Interaction

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The point of view expressed in the literature that gravitomagnetism has not yet been observed or measured is not entirely correct. Observations of gravitational phenomena are reviewed in which the gravitomagnetic interaction—a post-Newtonian gravitational force between moving matter—has participated and which has been measured to 1 part in 1000. Gravitomagnetism is shown to be ubiquitous in gravitational phenomena and is a necessary ingredient in the equations of motion, without which the most basic gravitational dynamical effects (including Newtonian gravity) could not be consistently calculated by different inertial observers.

1. INTRODUCTION

In the overview *Physics Through the 1960s*, the National Academy of Sciences (1986) review of opportunities for experimental tests of general relativity, they declare that "At present there is no experimental evidence arguing for or against the existence of the gravitomagnetic effects predicted by general relativity. This fundamental part of the theory remains untested." Similar points of view have been expressed elsewhere in promotion of various experiments designed to "see" gravitomagnetism.

In this paper I make two points on this issue, which together lead to a position contrary to the viewpoint summarized by the above statement.

1. The gravitomagnetic interaction is a consequence of the gravitational vector potential. This vector potential pays a crucial, unavoidable role in gravitation; without the gravitational vector potential the simplest gravitational phenomena—the Newtonian-order Keplerian orbit and the deflection of light by a central body—cannot be consistently calculated in two or more inertial frames of observation. Gravitation without the vector potential is an incomplete, ambiguous theory in the most fundamental sense.

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5. DRAGGING OF INERTIAL FRAMES AND MACH'S IDEAS

What seems to have especially caught the interest of physicists in searching for the spin–spin interaction in gravity is that this would seem to be a manifestation of ideas of Mach, who a century ago believed that inertia was caused, in some sense, by the universe's matter distribution. Lense and Thirring later showed that, indeed, in general relativity rotating matter would drag the inertial frame around at a slow rate which fell off with distance from the rotating matter,

$$\mathbf{\Omega} = \frac{G}{c^3}\left(\frac{\mathbf{J} - 3\mathbf{J}\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}}{r^3}\right) \tag{16}$$

$\mathbf{J}$ is the angular momentum of the spinning body and $\mathbf{r}$ is the distance to the point of space in question, $\mathbf{\Omega}(\mathbf{r})$ is the rotation rate and rotation axis for the inertial space at that point of space which is induced by the spinning source. Equation (16) follows from (12) with choice of PPN coefficients appropriate to general relativity, and the identification

$$\mathbf{\Omega} = -\frac{c}{2}\nabla\times\mathbf{h}$$

Looking at the general case, one can ask what is the complete effect of the gravitational vector potential in dragging inertial frames? This question can be addressed by calculating the contribution of $\mathbf{h}$ in establishing the geodesic coordinate frames (inertial frames). The general formula

$$[x^\gamma - x^\gamma_{(0)}]' = [x^\gamma - x^\gamma_{(0)}] + \tfrac{1}{2}\Gamma^\gamma_{\alpha\beta}[x^\alpha - x^\alpha_{(0)}][x^\beta - x^\beta_{(0)}] \tag{17}$$

in which $\Gamma^\gamma_{\alpha\beta}$ are the Christoffel symbols produced from first derivatives of the gravitational metric field, gives the transformation from original space-time coordinates x^γ to inertial (geodesic) coordinates $x^{\gamma'}$ in the vicinity of any chosen space-time point $x^\gamma(0)$. Examining solely the vector potential (g_{0i}) contribution to (17) yields

$$[\mathbf{r} - \mathbf{r}_{(0)}]' = [\mathbf{r} - \mathbf{r}_{(0)}] - c\left[\frac{1}{2}\frac{\partial\mathbf{h}}{\partial t}(t-t_0)^2 + \left(\frac{\nabla\times\mathbf{h}}{2}\right)\times(\mathbf{r}-\mathbf{r}_{(0)})(t-t_0)\right] \tag{18}$$

The gravitational vector potential produces in this general case a "dragging" of inertial space at each locality with both an acceleration of the inertial frame at rate

$$\mathbf{a}(\mathbf{r}, t) = -c\,\partial\mathbf{h}/\partial t \tag{19a}$$

and a rotation of the inertial frame at angular rate and axis

$$\mathbf{\Omega}(\mathbf{r}, t) = -\tfrac{1}{2}c\nabla\times\mathbf{h} \tag{19b}$$

If we return to the problem of light deflection by a body moving at speed w and employ the vector potential given by (7), we find that (19a) gives no contribution to the light ray deflection; however, (19b) produces a rotational dragging of inertial frames at a rate

$$\Omega(r, t) = (1+\gamma)\frac{GMDw}{c^2}\frac{1}{|\mathbf{r}-\mathbf{w}t|^3}$$

and in a counterclockwise sense. The time integral of this rotation rate over the entire trajectory of the light ray produces the total deflection or rotation angle

$$\delta\theta = -\frac{2w}{c}\theta_0$$

which is what is needed to obtain agreement with (5) as discussed in Section 2.

The periastron precession of the binary pulsar orbit discussed previously received contributions of inertial frame dragging from both (19a) and (19b). The situation can be viewed this way; part of the motion of the two bodies in the binary pulsar results from the "Coriolis" acceleration that each body experiences because the motion of the other body is producing rotational dragging of the inertial frame at the locality of each body in question.

Finally, the accelerated celestial body mentioned previously drags the inertial frames through (19a), with the resulting acceleration of inertial space being

$$\delta\mathbf{a}(\mathbf{r}, t) = -\left(2+2\gamma+\frac{\alpha_1}{2}\right)\frac{U(\mathbf{r}, t)}{c^2}\mathbf{a}$$

in which $U(\mathbf{r})$ is the Newtonian potential function of that body's mass distribution and $\mathbf{a}$ is the body's acceleration.

6. CONCLUSION

The gravitomagnetic interaction—the post-Newtonian gravitational interaction between moving masses—has been observed and measured in a number of different phenomena. The strength of this interaction is now known to an accuracy of 1 part in 1000. The gravitomagnetic interaction is also required in order to have a complete and consistent theory of gravity at all: even static source gravitational effects when viewed in another inertial frame require the gravitomagnetic interaction in order for basic consistency of a theory's equations of motion. Just as in electromagnetic theory, there is no absolute separation of "electric" and "magnetic" effects; such a division is inertial frame dependent.