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Gravity, Inertia, and Quantum Vacuum Zero Point Fields

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Over the past several years Haisch, Rueda, and others have made the claim that the origin of inertial reaction forces can be explained as the interaction of electrically charged elementary particles with the vacuum electromagnetic zero-point field expected on the basis of quantum field theory. After pointing out that this claim, in light of the fact that the inertial masses of the hadrons reside in the electrically chargeless, photon-like gluons that bind their constituent quarks, is untenable, the question of the role of quantum zero-point fields generally in the origin of inertia is explored. It is shown that, although non-gravitational zero-point fields might be the cause of the gravitational properties of normal matter, the action of non-gravitational zero-point fields cannot be the cause of inertial reaction forces. The gravitational origin of inertial reaction forces is then briefly revisited. Recent claims critical of the gravitational origin of inertial reaction forces by Haisch and his collaborators are then shown to be without merit.

1. INTRODUCTION

Several years ago Haisch, Rueda, and Puthoff[(1)] (hereafter, HRP) published a lengthy paper in which they claimed that a substantial part, indeed perhaps all of normal inertial reaction forces could be understood as the action of the electromagnetic zero-point field (EZPF), expected on the basis of quantum field theory, on electric charges of normal matter. In several subsequent papers Haisch and Rueda particularly have pressed this claim, making various modifications to the fundamental argument to try to deflect several criticisms. Notably, they have tried to structure their argument in terms of the electromagnetic energy and momentum flux putatively

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seen by an electric charge accelerating in the EZPF, for this allows them to sidestep the messy problem of how the EZPF couples to the accelerating charges. From their point of view, a good reason exists to do this. When the coupling of the field to electric charges is taken into consideration, the EZPF conjecture on the origin of inertial reaction forces can be shown to be unsupportable. The coupling of electromagnetic fields to electric charges cannot be made to generally mimic inertial reaction forces observed in normal matter.

Detailed and moderately elaborate arguments can be advanced to support the claim that the EZPF cannot be the cause of inertia (see Woodward and Mahood,[2] hereafter, WM, in this connection). But the fundamental argument that gives the lie to the EZPF argument is almost trivially simple. As Thomas Mahood and I recently pointed out (in WM), essentially all of the masses of the hadrons reside in the photon-like gluons that bind the electrically charged quarks together. Since the quarks are asymptotically free, thus coupling to the EZPF essentially independently of each other, and the EZPF does not couple to the electrically chargeless gluons that carry the bulk of the mass of hadrons, it follows immediately that the action of the EZPF on the quarks cannot account for the inertial reaction forces produced by hadrons when they are accelerated. (See also F. Wilczek[3] in this matter.) This conclusion is further reinforced when one considers the specifics of the constituent quarks and masses of the various hadrons (see WM), a point I will not belabor further here. Mass-energy, not electric charge, gravitates and is responsible for inertia.

Haisch and Rueda, now joined by Y. Dobyns, with their investment in the EZPF conjecture on the origin of inertia, naturally, are unenthusiastic about abandoning the EZPF idea and have attempted to rebut Mahood's and my (WM) critique (Dobyns, Rueda, and Haisch,[4] hereafter, DRH), claiming that our critique is flawed. Their claims of errors in our critique of the EZPF conjecture *per* se all have answers of course, but those answers are not critically important in view of the fact that the EZPF conjecture on the origin of inertia is demonstrably wrong, as just mentioned. Indeed, answers to most of their error claims involving the EZPF conjecture are already implicitly contained in WM. For these reasons, I will not respond to the part of their rebuttal that deals specifically with the EZPF conjecture on the origin of inertia. The first part of their paper—a general discussion of ZPFs and the energy density of the vacuum in quantum field theory, the possibility that a generalized non-gravitational ZPF scheme might account for both gravity and inertia, and a critique of the gravitational explanation of inertial reaction forces in general relativity theory (GRT)—however, does merit a response, for the gravitational origin of inertial reaction forces, contrary to their claims, is correct and physically significant. And if inertial reaction forces are due to the gravitational action of chiefly distant matter, as they are in GRT and is expected on the basis of "Mach's principle," then inertial reaction forces cannot be the result of the direct action of quantum vacuum fluctuations on elementary particles, contrary to the claims of DRH. These expanded claims made by DRH are addressed in this paper.

2. GRAVITY, INERTIA, AND VACUUM FLUCTUATIONS

DRH lead off their rebuttal to WM with a discussion of the "vacuum catastrophe," the fact that relativistic quantum field theory (RQFT) predicts that the vacuum, as a consequence of ZPFs, electromagnetic and others, should have an energy density that is idiotically larger (by a hundred orders of magnitude or more) than that observed (less than 10^{-29} gm/cm^3), affirming the commonplace that this is a real problem. To avoid the gravitational consequences of this preposterous energy density, DRH suggest that if it could be shown that gravity itself were a consequence of quantum vacuum fluctuations, then the vacuum fluctuational energy that causes the problem could be ignored on the grounds that the vacuum cannot act on itself. As a result, since gravity and inertia are linked via the Equivalence Principle (EP), they claim that the action of the ZPFs of RQFT on charged (for the EZPF conjecture, electrically charged) objects would lead to both Gravitational and inertial forces. Or, as DRH remark,[(4)]

> We may consider the possibility that the electromagnetic and other zero-point fields really do exist as fundamental theoretical considerations mandate, but that their zero-point energies do not gravitate because it is the actions of these fields on matter that generate gravitational forces (which are mathematically represented by the curving of spacetime). The zero-point energies do not gravitate because the zero-point fields do not, indeed cannot, act upon themselves.... The principle of equivalence, however, dictates that if gravitation is an effect traceable to the action of zero-point fields on matter, then so must the inertia of matter be traceable to zero-point fields.

As this passage makes clear, DRH no longer assert that the EZPF conjecture by itself can account for gravity and inertia. But they continue to claim that some sort of ZPF account of *gravity and inertia in terms of ZPFs other than that of the elusive*, *yet to be invented quantum gravity* is the correct way to understand these phenomena. As Mahood and I pointed out in WM, a ZPF interpretation of inertia might well be possible should a successful theory of quantum gravity ever be invented. But this is not the same as DRH's claim where gravity and inertia *emerge*, if you will, from the actions of other, non-gravitational fields on matter. Several reasons can be mustered to reject this claim, not the least of which is the fact that DRH's assertion that, "the electromagnetic and other zero-point fields really do exist as fundamental theoretical considerations mandate," at least as far as the electromagnetic field is concerned, notwithstanding widespread popular belief to the contrary, is arguably not correct.

Peter Milonni and several collaborators, for now approaching 30 years, have repeatedly shown that the vacuum fluctuation interpretation of quantum electrodynamics (QED) is just that: an interpretation. (See Milonni's outstanding book, *The Quantum Vacuum*, 1993,[5] on this fascinating topic.) It is not a physically necessary representation of the facts of reality. Indeed, with a reordering of commuting operators, the vacuum fluctuation picture of any particular phenomenon, say the Casimir effect or Lamb shift, can be rendered completely in terms of radiation reaction due to radiative coupling to other, distant matter. In this completely alternative interpretation the vacuum is devoid of energy since no fluctuations are present—just as gravity mandates that it must almost exactly be. Milonni prefers an interpretation where half of the interaction is vacuum fluctuations and half radiation reaction. The vacuum fluctuation part he takes to be responsible for "spontaneous" emission by atoms in the vacuum. The other half, due to radiation reaction, he points out is necessary to quench the spontaneous excitations of such atoms that would otherwise occur, but are never observed in fact. It is worth noting here that likewise in the full radiation reaction picture no spontaneous excitations occur, for there is no fluctuational energy in the vacuum to cause them. If one adopts this empty vacuum picture, however, then one has no simple, intuitively appealing explanation for spontaneous emission. Nonetheless, assuming that Milonni's arguments can be extended to the other quantum fields of the standard model, it seems that the formalism of RQFT may not demand that the vacuum be filled with the zero-point fluctuations of popular lore.

If radiation reaction is taken to be a fully "retarded" interaction, that is, all radiation appears to propagate forward in time and "causes" always seem to precede effects, we may ask: Is it possible to provide a physical explanation of spontaneous emission? Yes. But at a price. It has long been known, especially since the work of Wheeler and Feynman,[6, 7] that radiation reaction has more than one representation. In particular, instead of the conventional, fully retarded representation, one can adopt the "action-at-a-distance" picture where observed radiation consists of both "advanced" waves propagating from the present (and future) into the past and retarded waves headed the other temporal direction. When the amplitudes of the advanced and retarded waves emitted by an excited source are equal and the retarded waves emitted here in the present are fully absorbed out there in the future, the sum of the advanced and retarded waves yield observed phenomena, and the appearance of fully retarded interactions obtains notwithstanding the real presence of advanced waves. In this picture radiation is only initiated when the conditions of its absorption in the future are fully established. One may ask: How does an excited atom "know" when conditions of future absorption have been established? Well, quantum mechanics. As John Cramer[8] has shown, the classical action-at-a-distance theory of radiation of Wheeler and Feynman has a natural extension in quantum mechanics. In his "transactional interpretation" of quantum mechanics it is the wave function that, if you will, feels out the future and alerts the excited atom to emission possibilities. And when future absorption is established, emission takes place. You may find all this exceedingly bizarre. I certainly do. But the main point here is that this interpretation is fully supported by the formalism of radiation theory; and it leaves the quantum vacuum devoid of the ZPFs whose energy densities are not allowed by gravity.

Returning to DRH's speculation on the ZPF origin of gravity and inertia we ask: Is it possible that some clever combination of several non-gravitational ZPFs can do the trick? For two reasons, no. The first is a simple generalization of the central argument of WM for the case of EZPFs: The coupling of the non-gravitational fields to their charges in a sea of vacuum fluctuations very likely cannot be made to successfully mimic the universal coupling of gravity to mass-energy *in all circumstances*. Even were this possible, the "fine-tuning" involved would make the fine-tuning problems of standard theory pale into insignificance. The second, more fundamental reason to dismiss the DRH speculation is related to the fact that gravity, and concomitantly inertia, in this conjecture, are emergent features of reality since they arise from the action of ZPFs on matter, a local process. The problem with this conjecture is the virtue that DRH claim for their ideas: the locality of the process that produces the emergent phenomena. Why is this a problem? Because gravity, whatever its cause may be, is a long-range interaction. Note that this long-range force must act on the emergent property (that is, the mass-energy) of other objects, not the fluctuation filled vacuum at distant locations. For if gravity acted directly on the distant vacuum, then either the "vacuum catastrophe" is recovered because the vacuum acts back on the source of the gravity, becoming a source of gravity itself, or one must assume that the vacuum can be acted upon by distant gravitating objects without it reacting on the distant gravitating objects. In this latter case, we must abandon Newton's third law since we have action without reaction, and with it the conservation of energy and momentum.

Of course, one can simply assert that long-range gravitational interactions happen. But then the intuitive appeal of local action, the chief motivation of the ZPF conjecture on the origin of inertia in the first place, is sacrificed. And since gravity as a long-range force has been introduced, and as the EP asserts the equivalence of inertial and gravitational phenomena (see the DRH quote above and the next section below), one finds that inertial reaction forces are indeed due to the gravitational action of chiefly distant matter, just as Mahood and I argued in WM, not to the direct local action of a ZPF, notwithstanding that a local ZPF action on matter *might* be the cause of gravity. Thus, if one seeks to eliminate gravity as a fundamental force by making it an emergent consequence of the local action of non-gravitational ZPFs, the fact of the EP leads one inexorably back to the fact that inertial reaction forces are caused by gravity, emergent or not, not the direct action of local ZPFs. (See Note 1) So, in eliminating the gravitational consequences of vacuum energy by making gravity an emergent consequence of the local action of non-gravitational ZPFs on matter, we have insured that inertial reaction forces *cannot* be caused by the direct local action of non-gravitational ZPFs. The assertion about the EP requiring that inertial reaction forces be due to long-range gravity (in relativistically invariant gravity theories anyway) we will find is demonstrably true (in the next section below).

With all of this talk of the emergence of mass and gravity due to the local action of some unspecified non-gravitational ZPFs, one may be tempted to conjecture that the Higgs field of RQFT is the ZPF mechanism sought by DRH. After all, the action of the Higgs field is to confer restmass on the quanta of other fields, quarks, leptons, and the W and Z bosons in particular. Alas, this will not do. The Higgs field is unsuitable for precisely the same reasons that the EZPF conjecture on the origin of inertia must be abandoned. While photons and gluons are restmassless, they nonetheless gravitate because energy is a source of gravity. Indeed, the bulk of the masses of the hadrons reside in the gluons that bind them, not the quarks. But photons and gluons do not couple to the Higgs field. So the local action of the Higgs field cannot be the cause of emergent gravity.

I should mention here that the local, non-gravitational ZPF conjecture on the origin of mass and gravity cannot arise through the mechanism first proposed by HRP. That mechanism leads to local inertial reaction-like forces when objects are accelerated by external forces. Since long-range, emergent gravity must also produce inertial reaction forces—as mandated by the EP (now known to be true to a part in 10^{12} or so) and the formalism of relativistic gravity—any significant additional ZPF driven inertial reaction-like force, taken to be a contribution to inertia and added to the gravitational inertial reaction force, would produce violations of the EP. So any non-gravitational ZPF interaction that one might posit as the cause of mass and gravity must not display the directionality of the HRP conjecture. Rather it must be non-directional, as is the case for the Davies–Unruh effect for example. Evidently, getting rid of the energetic consequences of real vacuum ZPFs is a much trickier business than imagined by DRH. Exploration of the gravitational origin of inertial reaction forces, it would seem, merits further investigation.

3. GRAVITY AND INERTIA

Before turning to the specific claims of DRH regarding the gravitational origin of inertial reaction forces, I recapitulate and amplify a bit some of the material in WM on this point. I do this for two reasons. First, the material is needed in conjunction with the analysis of their claims in Sec. 4, and to have it here is convenient. Second, an alternate presentation of this material may make its significance somewhat more transparent than it was in WM.

Much of the confusion over the relationship between gravity and inertia can be traced to the idiosyncratic historical development of the theories of mechanics and gravity. In particular, notwithstanding that both Galileo and Newton were aware of the equivalence of gravitational and inertial mass, for them gravity was a force to be treated separately from, although within, the general framework of mechanics. Quite apart from the seemingly serendipitous equivalence of gravitational and inertial mass, problems, apparently unrelated to this equivalence, arose as soon as Newton became serious about working through the full formalism of his "system of the world." In particular, he discovered, likely in the 1680s, that gravity must be an *exactly* central force in order to recover Keplerian orbital motion for the planets. That is, the gravitational force exerted by the Sun on a planet must always act along the *instantaneous* line of centers of the bodies. Moreover, this must be true in all inertial frames of reference, as is the case for Newtonian gravitation under the Galilean group of transformations. And this, in turn, seemed to require that gravity act instantly over arbitrarily large distances. That is, in the idiom of that day, gravity is an "occult" force, for no one could imagine how a mechanism for instantaneous action could be invented. Almost no one, Newton included, could accept this. Not even Newton had a solution for this blatant defect of his system.

The instantaneity of the action of Newtonian gravity was a burr under the saddle of science until the advent of GRT. Many of the most eminent physicists of the 18th and 19th centuries tried to address this issue, all unsuccessfully. Indeed, notwithstanding the creation of GRT by Einstein, one finds serious discussions of the speed of propagation of gravity, the claim being made that it must vastly exceed the speed of propagation of light in vacuum, in the serious literature to this day. Were this true, of course, it would render the principle of relativity false, and with it both special relativity and GRT. But GRT actually settled this issue: gravity propagates in vacuum at the vacuum speed of light, *as it must if the principle of relativity is correct*. Historically this fact did not emerge in a clear-cut way because when the attempt was made to show that GRT yielded Newton's law of gravitation in an appropriate approximation, it was Newton's law replete with instantaneous action that people sought to recover. Since the "Newtonian limit" is obtained by treating gravity as a weak field phenomenon involving bodies moving at non-relativistic velocities, neither of which has anything explicitly to do with the speed of propagation of gravity, this seemed to leave open the question of the propagation speed of gravity.

Ironically, if relativistic invariance is demanded, the correct "Newtonian" approximation of GRT is a set of field equations with Maxwellian form. Ken Nordtvedt[(10)] made this clear in a superb paper about a dozen years ago. (For historical reasons, however, this is commonly referred to as the "post-Newtonian" limit or approximation.) Briefly, what Nordtvedt showed was that although the usual Newtonian approximation of GRT works for orbit calculations where the Sun is taken to be at rest in an inertial frame of reference, if one, invoking relativistic invariance, makes a Lorentz boost to any other inertial frame of reference and then calculates the orbit, the result is nonsense. The orbit rapidly blows up. This is true even if the frame has a velocity relative to the Sun that is non-relativistic. Nordtvedt went on to show that in order to recover Keplerian motion in inertial frames moving with respect to the Sun one must use the "post Newtonian" approximation of GRT where, at linear order in the mass, gravity is represented by a four-vector, rather than a scalar, potential. The three-vector part of the four-potential yields a "gravitomagnetic" field equation in addition to the usual "gravitoelectric" field equation that mimics Newton's Law of gravity in appropriate circumstances.

The physical reason why the three-vector part of the four-potential (and its associated gravitomagnetic field) must supplement the scalar potential (with its associated Newtonian gravitoelectric field) is that the gravitoelectric field in the moving frame recovered from the gradient of the scalar part of the potential does not point along the instantaneous line of centers of the bodies, owing to the retardation of the gravitoelectric field that arises from its finite propagation velocity. However, in the moving frame the Sun becomes a rather large mass current, so the predicted gravitomagnetic force and $\partial \mathbf{A}/\partial t$ contribution to the gravitoelectric force it exerts on a planet cannot be ignored. As Nordtvedt showed, when the effect of the three-vector part of the potential is added to the gradient of the potential that yields the Newtonian part of the gravitoelectric force, Lorentz boosted Keplerian motion results. Thus, arguably, the correct "Newtonian" approximation of GRT, because the propagation velocity of gravity must be less than or equal to *c*, is a set of field equations with Maxwellian form. I should add here that in the recent discussions of the propagation speed of gravity the vector potential need not be explicitly invoked to show that the force of gravity points along the instantaneous line of centers. As lbison, Puthoff, and Little[(11)] have pointed out in a particularly apposite paper, even in the limit where the induction field alone acts (that is, when no propagating disturbance is present in the electromagnetic field), the electric field lines of an inertially moving electric charge in Maxwellian electrodynamics point toward the instantaneous location of the charge in all inertial frames of reference. Similarly, in GRT the gravitational field lines of a comparable moving mass point toward the instantaneous position of the mass.

What does all this have to do with inertia? Well, Nordtvedt also pointed out in this paper that it is the three-vector part of the four-potential in the post Newtonian approximation of GRT that leads to all of the well known Machian "frame-dragging" effects predicted by GRT when nearby accelerating massive objects are present. The Newtonian part of the gravitoelectric part of these field equations that arises from the gradient of the scalar potential has nothing to do with the Machian effects. For those familiar with Dennis Sciama's outstanding paper on the origin of inertia,[(12)] this should come as no surprise at all, for it is the three-vector part of the four-potential of the Maxwellian field equations he adopted for gravity that leads to inertial reaction forces and other Machian effects. Nordtvedt did not extend his investigation of Machian effects to the global, cosmic scale and recover full inertial reaction forces, but especially in light of Sciama's work, as discussed in WM, this extension is straight-forward.

The fundamental points to be made in all this are:

- Gravitomagnetic fields and gravitoelectric fields that arise from the three-vector part of the four-potential *must* be present if ones theory of gravity is to be relativistically invariant and Keplerian orbital motion is to be recovered in all inertial frames of reference.
- Machian effects generally, and inertial reaction forces in particular, are a consequence of the three-vector potential. They do not arise from, or depend upon the gradient of the scalar part of the full four-potential that is associated with the gravitoelectric part of the interaction.

One more point needs to be discussed before I turn to DRH's critique of the gravitational origin of inertia. It is that in the approximation of GRT under consideration, special conditions on the value of the scalar gravitational potential must obtain. These conditions are most easily addressed with a little formalism.

Consider a test particle moving in a universe of constant matter density with some velocity $\mathbf{v}$ with respect to the rest frame of the uniformly distributed matter. Given the Maxwellian form of the post-Newtonian gravitational field equations, the gravitoelectric field $\mathbf{E}$ at the test particle will be:

$$\mathbf{E} = -\nabla\phi - (1/c)\,\partial\mathbf{A}/\partial t \tag{1}$$

where ϕ is the scalar part of the four-potential, c the vacuum speed of light, and $\mathbf{A}$ the three-vector part of the four-potential of the gravitational field. $\mathbf{A}$, by analogy with electrodynamics, is just the integral over all causally connected space (out to the particle horizon) of the matter current density, $\rho\mathbf{v}$, in each volume element divided by the distance $\mathbf{r}$ from the test particle to the volume element dV. To make the matter current density a proper source of gravity (a gravitational charge that is) it must be multiplied by Newton's constant of universal gravitation, G. So,

$$\mathbf{A} = -(1/c)\int (G\rho\,\mathbf{v}/\mathbf{r})\,dV \tag{2}$$

The scalar potential is:

$$\phi = -\int (G\rho/\mathbf{r})\,dV \tag{3}$$

In the instantaneous rest frame of the test particle the rest of the matter in the universe appears to move rigidly with velocity $-\mathbf{v}$. And as long as we can remove $\mathbf{v}$ from the integral in Eq. (2), arguably a valid step in the circumstances we are considering given the apparently rigid motion of the matter in the universe past the test particle, we have:

$$\mathbf{A} = \phi\mathbf{v}/c \tag{4}$$

This expression for $\mathbf{A}$ may be substituted into Eq. (1), yielding:

$$\mathbf{E} = -\nabla\phi - (\phi/c^2)\,\partial\mathbf{v}/\partial t \tag{5}$$

The gradient of ϕ vanishes at the test particle because of the constancy of the matter density, and there is no gravitomagnetic force present because the curl of **A** vanishes by symmetry. Indeed, **E** vanishes too if **v** is a constant. But if an external force acts to accelerate the test particle, then $\partial \mathbf{v}/\partial t$ is not zero, and the test particle experiences a gravitoelectric field produced by the gravitational action of the matter within the particle horizon. If ϕ/c^2 is equal to one, then the gravitoelectric force on the test particle (**E** times the test particle mass) is exactly the inertial reaction force the accelerating agent experiences.

The crucial point to be made here is that the coefficient of $\partial \mathbf{v}/\partial t$ in Eq. (5) must be equal to one *in all circumstances* if gravity is to properly account for inertial reaction forces. This means that ϕ must be exactly equal to c^2 for this to work (at least in this approximation which follows the form of Sciama's argument). This is the condition recovered if critical cosmic matter density is the fact of experience. Both recent studies of distant supernovae and fluctuations in the cosmic background radiation strongly suggest that cosmic matter density satisfies this condition. (Even though no one knows what most of the matter in question is.) It would seem to follow immediately that inertial reaction forces *must* be ascribed to gravity. But what if the value of ϕ changes, either because it is a function of cosmic epoch, or because of the presence of some nearby concentration of matter (for example, the Sun or a planet)? The locally measured value of c doesn't change. Its local invariance is demanded by the *principle* of relativity. So if ϕ measured locally were to depend on epoch or the presence of nearby "spectator" matter, then the gravitational contribution to inertial reaction forces might be more or less than the force required to satisfy Newton's third law, and inertial reaction forces could not be claimed to be exclusively gravitational in origin—leaving the door open for non-gravitational ZPF and/or other yet to be devised schemes.

Evidently, we require some compelling argument that ϕ must always be exactly equal to c^2 if the exclusively gravitational origin of inertia is to be demonstrated. It turns out that $\phi \equiv c^2$ is a *necessary* consequence of the *principle* of relativity. To show this, we first note that the invariance of c demanded by the principle of relativity is not a global invariance. Indeed, the value of c measured by an observer at some spatial location in some arbitrarily specified coordinate frame of reference—the "coordinate" vacuum speed of light—depends on whether matter is located near to the place where the speed is measured by the distant observer. That is, in the presence of a gravitational field, the speed of light measured by the distant observer where the gravitational field strength is different will differ from the speed measured by the observer at his/her location. For example, the speed of light near the event horizon of a black hole measured by a distant observer is very nearly zero. But an observer near the event horizon will measure the usual value of 3×10^{10} cm/sec. This is a consequence of the fact that the condition on c required by the principle of relativity is that it be a *locally measured invariant.*

In order to establish that *if* $\phi \equiv c^2$ *at any event in spacetime*, *then* $\phi \equiv c^2$ *is necessarily true for every spacetime event*, all we need show is that ϕ, like c, must be a *locally measured invariant*. Carl Brans,[13] in 1962, was the first to show explicitly that the principle of relativity requires that ϕ be a locally measured invariant. I will not recapitulate his argument here in any detail. Suffice it to say, he was interested in the effect of "spectator" matter on measurements made in a small, shielded laboratory. If the value of ϕ is not a locally measured invariant, then one might reasonably expect that the "spectator" matter would give ϕ in the laboratory a different value from that in a laboratory deep in outer space. Now consider two test bodies with identical masses when measured in the same place. We place one in each of the two labs. If ϕ has different values in the two labs, then the masses of the test bodies will differ because the gravitation potential energy each possesses will be different. If we now perform the classic EP experiment we will find that they both accelerate relative to the labs at exactly the same rate notwithstanding their different masses. So it appears from their accelerations that the EP is not violated. But when they strike the lab floors, they will make different size dents because their masses are different. Thus, if the *locally measured* value of ϕ is different in the two labs, we have a way to discriminate the action of gravitational fields from the "fictitious" gravity field produced by viewing inertial motion in an accelerated frame of reference.

This, plainly, is a violation of both the EP and the principle of relativity. Thus, if the EP and the principle of relativity are true, then ϕ must be a locally measured invariant, in which case the dents would be the same. Brans didn't consider dents. Rather, he considered the accelerations of electrically charged test particles induced by identical electrical fields in the two labs, *which amounts to the same thing*. If the masses of the test particles differ due to different gravitational potentials in the two labs, then their resulting accelerations produced by the electric fields will differ too because their charge to mass ratios differ. Brans showed that in GRT the accelerations are the same, as one would expect since the principle of relativity and the EP are the foundations upon which GRT is built. It therefore follows that in GRT ϕ must be a locally measured invariant, just as is c. We now note that it is an experimental fact that critical matter density presently obtains, so $\phi \equiv c^2$ (up to a factor of 4 in the full GRT calculation that does not affect any of these arguments). And *we may conclude with certainty that all inertial reaction forces arise exclusively through the gravitational action of all of the matter in the causally connected part of the universe if the principle of relativity and the EP are correct*. Moreover, this is true for all epochs and notwithstanding the presence of spectator matter.

4. THE DRH CRITIQUE OF GRAVITATIONAL INERTIA

DRH open their critique of gravitational inertia by noting that in GRT those forces we usually ascribe to gravity do not result from, say, the Earth exerting a "force" on you (and me) that causes us to be attracted to the Earth. Rather, the presence of the Earth causes a local deformation of the spacetime geodesics so that local inertial frames of reference are those that we would normally identify as being in a state of "gravitational free-fall." The "force" of gravity that we experience standing on the surface of the Earth, in fact, is a consequence of our being constantly accelerated with respect to these frames that are in a state of free-fall. Since the force in question we interpret as the local action of gravity arises from an acceleration relative to local geodesic motion, we see that the force is in fact an inertial reaction force, rather than what we normally think of as a gravitational force. This seemingly peculiar state of affairs, DRH point out, is a consequence of the EP. Is this right? Yes! Not only are all inertial reaction forces of gravitational origin in GRT, all "forces" that we normally regard as gravitational, given the geometrical physical content of GRT, are actually inertial reaction forces, just as DRH claim. But, having correctly sorted this out, they then fall into a fundamental *non-sequitur*. In their words:

> In other words, the Principle of Equivalence asserts that gravitational "forces" as conventionally measured are inertial reaction forces—pseudo-forces, as these are sometimes called. We thus see that any attempt to identify gravity as the- source of inertia, within the context of GRT, suffers from an essential circularity. At the level of ordinary discourse, this is almost trivially obvious. We consider an extrinsic theory of inertia which claims that inertial reaction forces are gravitational forces. But the equivalence principle requires that gravitational forces are inertial reaction forces, so applying equivalence to the theoretical claim we see it reduce to the uninformative declaration that inertial reaction forces are inertial reaction forces.

Concerned that their readers might think this argument "linguistic play," DRH go on to amplify and reiterate this claim several times. In the course of these remarks they do not apprehend the *non sequitur* in their argument (though once or twice they come close to doing so). Ultimately, they boldly assert:

> *General relativity, in reducing gravity to a consequence of geometry, offers a very hostile background to a gravitational theory of extrinsic inertia. GTR shows how mass distorts spacetime, and allows one to calculate the trajectories unconstrained bodies will follow in the resulting distorted spacetime. It does not explain why a body, constrained by non-gravitational forces to travel on some trajectory that is not a geodesic, exerts an inertial reaction force proportional to its mass.*

The *non-sequitur* in their circularity claim, evidently, is a consequence of their belief that this last statement about non-geodesic motion is correct. It is not correct. The field equations of GRT do indeed allow one to calculate the effect of gravity in the circumstance of non-geodesic motion. The effect of gravity is precisely the inertial reaction force exerted on the agent causing the motion to be non-geodesic. By 1921, in a discussion of Mach's principle, Einstein had already published the appropriate post-Newtonian (Maxwellian) approximation field equations needed to make the calculation in question, and they remain widely available in his, *The Meaning of Relativity*.(14) The only thing missing from Einstein's account is the realization that, in the notation used above, $\phi \equiv c^2$, with both ϕ and c being *locally measured* invariants. Sciama identified the $\phi \equiv c^2$ condition nearly fifty years ago; and the observed fact of critical matter density, which leads to this condition, is a commonplace these days. Einstein thought that spectator matter could affect nearby objects (in which case ϕ would not be a locally measured invariant). Indeed, he claimed this seeming gravitational induction of *mass* as a Machian feature of GRT. Brans, however, corrected this mistake now nearly 40 years ago. So, when all of the matter and matter currents in the causally connected part of the universe are taken into account (as shown in the previous section for the simple case of the linear acceleration of a test body) the action of the matter and matter currents via the gravitational field give back exactly the inertial reaction force experienced by the agent accelerating the body.

DRH have appreciated that the local invariance of ϕ is important. Unfortunately, missing the correct physical significance of the local invariance of ϕ in GRT, they make claims that are erroneous. As an example:

> ... this new [i.e., 1962] perspective on ϕ shows that the Nordtvedt [i.e., GRT] frame-dragging effect... is, rather than a support of the WM inertia theory [i.e., GRT], absolutely fatal to it. If ϕ is a locally measured invariant due to the action of the entire cosmos, no local concentration of matter can affect ϕ, which leads to the startling conclusion that *no body smaller than the Universe as a whole can produce any frame dragging effects whatsoever*! WM [i.e., GRT] require[s] this locally invariant character for ϕ in order to avoid having inertia behave unacceptably (that is, in a manner contrary to long-established observation) in the vicinity of gravitating masses. Yet the price of this local invariance is the disappearance of all local frame-dragging effects.

Nonsense. Frame dragging effects arise through the action of the three-vector potential, **A**, as is made clear in the previous section above and in WM [especially Sec. 4.1]. It is obvious on simple inspection of Eqs. (2) through (4) that **A**, unlike ϕ, is *not* a locally measured invariant since it depends on the integration over all space of the [retarded] matter *currents*. GRT, even at linear order in the mass and in the post-Newtonian limit, correctly predicts that a large, nearby matter current will affect **A**, changing it from what would be expected on the basis of the bulk relative motion of the chiefly distant matter in the universe alone, notwithstanding that ϕ is a locally measured invariant. In turn, $\partial\mathbf{A}/\partial t$ is changed, giving back faithfully local frame dragging effects. We can isolate the effect of local matter currents, should we so choose, by the simple expedient of breaking the integration that yields **A** up into suitable parts.

Consider, for example, the simple case of linear accelerative frame dragging caused by the acceleration of a spherical shell of matter on point located at the center of the shell. Using the formalism introduced above, we calculate the value of **A** at the center of the shell. It has two parts, one due to the uniformly distributed matter throughout space, and the other due to the shell. Because of the symmetry of the specified circumstances, as was possible in the simple case of perfectly uniform matter density considered above, we find that the two parts of the vector potential can be written as:

$$\mathbf{A}_{\mathrm{u}} = -(1/c)\int (G\rho\,\mathbf{v}/\mathbf{r})\,dV = \phi_{\mathrm{u}}\mathbf{v}_{\mathrm{u}}/c \tag{6}$$

$$\mathbf{A}_{\mathrm{s}} = -(1/c)\int (G\rho\,\mathbf{v}/\mathbf{r})\,dV = \phi_{\mathrm{s}}\mathbf{v}_{\mathrm{s}}/c \tag{7}$$

where the subscripts u and s identify the contributions due to the rest of the universe and the shell of matter respectively. The volume integration for $\mathbf{A}_{\mathrm{u}}$ is to be carried out everywhere except at the location of the shell; and the volume integration for the shell is limited to the space occupied by the shell. Now, ϕ_{u} is approximately, but not exactly equal to ϕ of Eqs. (4) and (5). Indeed, the locally measured invariance of ϕ requires that:

$$\phi_{\mathrm{u}} + \phi_{\mathrm{s}} \equiv \phi \tag{8}$$

everywhere when measured locally. As in the case considered in the previous section, the curl of **A** vanishes by symmetry and there is no gravitomagnetic field at the test particle. When we calculate the gravitoelectric field seen by a test particle located at the center of the spherical shell of matter, however, we find:

$$\begin{aligned}\mathbf{E} &= -\nabla\phi - (1/c)\,\partial\mathbf{A}/\partial t = -(1/c)\,\partial(\mathbf{A}_{\mathrm{u}} + \mathbf{A}_{\mathrm{s}})/\partial t \\ &= -(\phi_{\mathrm{u}}/c^2)\,\partial\mathbf{v}_{\mathrm{u}}/\partial t - (\phi_{\mathrm{s}}/c^2)\,\partial\mathbf{v}_{\mathrm{s}}/\partial t \end{aligned} \tag{10}$$

For convenience we choose the instantaneous frame of rest of the test particle at the center of the spherical shell of matter. Since the universe is not being accelerated relative to the test particle by some "external" force (or, equivalently, no non-gravitational force acts directly on the test particle),

$\partial \mathbf{v}_u/\partial t = 0$ and $\mathbf{E}_u = 0$ too. But, by hypothesis, the spherical shell of matter is being accelerated by some external force, so if the test particle is constrained by another external force to move with constant velocity with respect to the rest of the universe, it will experience a gravitoelectric field produced by the accelerating spherical shell of matter given by:

$$\mathbf{E} = -(\phi_s/c^2)\,\partial \mathbf{v}_s/\partial t \tag{11}$$

If we remove the constraining force, the test particle, now in a state of local free fall, will accelerate relative to coordinates moving at constant velocity with respect to the "fixed stars" due to the action of the **E** field given by Eq. (11). Because of the universal coupling of gravity to matter, all other nearby objects will experience the same acceleration as the test particle. And we see, in the geometrical picture of GRT, that local inertial frames are "dragged" by the gravitational action of the accelerating spherical shell of matter, just as one would expect. Moreover, we also see that geodesic motion is "enforced" by the gravitationally induced inertial reaction force that acts whenever other external forces cause the motion of the test particle to be non-geodesic.

5. CONCLUSION

It is tempting to go point-by-point through the rest of DRH's arguments, showing why each putatively refutatory claim they make regarding the gravitational origin of inertial reaction forces is flawed. But this paper is already quite long. And approached with an understanding of what GRT *actually says*, it is not difficult to identify the errors in their arguments. Suffice it to say that the local non-gravitational ZPF conjecture on the origin of *inertia* is wrong, though the origin of mass and thus gravity as an emergent phenomenon due to the local action of non-gravitational ZPFs is still an open question. GRT does explain the origin of inertial reaction forces that agents acting to accelerate bodies experience as the gravitational action of chiefly distant matter on the accelerating bodies. Local, non-gravitational ZPFs cannot contribute to these inertial reaction forces without violating the EP, now known to be true to roughly a part in a trillion. Why is all this important? Well, put into practical terms, what this means is that should you be looking for a way to manipulate inertia to achieve rapid spacetime transport, schemes based on non-gravitational ZPFs are a waste of time and money unless the cause of mass and gravity (but not inertia) can be demonstrated to be local, non-gravitational ZPFs, and some way of screening objects from the ZPFs can be devised. Given the fact that the ZPFs, in this view, populate the vacuum everywhere and only interact locally, the practical prospect of achieving such screening is arguably insuperable.

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Note. 1. I can't resist mentioning here a paper by Richard Cook[9] where he shows that if one assumes a radiation reaction type inertia force as fundamental, the *ziterbewegung* of massive objects (caused by the action of vacuum fluctuations?) leads to a Newtonian force law for gravity (Cook, 1976). Since Cook takes as his starting point the "Einstein–Sciama" [his terminology] force law with a $1/r$ dependence characteristic of radiative interactions, a long-range force law is assumed. So this is not the ZPF origin of gravity sought by HRP. But it is really neat nonetheless. Cook has been one of Milonni's collaborators.

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