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A NEW EXPERIMENTAL APPROACH TO MACH'S PRINCIPLE AND RELATIVISTIC GRAVIATION

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As noted many years ago by Sciama, and more recently by Nordtvedt, Lorentz invariant (relativistic) gravitation at linear order involves a vector potential that is required to properly account for large inertial effects as well as the correct prediction of the classical tests of general relativity theory (GRT). It is pointed out that the linear-order vector aspect of the gravitational potential makes possible a simple, powerful and inexpensive technique for testing the predictions of GRT and associated issues. An experiment using this technique gives preliminary results that, to order of magnitude, corroborate GRT.

Key words: general relativity, Mach's principle, experimental gravitation.

1. INTRODUCTION

It is widely thought that the predicted effects of GRT are so small as to make laboratory exploration of such effects other than the gravitational redshift and, with great difficulty, the detection of gravitational waves practically impossible. It is also believed by many that Mach's principle is not contained in GRT. But the Machian aspect of GRT makes possible an experimental test of relativistic gravitation not yet explored.

Arguably one of the clearest expositions of Mach's principle—that local inertial effects are produced via the gravitational interaction of objects with the large scale distribution of matter in the universe—was made nearly forty years ago by D. Sciama [1]. Sciama claimed that, to account for inertial effects in Machian terms, a vector potential theory of gravitation was required. Briefly recapitulated, Sciama's argument went as follows.

If we consider a test particle in the universe which we take to have a constant active gravitational matter density ρ_c, then the particle will have a gravitational potential energy due to its interaction with the rest of the matter in the universe given by

$$\phi = m\Phi \approx \frac{2\pi GMm}{R}, \tag{1}$$

where G is the Newtonian constant of gravitation and M the mass obtained by integrating ρ_c out to the event horizon located at R. The gravitational field $\boldsymbol{F}$ at the test particle, if it is at rest in the universe, is $\boldsymbol{F} = -\nabla\Phi = 0$; but if the test particle is moving with respect to the universe, then

$$\boldsymbol{F} = -\nabla\Phi - c^{-1}\frac{\partial \boldsymbol{A}}{\partial t} \tag{2}$$

where $\boldsymbol{A}$ is the gravitational vector potential.

Now $\boldsymbol{A} = (\Phi/c)\boldsymbol{v}$, $\boldsymbol{v}$ being the relative velocity of the test particle and the universe, so $\boldsymbol{F} = 0$ for both rest and constant $\boldsymbol{v}$. But, if the test particle and universe are relatively accelerated, we have

$$\boldsymbol{F} = -\left(\frac{\Phi}{c^2}\right)\frac{\partial \boldsymbol{v}}{\partial t}. \tag{3}$$

If $\Phi/c^2 = 1$, a condition that only obtains if ρ_c is exactly the "critical" density, then Eq. (3) gives a gravitational field that produces the observed inertial reaction force on the accelerated test particle. (This is arguably the most compelling argument for the existence of "dark" matter.)

Sciama's results are also obtained from linear-order GRT. This may be shown by taking the divergence of Eq. (2) and invoking a suitable gauge condition to express $\boldsymbol{A}$ in terms of Φ (see below). One recovers the inhomogeneous wave equation for Φ, just as one finds in linear-order GRT. It is well-known that non- and anti-Machian solutions of the GRT field equations exist. Indeed, all solutions for which "critical" cosmic matter density does not obtain fail to satisfy Mach's principle as defined here (gravitational induction of inertia). But if "critical" cosmic matter density obtains, then relativistic gravitation, GRT in particular, gives back Mach's principle.[1] To show this we consider the Nordtvedt effect.

2. THE NORDTVEDT EFFECT AND MACH'S PRINCIPLE

As Nordtvedt has remarked [3], linear-order relativistic gravitational theory predicts mass shifts in accelerated objects that are much larger than one might expect. For example, if a body composed of interacting discrete particles with masses M_i is given an acceleration $\boldsymbol{a}$ due to an external force, a change in its inertial mass δM is induced by the acceleration and

$$\delta M = \sum_i M_i \delta \boldsymbol{a}_i \approx \left(\frac{4G}{c^2}\right) \sum_{ij} \left(\frac{M_i M_j}{r_{ij}}\right) \boldsymbol{a}, \tag{4}$$

where r_{ij} is the distance separating the ith and jth particles and a small additional term in the double sum has been dropped.[2] In the case of the Earth, because it is accelerated, this effect leads to a mass shift of several parts in 10^9. For the case of a test particle in a homogeneous universe we take M_i as the test particle, drop the sum over i and note that the sum over j of GM_j/r_{ij} within the event horizon is just Φ in Eqs. (1) to (3). Thus

$$M_i \delta \boldsymbol{a}_i \approx \left(\frac{4\Phi}{c^2}\right) M_i \boldsymbol{a}. \tag{5}$$

Since $\delta \boldsymbol{a}_i = \boldsymbol{a}$, self-consistency of the Nordtvedt effect requires that $4\Phi/c^2 \approx 1$, i.e., that "critical" cosmic matter density obtains.

If one examines the issue from the point of view of the dragging of inertial frames of reference, as Nordtvedt remarks [3], an accelerated massive body drags inertial space as

$$\delta \boldsymbol{a}(\boldsymbol{x}) = -\frac{4U(\boldsymbol{x})\boldsymbol{a}}{c^2}, \tag{6}$$

where U is the Newtonian gravitational potential at $\boldsymbol{x}$. If $U = \Phi$ and $4\Phi/c^2 \approx 1$, then $\delta \boldsymbol{a} \approx \boldsymbol{a}$, and inertial frames of reference are "dragged" with the universe if it, as a whole, is accelerated and only accelerations relative to the dragged inertial frames of reference are detectable. Thus the gravitational self-energy induced inertial mass shift identified by Nordtvedt and the dragging of inertial frames in GRT require "critical" cosmic matter density to be consistent. And that consistency produces Machian conclusions equivalent to the conclusion of Sciama's argument.

It is worth noting that no effect analogous to the Nordtvedt effect occurs in electrodynamics. This can be traced to the fact that the self-energy of a gravitating system includes a field/source interaction term $\rho\Phi$ that is not present in electrodynamics [4]. In electrodynamics the entire interaction energy can be ascribed to the field. In gravitation this cannot be done. If the energy in the gravitational field is taken as positive (as required by binary pulsar observations), then the $\rho\Phi$ term must be present since the total self-energy of a gravitating system is negative (i.e., work must be done to remove the sources to infinity).

Nordtvedt's mass shift prediction suggests that one ask what the effect is on the mass of a test particle if the whole universe is accelerated. This may seem to be the stuff of thought experiment only, but the principle of relativity tells us that, if the test particle and universe are relatively accelerated (by applying an external force to the test particle), we may take the test particle as at rest and regard the universe as accelerated. to get a rough estimate [terms $(v/c)^2$ and higher neglected] of the mass shift expected in a test particle of density ρ, in a universe of density ρ_c for linear order gravitation, we take the divergence of both sides of Eq. (2) to get

$$\nabla \cdot \boldsymbol{F} = -\nabla^2\Phi - c^{-1}\,\frac{\partial(\nabla \cdot \boldsymbol{a})}{\partial t} = -4\pi G\rho. \tag{7}$$

Invoking the Lorentz gauge condition, we then have

$$\nabla \cdot \boldsymbol{F} = -\nabla^2\Phi - c^{-2}\,\frac{\partial^2\Phi}{\partial t^2} = -4\pi G\rho, \tag{8}$$

the inhomogeneous wave equation for Φ with sources $G\rho$. In stationary circumstances, i.e., inertial motion of the test particle, the $\partial^2\Phi/\partial t^2$ term in Eq. (8) vanishes and the Poisson equation is recovered. But, if the test particle is accelerated, in general the $\partial^2\Phi/\partial t^2$ term no longer vanishes at the test particle.

In electrodynamics, when dealing with equations like Eq. (8), one stipulates a distribution of sources and initial conditions and then solves for Φ, getting well-known results expressed in terms of a retarded source distribution. In this case, Eq. (8) is linear, since electric charge is a Lorentz-invariant scalar. In the case of gravitation, this is not true even at linear order, because of the field/source self-interaction contribution to the sources of the gravitational field. But here we do not seek the usual solution for field in terms of stipulated sources. Instead we look for the effect of the field of distant matter on a source—the test particle.

At our test particle, when its interaction with the rest of the matter within the event horizon is taken into account, Φ is proportional to the local self-energy density, $\rho\Phi_c$. Since Φ_c is essentially constant, when the potential $\Phi \propto \rho\Phi_c$ is used in the $\partial^2\Phi/\partial t^2$ term in Eq. (8), one gets

$$\nabla \cdot \boldsymbol{F} = -\nabla^2\Phi - \left(\frac{\Phi_c}{c^2\rho}\right)\frac{\partial^2\rho}{\partial t^2} = 4\pi G\rho. \tag{9}$$

The factor of ρ in the brackets must be included to normalize this term and make it dimensionally correct. Note that the $\partial^2\rho/\partial t^2$ term in this equation, if $\Phi_c/c^2 = 1$, is just the time-like component of the four-divergence of the four-vector generalization of $\boldsymbol{F}$ required by local Lorentz invariance. That is,

$$\frac{\partial^2\rho}{\partial t^2} = \frac{\partial(\boldsymbol{f}\cdot\boldsymbol{v})/\partial t}{c^2},$$

where $\boldsymbol{f}\cdot\boldsymbol{v}$ is the rate at which gravitational forces do work on their sources per unit volume. It is important to note here that inertial reaction forces are included in these gravitational forces, since they are the gravitational forces on local matter produced by all matter within the event horizon.

Since the four-divergence of the four-vector generalization of $\boldsymbol{F}$ must equal a Lorentz invariant scalar it follows that $\nabla^2\Phi = 4\pi G\rho_0$ with ρ_0 the rest matter density. Thus Eq. (9) may be written

$$\nabla \cdot \boldsymbol{F} = -4\pi G\rho_0 - \left(\frac{\Phi_c}{c^2\rho}\right)\frac{\partial^2\rho}{\partial t^2} = 4\pi G\rho. \tag{10}$$

Now $\partial^2\rho/\partial t^2 \propto \partial(\boldsymbol{f}\cdot\boldsymbol{v})/\partial t$ in general is not zero if our test particle is accelerated and Eq. (10) therefore is nonlinear for non-inertial motions. Writing $\Phi_c/c^2 = \beta(\approx 1)$ and $\rho = E/c^2$, it follows from Eq. (10) that

$$\rho = \rho_0 + \left(\frac{\beta}{2\pi G c^2}\right)\frac{\partial^2 E}{\partial t^2}, \tag{11}$$

where a factor of two correction from GRT that arises from the distribution of gravitational energy between field and sources has been included.[3]

3. AN EXPERIMENT

The second term in Eq. (11) says that, if we vary the energy density in the test particle, we can produce transient fluctuations in its active gravitational mass (and via the equivalence principle its passive gravitational and inertial masses). For example, if we apply an external, oscillating electric field to a dielectric test particle, the resulting acceleration of its parts will stimulate inertial reaction forces. If those reaction forces are time varying, they will produce, through field/source coupling, a mass fluctuation which has a density given by the $\partial^2 E/\partial t^2$ term in Eq. (11). For periodic accelerations this term time-averages to zero. But instantaneously it yields "anomalous" results. This type of behavior was noted many years ago by Nordtvedt [5].

Do the transient mass fluctuations predicted in Eq. (11) actually occur? $(Gc^2)^{-1} = 1.67 \times 10^{-14}$(cgs) is a rather small number. But $\partial^2 E/\partial t^2$ can be made very large in suitable apparatus. Values of $\partial^2 E/\partial t^2$ in the range of 10^{10} to $10^{12}\,\mathrm{erg/cm^3 s^2}$ can be attained without great difficulty in laboratory apparatus, leading to predicted peak transient mass fluctuations on the order of milligrams. This is a testable prediction.

Energy density fluctuations can be generated in a variety of ways, but, when the requirement that the system in which the fluctuations take place is to be weighed with high sensitivity and fast time-response is added, practical considerations limit the choice of approach. Arguably one of the simplest approaches is to weigh capacitors as they are charged and discharged. An apparatus designed to do this is shown in schematic longitudinal section in Fig. 1. Arrays of high voltage capacitors are mounted in a shielded case that rests on a sensitive force transducer that gives a weight signal. An accelerometer is present to monitor seismic and electromagnetic signals that might be mistaken for the effect sought. Both pulsed DC and AC high voltage power supplies can be used with a device of this sort. (High voltage is used to minimize currents in the device since the transducers use magnetoresistive elements.) And AC voltages can be driven near the mechanical resonant frequency of the force transducer to amplify the weight signal.

Several steps can be taken to improve the sensitivity of this device and suppress spurious signals. Since the expected effect is small and fast time response is required to see it, signal averaging must be used. In the signal averaging process, if half of the data are taken with an applied voltage of one polarity and the other half with the opposite polarity, then polarity dependent electromagnetic coupling effects will be averaged away. Similarly, if half the data are taken with the capacitors in one position and the other half with the capacitors inverted, then electromechanical effects due, for example, to electrostriction will be averaged away.

The response of the system to the high voltages can be eliminated from the results by replacing the capacitors with small capacitance dummy capacitors, repeating the data acquisition sequence,

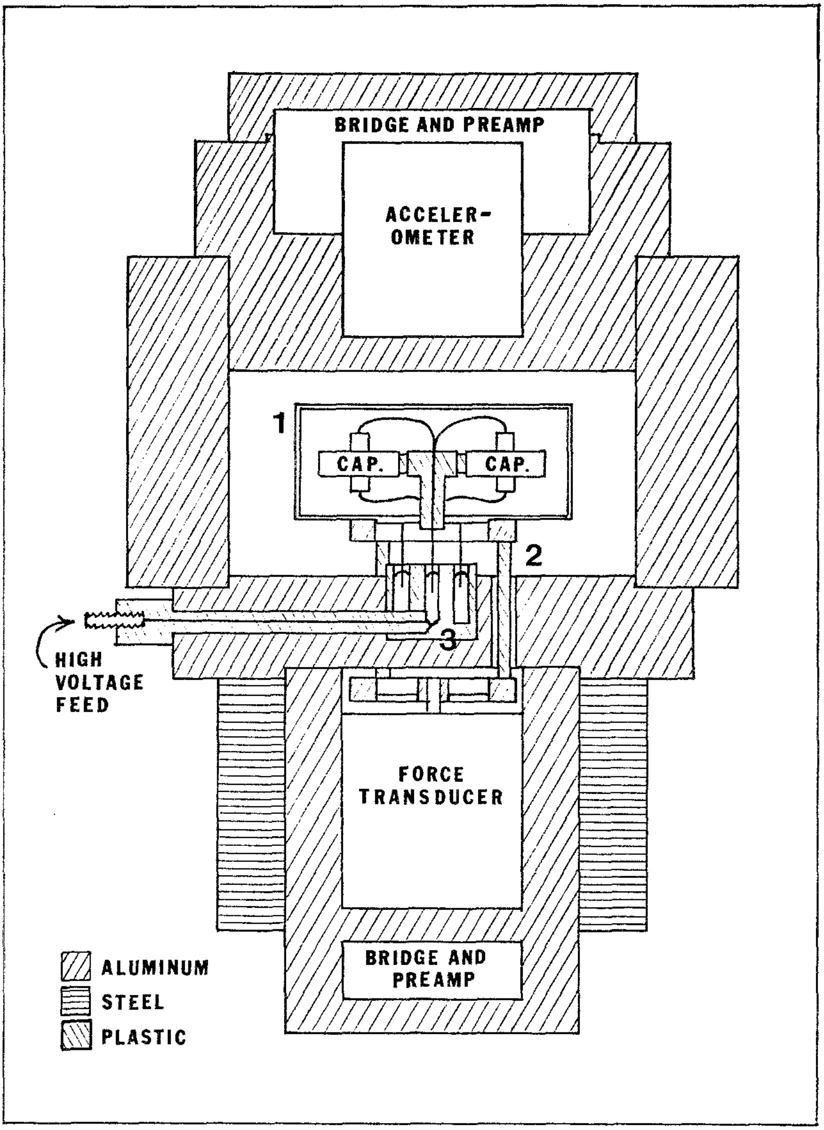

Fig. 1. A schematic longitudinal section of an apparatus designed to weigh arrays of high voltage capacitors as pulsed and sinusoidal voltages are applied to them. The capacitors are mounted in a thin-walled aluminum case (1), lined inside and out with μ-metal foil, that rests on rings and rods (2) attached to a Unimeasure U80 position transducer fitted with a P-100 pressure adapter. Electrical connections to the capacitors and their case are made via mercury contacts (3). The accelerometer is based on a U80-D transducer. The entire device is suspended by elastic shock cords to isolate it from seismic noise.

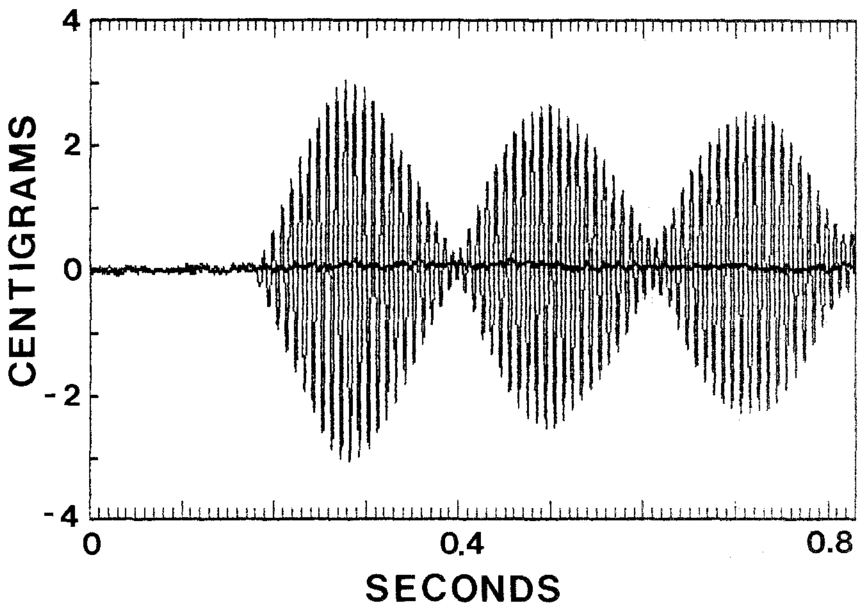

Fig. 2. Net results averaged for several pairs of Ceramite (amorphous barium titanate dielectric) disk capacitors (about 0.012 μf per pair) with a 52 Hz, 3.0 kv peak-to-peak applied voltage. The trace that runs through the force transducer signal is the corresponding accelerometer signal. The beat pattern in the result is produced by driving the power frequency near the mechanical resonance frequency of the transducer to amplify the mass fluctuation signal.

and subtracting the dummy results from those for the high capacitance samples. Likewise, the effects of the charging currents can be ascertained by moving the capacitors to a place in the circuit external to the device and replacing them with conductors. And several other tests can be done to insure that spurious signals do not contaminate the results.

I have tested several types of commercial high voltage capacitors in a device like that shown in Fig. 1. Both KD components and Ceramite disk capacitors, with amorphous barium titanate in a ceramic binder for the dielectric, and plastic capacitors, axial capacitors made with aluminized Mylar and paper in oil encapsulated with glass, were tested. The plastic capacitors were chosen because they contain no ferroelectric substances. All of the capacitors show a net effect of the sort expected. Preliminary results obtained with the Ceramite disk capacitors excited with an AC voltage chosen so that its power frequency was near the transducer resonant frequency are shown in Fig. 2. The measured values of β for the capacitors are typically about 2 with a 2σ error of ± 1. To order of magnitude these results are consistent with linear-order relativistic gravitation.

Allowing that the bulk of the observed effect is that predicted, one may ask why the measured values of β are not smaller. Several potential explanations can be offered: Approximations made in obtaining Eq. (11) account for the discrepancy. For example, the constituents of the solid dielectric in the capacitors are not free test particles. The force transducer, passively calibrated with a small mass, has a more sensitive small signal response than assumed (the most likely explanation), or other experimental error.

But the issues just raised, and others, are matters for further investigation. The important point here is that this experimental technique enables one to explore relativistic theories of gravitation and Mach's principle in a new and powerful way that should shed light on aspects of their nature not accessible with techniques already employed.

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NOTES

1. If one demands a theory that satisfies Mach's Principle irrespective of the particular value of ρ_c, one must go to a theory that contains GRT with "critical" cosmic matter density as a special case. Such a theory (an Einstein-Cartan theory with teleparallelism) has been developed by Treder [2].
2. Equation (4) here is Nordtvedt's Eq. (14), in Ref. 3, with GRT PPN parameters chosen.
3. The exact value of this correction factor that depends on the way in which energy is distributed between field and sources in turn depends on how the source term for the gravitational field equations is constructed. At least two different source terms that give correct predictions for the various tests of GRT exist. In this connection see Peters [4]. This ambiguity does not mean that it is impossible in principle to determine how energy is distributed between sources and field. Indeed, if one posits the existence of "critical" cosmic matter density, this experiment can decide the issue.