MACH'S PRINCIPLE
AND THE ORIGIN OF INERTIA

Edited by M. Sachs and A.R. Roy

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Preface

This volume consists of a selection of refereed invited lectures presented at the *International Workshop on Mach's Principle and the Origin of Inertia* which was held at the Indian Institute of Technology, Kharagpur, India, as a part of the golden Jubilee celebrations of the Institute, from February 6 to 8, 2002. The Workshop was organized by the Centre for Theoretical Physics of the Institute with Prof. A.R. Roy as the Organising Secretary. It also includes a few contributions from experts who could not attend the Workshop.

Inertia is one of the main physical properties of all bodies. Its origin poses problems of a fundamental nature. Ernst Mach proposed that the inertia of any body is caused by its interaction with the rest of the Universe. The idea reflects a deep connection between the cosmos at large and its individual constituent bodies, thereby implying a holistic conception of nature. On the whole, Ernst Mach had a seminal influence on the evolution of Physics in the 20th Century, and will influence the future development of Physics in the 21st Century. It is interesting that many of his ideas play a role (directly and indirectly) in opposite positions within contemporary physics. His epistemological viewpoint of positivism clearly influenced the basis of quantum mechanics. On the other hand his non-atomistic model of matter and the accompanying interpretation of inertial mass (the "Mach Principle") influenced the holistic approach of the continuous field concept of the theory of general relativity, as a general theory of matter.

The contributions to these proceedings demonstrate Mach's influence on contemporary thinking. For we see here the views of an international group of scholars on the implications of Mach's principle in physics and astrophysics. We believe the ideas presented here could indeed affect future paths of study in physics for many generations to come.

The Editors take this opportunity to put on record their gratitude to Prof. J. V. Narlikar, IUCAA, Pune, for kindly agreeing to write the Dedication note, and to Roy Keys of Apeiron, Montreal whose painstaking efforts have made the publication of this volume possible.

M. Sachs and A.R. Roy

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The organizing secretary also wishes to express his gratitude and thanks to Professor Amitabha Ghosh, the then Director of IIT Kharagpur for his constant encouragement at various stages and for his active participation in the workshop.

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A.R. Roy
Organising Secretary

A Tribute to Fred Hoyle,
who contributed so much toward our understanding of Mach's Principle...

I had the privilege of working with Fred Hoyle on ideas inspired by Mach's Principle. It is an honour to be asked to dedicate this book, based on the proceedings of a workshop on Mach's Principle, to his memory.

Fred and I had become interested in Mach's principle in 1962, when we discovered that if there is a scalar field generating matter, then such a process drives the universe toward a homogeneous and isotropic steady state. In this case all initial inhomogeneities are wiped out. This result explained why the local inertial frame is such that with respect to it the distant parts of the universe are non-rotating, an observation which was responsible for Machian ideas.

However, in 1961 Fred got interested in the Wheeler-Feynman theory in the cosmological context. In a way the theory, which is the relativistically invariant way of describing action-at-a-distance electrodynamics, could be called the Mach's Principle of electrodynamics. During the years 1963-71, Fred and I worked on making the Wheeler-Feynman theory generally covariant and also developed its full quantum version. The notion of an 'Electromagnetic Response of the Universe' to any local electromagnetic experiment comes through powerfully in such a framework—a notion which, I am sure, would have delighted Ernst Mach.

Finally, while carrying out this analysis, Fred and I also began to wonder if the general relativistic framework could be augmented analogously for gravitation. Indeed the answer to this inquiry lay in a definition of inertia as an action-at-a-distance phenomenon, with the mass of a particle being defined as a sum of inertial contributions of all other particles in the universe. This formulation at the starting end embodied Mach's philosophy and at the other end, led to the equations of general relativity. Fred was particularly happy that this brought together two leading thinkers, Mach and Einstein within a single framework. This new theory of gravity opened up the possibility of explaining the puzzling phenomena of anomalous redshifts.

Last year the world of Science lost a highly creative personality when Fred Hoyle passed away. Mach's Principle was something that had interested him deeply, and he would have enjoyed attending the Kharagpur Workshop. It is fitting that the Workshop was dedicated to him, and it therefore gives me great pleasure to dedicate these proceedings to his memory.

Jayant Narlikar
Ganeshkhind, Pune

The Mach Principle and the Origin of Inertia from General Relativity

Mendel Sachs*

There has been a great deal of discussion during the 20th century on the possible entailment of the Mach principle in general relativity theory. Is it a necessary ingredient? Additionally, there has been the question of the origin of the inertia of matter in general relativity—does inertia originate from the foundation of general relativity theory as an underlying theory of matter? I wish to demonstrate in this lecture that indeed both of these features of matter are intimately related to the conceptual and mathematical structures of the theory of general relativity.

The Theory of General Relativity

The first thing that we must do, then, is to clearly define terms. What do we mean by "the theory of general relativity"? I should like to preface this discussion with the comment that the title of the theory of relativity should be: "the theory of general relativity" (or the "theory of special relativity") rather than the more commonly used title: "the general theory of relativity," (or the "special theory of relativity") since it is the 'relativity' that is general (or special) and not the theory! There is indeed one theory of relativity, whether it is in the 'special' or the 'general' form, based on the single 'principle of covariance' (also called the 'principle of relativity'). The adjectives 'special' or 'general' refer to the types of relative motion of the frames of reference in which the laws are to be compared from the perspective of any one of them. When the relative motion is inertial, we have special relativity and when it is generally nonuniform we have general relativity. Thus it is the 'relativity' that is special or general, not the theory—which is a single concept!

The 'principle of covariance' is the underlying axiom that defines this theory. It is an assertion of the objectivity of the laws of nature, asserting that their expressions are independent of transformations to any frame of reference in which they are represented, with respect to any arbitrary observer's perspective (frame of reference). This implies an entailment of all possible frames of reference; thus it implies that any real system of matter is a *closed system*. Of course, when any local component of the closed system is sufficiently weakly coupled to the rest of the system, say to the rest of the universe or to any smaller subsystem of matter, then one may use the mathematical approximation in which all that there is to represent, mathematically, is the localized material system. In the next approximation, the rest of the system could perturb it. But for the actual *unapproximated* closed system, the implication is that there is no singular, separable 'thing' of matter. Any constituent matter is always

* Department of Physics, University at Buffalo, State University of New York

relative to other components that together with it make up the entire closed system—not as singular 'parts', but rather as modes of a single continuum.

In fundamental terms, then, the principle of covariance implies, ontologically, a *holistic model*, wherein there are no individual, singular, separable things; the closed system is rather a single system without independent parts! This is also an implication of the definition of the inertial mass of matter, according to the *Mach principle*, which we will discuss in detail later on.

The model of matter we have come to, then, from the principle of covariance of the theory of general relativity, is one of *holism*. What we observe as individual separable 'things', that we call 'elementary particles' or 'atoms' or 'people' or 'galaxies', are really each correlated modes of a single continuum. The peaks of these modes are seen to move about and to interact with each other. But indeed they are not independent, separable things, as they are all correlated through the single matter continuum, of which they are its manifestations. This single continuum is, in principle, the universe.

The Mach Principle

We have seen that the qualities of localized matter, such as the inertial mass or electric charge of 'elementary particles', are really only measures of their interactions within a closed system of matter, between these entities and the rest of the system. Thus their values are dependent, numerically, on the rest of the matter of the closed system, of which they are elementary, inseparable constituents. Their masses and electric charges are then measures of coupling within a closed system, not intrinsic properties of 'things' of matter. The dependence of the inertial mass of localized matter, in particular, on the rest of the matter of the 'universe', is a statement of the Mach principle.

It should be emphasized, however, that what Mach said about this was not the commonly stated definition of the principle. The latter is the assertion that only the distant stars of the universe determine the mass of any local matter. In contrast to this, in his *Science of Mechanics* [1] Mach said that *all* of the matter of the universe, *not only the distant stars*, determines the inertial mass of any localized matter.

I have found in my research program in general relativity, that the primary contribution to the inertial mass of any local elementary matter, such as an 'electron', are the nearby particle-antiparticle pairs that constitute what we call the 'physical vacuum'. (The main developments of this research are demonstrated in my two monographs: *General Relativity and* Matter [2], and *Quantum Mechanics from General Relativity* [3]). A prediction of this research program is that the main influence of these pairs on the mass of, say, an electron comes from a domain of the 'physical vacuum' in its vicinity, whose volume has a radius that is the order of 10^{-15} cm. Of course, the distant stars, billions of light-years away, also contribute to the electron's mass, though negligibly, just as the Sun's mass contribution to the weight of a person on Earth is negligible compared with the Earth's influence on this person's weight! Nevertheless, it was Mach's contention that in principle *all* of the matter of the closed system—the nearby as well as far away constituents—determines the inertial mass of any local matter.

Newton's Third Law of Motion

I believe that the first indication in physics of the holistic view of a closed material system came with *Newton's third law of motion*. I see this law as a very important precursor for the holistic aspect of Einstein's theory of relativity. The assertion of this law is that for every action (force) exerted on a body A, by a body B (that is located somewhere else), there is an equal quantity of (reactive) force exerted by A, oppositely directed, on B. According to this law of motion, then, the minimal material system must be the two-body system A-B. A or B, as individual, independent 'things', then loses meaning since, with this view, the limit in which A (or B) is by itself an entity in the universe does not exist!

One other mathematical feature (that was not noted by Newton) that is implied by his third law of motion is that the laws of motion of matter must be fundamentally *nonlinear*. For if A's motion is caused by a force exerted on it by B, which in turn depends on B's location relative to A, then the reactive force exerted by A on B, according to Newton's third law of motion, causes B to change its location relative to A. Consequently B's force on A is changes. Thus A's motion would be changed from what it was without the reactive force on B. We must conclude, then, that A's motion affects itself by virtue of the intermediate role that is played by B in the closed system A-B. The mathematical implication of this effect is in terms of *nonlinear laws of motion* for A (as well as for B). Thus we see that, at the foundational level, the model of matter, even in Newton's classical physics of 'things', must be in terms of a closed system that obeys nonlinear mathematical laws of motion.

The Generalized Mach Principle

As we will see later on, the principle of covariance of the theory of general relativity implies that the basic variables of the laws of matter must be continuous, nonsingular fields, *everywhere*. The laws of matter must then be a set of coupled, nonlinear field equations for all of the manifestations of the closed continuum that in principle is the 'universe'. Thus we see here how the Mach principle is entirely intertwined with the theory of general relativity, regarding the logical dependence of the inertial mass of local matter on a closed system.

The theory of general relativity goes beyond the Mach principle. It implies that all of the qualities of local matter, not only its inertial mass, are measures of dynamical coupling between this 'local' matter and the rest of the closed material system, of which it is a constituent. I have called this "the generalized Mach principle" [2]. Thus the foundational aspects of the theory of general relativity imply an ontological view of holism wherein all remnants of 'atomicity' are exorcised. With this view, the 'particle' of matter, as a discrete entity, is a fiction. What these 'things' are, in reality, are manifestations (modes) of a single matter continuum.

Let us now discuss the role of space and time in general relativity theory. We will then go on to show how the inertial mass of elementary matter emerges from the field theory of general relativity. Finally, it will be seen how the formal expression of quantum mechanics (the Hilbert space formalism) emerges as a linear approximation for a nonlinear, generally covariant field theory of the inertia of matter.

The Role of Space and Time in Relativity Theory

The assertion of the *principle of covariance* entails two scientific (*i.e.*, in principle refutable) assertions. One is the existence of laws of all of nature. This is the claim that for every effect in nature there is a logically connected cause. This assertion is sometimes referred to as the 'principle of total causation'. These relations between causes and effects are the laws of nature that the scientists seek.

The second implication of the *principle of covariance* is that the laws of nature can be comprehended and expressed by us. This is, of course, not a necessary truth. But as *scientists*, we have faith in its veracity. The *expressions* of the laws of nature are where space and time come in. In this view, space and time are not entities in themselves. Rather they provide the 'words' and the logic of a language that is invented for the sole purpose of facilitating an expression of the laws of nature. *It is important to know that the concepts entailed in the laws of nature underlie their language expressions—in one expression or another*.

The space and time parameters and their logic then form an underlying grid in which one maps the field solutions of the mathematical expressions of the laws of nature. The logic of the languages of the laws of nature is in terms of geometric and algebraic relations, as well as topological relations in some applications. With the assumption that a space and time grid forms a continuous set of parameters, the solutions of the laws of nature are then continuous functions of these parameters. These are the 'field variables'. They are the solutions of the 'field equations', field relations that are continuously *mapped* in space and time. According to the *principle of covariance*, the field equations must maintain their forms when transformed to *continuously connected* spacetime frames of reference.

It might be mentioned here, parenthetically, that there is no logical reason to exclude a starting assumption that the language of spacetime parameters is a discrete, rather than a continuous grid of points. In this case the laws of nature would be in the form of difference equations rather than differential equations. However, the implications of the spacetime parameters as forming a continuum, in the expressions of the laws of nature, as continuous field equations, agrees with all of the empirical facts about matter that we are presently aware of. Thus, we assume at the outset that the spacetime language is indeed in the form of a continuous set of parameters. Its geometrical logic in special relativity is Euclidean and in general relativity it is Riemannian. The algebraic logic is in terms of the defining symmetry group of the theory of relativity; it is a *Lie group*—a set of continuous, analytic transformations. The reason for the requirement of analyticity of the transformation group will be discussed in the next paragraph. The Lie group in special relativity is the 10-parameter Poincaré group, in general relativity it is the 16-parameter Einstein group.

A requirement of the spacetime language, stressed by Einstein, as mentioned above, is that the field solutions of the laws of nature—the solutions of the 'field equations'—should be *regular*. This is to say, they should not only be continuous but also analytic (continuously differentiable to all orders, without any singularities) *everywhere*. I am not aware that Einstein gave any explicit reason for this requirement in his writings. However, I believe that it can be based on the *empirical requirement* that the (local) flat spacetime limit of the general field theory in a curved spacetime, must include laws of conservation—of energy, linear momentum and angular momentum. For, according to *Noether's theorem* [4], the analyticity of the field solutions is a necessary and a sufficient condition for the existence of these conservation laws. Strictly, there are no conservation laws in general relativity because, covariantly, a 'time rate of change' of some function of the spacetime coordinates in a curved spacetime cannot be separated from the rest of the formulation that can go to zero. Thus, the *laws of conservation* apply strictly only to the local domain. The conservation laws are then a local limit of global laws in general relativity. In the latter global field laws, the time rates of change can no longer be separated, by itself, from a four-dimensional differential change of functions mapped in a curved spacetime. That is to say, in the curved spacetime the continuous transformations of a purely time rate of change of a function of the space and time coordinates, from its frame of reference where it may appear by itself, to any other continuously connected frame of reference, leads to a mixture of space and time differential changes. In this case we cannot refer to an *objective* conservation (in time alone) of any quantity, in the curved spacetime.

Thus we see that, based on the foundations of the theory of general relativity, we have a closed, nonsingular, holistic system of matter. It is characterized by the continuous field concept wherein the laws of nature are expressed in terms of nonlinear field equations that maintain their forms under transformations between any continuously connected reference frames of spacetime (or other suitably chosen) coordinates. Their field solutions—the 'dependent variables'—are *regular functions* of the space and time parameters, that is to say they are continuous and analytic (nonsingular) *everywhere*. The space and time parameters and their logical relations form the language of the 'independent variables' in which the field variables are mapped. The generalized Mach principle is then a built-in (derived) feature of this holistic field theory in general relativity.

Inertia and Quantum Mechanics from General Relativity

Thus far I have argued that the (generalized) Mach principle is automatically incorporated in the (necessarily) holistic expression of the theory of general relativity, as a general theory of matter. I now wish to show how, in particular, the inertial mass of matter enters this theory of matter in a fundamental way. I will try to avoid, as much as possible, the mathematical details of this derivation. They are spelled out in full in my two monographs [2,3].

In my view, the revolutionary and seminal experimental discovery about matter that relates to the basic nature of its inertia was made 75 years ago, when it was seen that, under particular conditions, particles of matter, such as electrons, have a wave nature. These were the experimental discoveries of electron diffraction by Davisson and Germer, in the US, and independently by G.P. Thomson, in the UK [5]. What they observed was that electrons scatter from a crystal lattice with a diffraction pattern, just as the earlier observed X-radiation does. The 'interference fringes' of the diffraction pattern emerge when the momentum, p, of the electron is related to the *de Broglie wavelength* $\lambda = h/p$, where h is Planck's constant, and the magnitude of p is such that λ is the order of magnitude of the lattice spacing of the diffracting crystal. (This relation between a (discrete) particle variable—its momentum p—and a (continuous) wave variable—its wavelength λ—was postulated by Louis de Broglie, three years before the experimental discovery. [6])

The (discrete) particle, electron, was discovered 25 years earlier by J.J. Thomson (the father of G.P. Thomson) is his cathode ray experiments. Yet, the conclusion about the discreteness of the electron from the cathode ray experiment was indirect. This is because one never sees a truly discrete object (in any observation)! What one sees, such as in J.J. Thomson's experiment, is a localized but *slightly smeared* 'spot' on the phosphorescent face of the cathode ray tube. One then extrapolates from this 'spot' to the existence of an actual discrete point where the electron is said to land on the screen. Nevertheless, a close examination of this *smeared spot* would reveal that inside of it, there is indeed a diffraction pattern! Thus, another possible interpretation of the experiments whereby one thinks that one is seeing the *effects* of a discrete particle is that what is actually seen is a 'bunched' continuous wave—that there is no discrete particle in the first place!

The discovery of the wave nature of the electron was a momentous and revolutionary discovery for physics. It signified a possible *paradigm change* in our ontological view of matter, from the *atomistic*, particularistic model that has held since the ancient times, to a continuum, *holistic* model. In the former view, macroscopic matter is viewed as a collection of singular, elementary bits of matter that may or may not interact with each other to effect the physical whole. In contrast, in the continuum, holistic view, there is a single continuous matter field. What is thought of as its individual constituents is in this view a set of manifestations (modes) of this continuum, that is, in principle, the universe! These manifestations may be electrons, or trees or human beings or galaxies, *etc.* They are all correlated aspects of a single continuum—they are of its infinite set of modes, rather than things *in it*.

In the 1920s, when the continuous wave nature of the electron was discovered, the physics community was not willing to accept this paradigm change, from particularity to holism and continuity of the material universe. Instead, mainly under the leadership of N. Bohr, M. Born and W. Heisenberg, (the *Copenhagen school*), they opted to declare a philosophical view of *positivism*. The view was to assert that if an experiment, using macroscopic equipment, should be designed to look at micromatter, such as the electron, as a (discrete) particle, as in the cathode ray experiment, this is what the electron would be then. But if a different sort of experiment were designed to look at the electron as a (continuous) wave, as in the electron diffraction study, this is what it would be under those circumstances. In other words, the type of measurement that is made on it by a macroscopic observer determines the nature of the electron (or any other material elementary particle), even though the continuous wave and discrete particle views logically exclude each other! This *positivistic* epistemological concept claims that all that can be claimed to be meaningful is what can be experimentally verified at the time a measurement is carried out. Thus it is said that both the 'wave' aspect and the 'particle' aspect of the electron are true, though in different types of measurements. This is called "wave-particle dualism." It is the basis of the theory called "quantum mechanics," that was to follow for describing the domain of elementary particles of matter.

Inertial Mass from General Relativity

The *correspondence principle* has been an important heuristic in physics throughout its history. I now wish to use this principle in order to show that the most primitive expression of the laws of inertial mass can be seen in a generalization in general relativity of the quantum mechanical equations in special relativity. We will then extend the quantum mechanical equations in special relativity to derive the field equations for inertia in general relativity.

The equations we start from are the *irreducible* form of quantum mechanics in special relativity—the two-component spinor form (called the Majorana equations). This is irreducible in terms of the underlying symmetry group of special relativity—the Poincaré group. The latter is a set of only continuous transformations (*i.e.*, without any discrete reflections in space or time) that leave the laws of nature covariant in all inertial frames of reference, from the perspective of any one of them. It is the following set of two coupled two-component spinor equations: (units are chosen with $h/2\pi = c = 1$)

$$\left(\sigma^{\mu}\partial_{\mu}+I\right)\eta=-m\chi \quad (1a)$$

$$\left(\sigma^{\mu *}\partial_{\mu}+I^{*}\right)\chi=-m\eta \quad (1b)$$

To restore reflection covariance, one may combine the two spinor field equations (1ab) to yield the single four-component Dirac equation in terms of the bispinor solution, where the top two components are $(\eta+\chi)$ and the bottom two components are $(\eta-\chi)$.

But the more primitive form of the quantum mechanical equations in special relativity, based on the irreducible representations of the underlying Poincaré symmetry group—a continuous group without reflections—is in terms of the coupled two-component spinor equations (1ab).

In the wave equation (1a) I is the interaction functional that represents the dynamical coupling of all other matter components of the closed system to the given matter field (η,χ), in accordance with the (generalized) Mach principle. $\sigma^{\mu}\partial_{\mu}$ is a first order differential operator, $\sigma^{\mu}=(\sigma^{0};\sigma^{k})$, where σ^{0} is the unit 2-matrix and σ^{k} (k = 1, 2, 3) are the three Pauli matrices. (The set of four matrices σ^{μ} correspond with the basis elements of a quaternion.) Thus, the operator $\sigma^{\mu}\partial_{\mu}$ is geometrically a scalar, but algebraically it is a quaternion. I^{*} is the time reversal (or space inversion) of I and $\sigma^{\mu *}=(-\sigma^{0};\sigma^{k})$ is the time reversal of σ^{μ}.

The spinor field equations (1ab) are the *irreducible* form of the quantum mechanical equations in special relativity. In the limit as $v/c\to 0$, where v is the speed of a matter component relative to an observer and c is the speed of light, these equations (and the four-component Dirac equation) reduce to the nonrelativistic Schrödinger equation for wave mechanics.

Our goal is to *derive* the inertial mass of matter m from a theory of matter in general relativity. This is instead of inserting m into the equations, later to have its numerical values adjusted to the data, as it is done in the conventional formulation of quantum mechanics in special relativity. We accomplish this by 1) setting the right-hand sides of equations (1ab) equal to zero and 2) globally extending the left-hand sides of these equations to their covariant expression in a curved spacetime.

Regarding the latter step, we extend the ordinary derivatives of the spinor fields to *covariant derivatives* as follows:

$$\partial_{\mu}\eta\to\left(\partial_{\mu}+\Omega_{\mu}\right)\eta\equiv\eta_{;\mu} \quad (2)$$

where Ω_{μ} is the "spin affine connection" field. It must be added to the ordinary derivative of a two-component spinor in order to make the spinor field (η,χ) integrable in the curved spacetime. Its explicit form is:

$$\Omega_{\mu}=\frac{1}{4}\left(\partial_{\mu}q^{\rho}+\Gamma^{\rho}_{\tau\mu}q^{\tau}\right)q^{*}_{\rho}$$

where $\Gamma^{\rho}_{\tau\mu}$ is the ordinary affine connection of a curved spacetime.[2] The quaternion field $q^{\mu}(x)$ is defined fundamentally in terms of the invariant quaternion metric of the spacetime, $\mathrm{d}s=q^{\mu}\mathrm{d}x_{\mu}$ of the (factorized) Riemannian (squared) differential metric invariant, $ds^2 = g^{\mu\nu}dx_\mu dx_\nu$. The quaternion field q^μ is a 16-component variable that is, geometrically, a four-vector, but each of its components is quaternion-valued. It was found to be a solution of a factorized version of Einstein's field equations. It replaces the metric tensor $g^{\mu\nu}$ of Einstein's formalism.[2] The quaternion q_ρ^* is the quaternion conjugate (time-reversal) to q_ρ.

Thus with $m = 0$ and the global extension of the left-hand side of eq. (1a) as indicated above, the matter field equation becomes:

$$q^\mu \eta_{;\mu} \equiv q^\mu \left(\partial_\mu + \Omega_\mu\right)\eta + I\eta = 0$$

Transposing terms we then have:

$$\left(q^\mu \partial_\mu + I\right)\eta = -q^\mu \Omega_\mu \eta \tag{3}$$

If the explicit inertial mass is to be derived from first principles in general relativity, then using the correspondence principle, compared in the special relativity limit with $m\chi$ in eq. (1a), it must come from the spin-affine connection term on the right side of eq. (3). Indeed a mathematical analysis showed that there is a *mapping* between the time-reversed spinor variables as follows:[2]

$$q^\mu \Omega_\mu \eta = \lambda\left[\exp(i\gamma)\right]\chi \tag{4}$$

where $\lambda = (1/2)[|\det\Lambda_+| + |\det\Lambda_-|]^{1/2}$ is the modulus of a complex function and $\gamma = \tan^{-1}[|\det\Lambda_-|/|\det\Lambda_+|]^{1/2}$ is its argument, where $\Lambda_\pm = q^\mu\Omega_\mu \pm$ h.c., and 'h.c.' stands for the 'hermitian conjugate' of the term that precedes it and 'det' is the determinant of the function.

Finally, applying the requirement of gauge invariance to the field theory, with the gauge transformations:

first kind: $\eta \to \eta \exp\left(-\frac{i\gamma}{2}\right), \quad \chi \to \chi \exp\left(\frac{i\gamma}{2}\right)$

second kind: $I \to I + \frac{i}{2} q^\mu \partial_\mu \gamma,$

the phase factor in eq. (3), (using eq. (4) on the right-hand side) is automatically transformed away. The field equation (3)—the global extension in general relativity of eq. (1a)—then takes the form:

$$\left(q^\mu \partial_\mu + I\right)\eta = -\lambda\chi \tag{5a}$$

Its time-reversed equation (the global extension of (1b)) is:

$$\left(q^{\mu *} \partial_\mu + I^*\right)\chi = -\lambda\eta \tag{5b}$$

Gauge covariance is a necessary and sufficient condition for the incorporation of the laws of conservation in the field laws, in the asymptotically flat spacetime limit of the theory. Thus the empirical facts about the existence of conservation laws of energy, linear and angular momentum, in the (asymptotically flat) special relativity limit of the theory, dictate that gauge covariance is a necessary symmetry, in addition to the continuous group symmetry in general relativity (the “Einstein group”) of the field theory.

We see, then, in using the correspondence principle, comparing the generally covariant field equations (5ab) with the asymptotically flat special relativity limit (1ab), that the function λ plays the role of the inertial mass of matter, m. Thus we may interpret the generally covariant equations (5ab) as the defining field relations for the inertial mass of matter.

As we asymptotically approach the flat spacetime limit, equations (5ab) approach equations (1ab) and the generally covariant solutions (η,χ) approach the flat spacetime elements of the Hilbert function space $\{\eta_1,\ldots\eta_k, ..; \chi_1,\ldots\chi_k, \ldots\}$, with the condition of square integrability (and normalization) imposed on these spinor variables. In this (Hilbert space) limit of the formalism, the expectation values of the positive-definite field λ is the set of squared eigenvalues (the mass spectrum formula):

$$\lambda_k^2 = \left| < \eta_k \right| \left(-q^{\mu *}\Omega_\mu^+ \right)_a \left(q^\mu \Omega_\mu \right)_a \left| \eta_k > \right|$$

where the subscript ‘*a*’ denotes the asymptotic value of the term in parentheses as the flat spacetime limit is *approached*, and the ‘dagger’ superscript denotes the ‘hermitian conjugate’ function.

A few points about the inertial mass field λ should be noted. First, in the actual flat spacetime limit, the spin affine connection Ω_μ vanishes so that in this limit $\lambda_k = 0$. The vanishing of the spin affine connection field occurs only for the vacuum—the absence of all matter, everywhere. Thus the derivation from general relativity of the vanishing of the inertial mass $\lambda_k = 0$, where there is no other mass to couple to, is in accordance with the statement of the Mach principle.

A second important point is that, as the modulus of a complex function, λ is positive-definite. This implies that any macroscopic quantity of matter, being made up of these ‘elementary’ units of matter with positive mass, must itself have only positive mass. The implication is that, *in the Newtonian limit of the theory*, the gravitational force has only one polarization. It is either under all conditions repulsive or under all conditions attractive. In view of the locally observed attractive Newtonian gravitational force, it must then under all circumstances be attractive. This conclusion is in agreement with all of the empirical data on Newton’s force of gravity. It has never been derived before from first principles, either in Newton’s classical theory of gravitation or in the tensor formulation of Einstein’s theory of general relativity. This result implies that in the Newtonian limit of the theory there is no anti-gravity, *i.e.*, no gravitational repulsion of one body from another.

The Oscillating Universe Cosmology

In the generally curved spacetime of the theory of general relativity, the role of the gravitational force is not directly related to the mass of matter, as it is in

Newton's theory. As we see in the geodesic equation in general relativity (the equation of motion of a test body) the 'force' acting on a body relates to the 'affine connection' of the curved spacetime. The latter is a *non-positive-definite* field. Thus, the general prediction here is that under particular physical circumstances (of sufficiently dense matter and high relative speeds between interacting matter) the 'gravitational force' can be repulsive. Under other physical circumstances (of sufficiently rarefied matter density and low relative speeds of interacting matter) the gravitational force can be attractive.

This result in general relativity, applied to the problem of the universe as a whole, implies an oscillating universe cosmology. At one inflection point, the matter components of the universe begin to repel each other, dominating the attractive components of the general gravitational force, thence leading to the *expansion phase* of the universe, with the matter continuously decreasing its density. Then, when the matter of the universe becomes sufficiently rarefied, and relative speeds between interacting matter is sufficiently low, another inflection point is reached where the attractive component of the gravitational force begins to dominate and initiates the *contraction phase* of the universe. This continues with ever-increasing matter density until the conditions are ripe again for the repulsion of matter to dominate. The universe then reaches the inflection point once again for a turn around from contraction to expansion. The expansion phase starts again, until the next inflection point when the attractive force takes over once more, and so on *ad infinitum*.

The answer to the question: How did the matter of the universe get into the maximum instability stage at the last 'big bang' (the beginning of the present cycle of the oscillating universe) is then: Before the last expansion started, the matter of the universe was contracting toward this physical stage. This view of the oscillating universe denies the idea of a mathematical singularity at the inflection point—at the *beginning* of any particular cycle of the oscillating universe—that is commonly believed by present-day cosmologists who adhere to the 'single big bang' model.

This cosmology also rejects the present-day model wherein there is an absolute time measure—the 'cosmological time'—measuring the time since the last big bang happened. The latter view of absolute time is incompatible with Einstein's theory of relativity, wherein there is no absolute time measure. It is replaced in relativity theory with a totally covariant description of the universe wherein the time measure (as the space measure) is a function of the reference frame from which it is determined. The universe itself cannot be expressed in terms of an absolute reference frame. In the theory of relativity, there are no absolute frames of reference or time measures.

Summary

I have argued that the basis of the theory of general relativity implies that any material system is necessarily a *closed system*. This in turn implies a *holistic* *model* of matter, whereby there are no separable, individual particles of matter. It is a view that is compatible with a continuum, rather than in terms of a collection of discrete particles of matter.

The first empirical evidence for this continuum view was the discovered wave nature of matter in the experiments on electron diffraction in the 1920s. The "matter waves" (as they were named by their discoverer in theory, Louis de Broglie) may then be viewed as the infinite number of *correlated* manifestations (modes) of a single continuous whole—in principle the universe. This implies that the inertial mass of any local matter is not intrinsic, but rather it is dependent on all of the other matter of the closed universe (the Mach principle). It also follows that *all other* physical properties of matter, as well as inertial mass, such as electric charge, are not intrinsic, but are also measures of coupling within the closed system of matter. This is the "generalized Mach principle."

It was seen that the formal expression of quantum mechanics in special relativity relates, by means of a *correspondence principle*, to a generally covariant field theory of inertia, in general relativity. The formal expressions of quantum mechanics in special relativity, in accordance with the irreducible representations of the Poincaré group, are a set of two coupled two-component spinor equations. Each is a time reversal (or space reflection) of the other. The mass parameter is conventionally inserted in a way that appears as a mapping between the two sorts of (reflected) spinors. Removing the mass term in the latter expression, and globally extending the rest of the equation to a curved spacetime, based on the symmetry of the Einstein group of general relativity, leads to the covariant field theory of inertial mass. With the added symmetry of gauge invariance, the field equation is recovered with a *mass field* appearing where the mass parameter was initially inserted.

The asymptotic limit toward the flat spacetime of the latter (nonlinear) field equations in general relativity for inertia is the formal structure of (linear) quantum mechanics—as a linear approximation. This analysis has then led to the derivation of quantum mechanics (the Hilbert space structure) as a linear approximation for a generally covariant field theory of the inertia of matter.

References

[1] E. Mach, *The Science of Mechanics* (Open Court, La Salle, 1960), p. 267, *transl.* T. J. McCormack.

[2] M. Sachs, *General Relativity and Matter* (Reidel Publishing Co., Dordrecht, 1982).

[3] M. Sachs, *Quantum Mechanics from General Relativity* (Reidel Publishing Co., Dordrecht, 1986).

[4] Noether's theorem is derived in E. Noether, *Goett. Nachr*. **235** (1918). It is explicated further in: C. Lanczos, *The Variational Principles of Mechanics* (Toronto, 1966), Appendix II and in R. Courant and D. Hilbert, *Mathematical Methods of Physics*, Interscience, 1953. The application to fields is demonstrated in N.N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields*, 3rd edition, Interscience, 1980, Sec. 1.2.

[5] C.J. Davisson and L.H. Germer, *Phys. Rev*. **30**, 705 (1927), G.P. Thomson, *Proc. Roy. Soc* (London) **A117**, 600 (1928).

[6] L. de Broglie, *Recherches d'un Demi-Siecle* (Albin Michel, Paris, 1976).

Extension of Mach's Principle and Cosmological Consequences

Amitabha Ghosh*

One of the primary features of Mach's principle is that inertial force is simply the manifestation of interaction of an accelerating object with matter present in the rest of the universe. An attempt to quantify this principle was made by Sciama when he proposed a model of inertial induction which leads to an extra force (over and above the Newtonian gravitational interaction) when a mass accelerates with respect to another massive object. The model is partially successful as it can be shown that the sum total of the force due to inertial induction of an accelerating object of mass *m* and acceleration *a* with the matter in the rest of the universe is of the order of *ma*. However, an exact equivalence of gravitational and inertial mass can only be achieved for a certain precise combination of independent universal parameters. In this paper an extension of Mach's principle has been proposed which leads to inertial induction between two objects that also depends on relative velocity. It has been demonstrated that this results in a feedback mechanism, and exact equivalence is automatically achieved without the need for fine-tuning of universe parameters. Furthermore, a cosmic drag (of very small value) acts on bodies moving with even constant velocity with respect to the mean rest frame of an infinite, quasi-statistic universe.

1 Introduction

One of the most elusive concepts in the field of physical sciences is "Mach's Principle." From the dawn of the Newtonian era the question that has continued to bother the scientists and philosophers is whether motion of an object has meaning only in relation to other objects in the universe: is an object's motion in empty space also meaningful? In the other words, is the inertia of an object an intrinsic property of matter (irrespective of the presence of other matter in the universe) or is it nothing but the manifestation of the interaction of the moving object with the other matter present in the rest of the universe? Newton proposed his famous bucket experiment [1] to demonstrate the absolute nature of rotational motion with space, but some contemporary philosophers felt that the effect of relative motion of water and the bucket was too small to be detected. In the late 19th century Ernst Mach [2] reopened the question, making a great impression on Einstein. The issue remains unsettled to this day.

2 Mach's Principle

"Mach's Principle" has thus remained a philosophical principle. It proposes that the inertia of an object to acceleration is due to the resistance generated by its interaction with the matter present in the rest of the universe. Any further progress requires a quantitative law to implement "Mach's Principle." The first attempt in this direction was made by Sciama in 1953 [3]. According to his

* Indian Institute of Technology, Kanpur, India – 208016

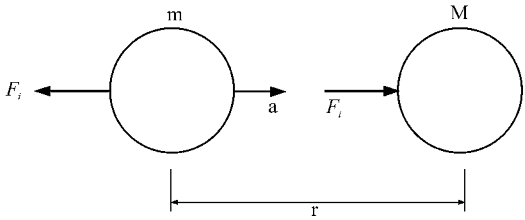


Figure 1: Acceleration dependent inertial induction

quantified rule for Mach's Principle, the interactive force between two objects (Fig. 1) moving relative to each other is given by

$$F_i = G\left(\frac{mM}{c^2 r}\right)a \tag{1}$$

where F_i is the interactive force over and above the Newtonian gravitational force, M, m are the gravitational masses of the two bodies, G is the gravitational constant, c is the velocity of light, r is the distance between the two objects, and a is the acceleration of m with respect to M, as shown in Fig. 1.

The phenomenon depicted in Fig. 1 was termed "inertial induction" by Sciama. Summing the forces due to inertial induction of an accelerating object (gravitational mass m) with all matter present in the rest of the universe we get

$$F_i = \Sigma\Delta F_i = \Sigma G\left(\frac{m\Delta M}{c^2 r}\right) \quad a = \lambda ma, \tag{2}$$

where $\lambda \sim 1$. Thus Newton's second law is derived approximately, and this is considered to be strong evidence in favour of Mach's Principle. However, two major arguments are usually raised against Mach's Principle.

According to the critics of Mach's Principle, this mechanism involves instantaneous action-at-a-distance. This is so because an acceleration is instantaneously felt by all matter in the rest of the universe. Moreover, exact equivalence of the gravitational and inertial masses requires λ exactly equal to unity, not approximately. This is possible if the various parameters of the universe, and G are exactly tuned in a manner so that $\lambda = 1$, which is very doubtful.

The first objection, on the grounds of instantaneous action-at-a-distance, can be answered easily. Since the matter in the rest of the universe already exists, whatever influence it has at the location occupied by a particle also exists. So the accelerating object feels this influence instantaneously. However, when a particle is given an acceleration, its effect on a distant mass will be felt later due to a finite speed with which the "inertial induction" propagates. (We assume this speed to be the same as the speed of light, c).

The second objection is a more serious one, and has deep cosmological and philosophical implications. The only way an exact equivalence can be achieved without fine-tuning of the various parameters and properties of the universe is to have a servomechanism, or feed-back mechanism, associated with the inertial induction phenomenon.

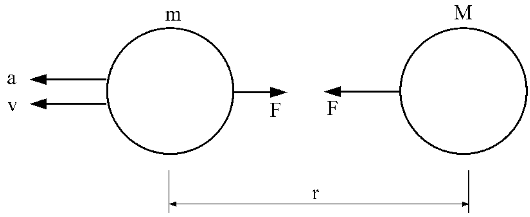


Figure 2: Velocity dependent inertial induction

It should be noted here that "inertial induction" is nothing but a dynamic gravitational phenomenon, where the force between the gravitating massive objects depends not only on the distance between them, but also on their relative motion. Since the term $G(mM/c^2r)a$, introduced by Sciama, represents the effect of relative acceleration, it can be considered to represent a phenomenon we may call "acceleration dependent inertial induction."

In the next section an extension of Mach's Principle is proposed which can provide the needed feed-back mechanism. A number of interesting consequences also result from the proposed extension.

3 Velocity Dependent Inertial Induction: Extension of Mach's Principle

In the original version of Mach's Principle, quantified by Sciama in his model of inertial induction, an extra force (of gravitational nature) acts depending on the relative acceleration. When summed over for all the matter present in the universe, this force is assumed to give rise to the well-known force *ma*. However, there is also an interactive force depending on the relative velocity of the two interacting objects. Thus, we have proposed [4] that the total gravitational interactive force between two objects (Fig. 2) can be represented by

$$F = G\left(\frac{mM}{r^2}\right) + G\left(\frac{mM}{c^2r^2}v^2\right) + G\left(\frac{mM}{c^2r}\right)a$$

where the first term on the L.H.S. represents the Newtonian gravitational attraction on *m* by *M*. The third term represents the acceleration-dependent inertial induction on *m* due to its relative acceleration *a* with respect to *M*. The second term represents a proposal for velocity-dependent inertial induction on *m* due to its relative velocity with respect to *M*. Thus, in a sense the scope of Mach's Principle is extended to the interaction of a body with the matter present in the rest of the universe which also depends on the body's relative velocity with respect to the mean-rest-frame of the universe.

The last sentence of the above paragraph implies that there exists a mean-rest-frame for the matter present in the universe. Consequently, we must conceive of a quasi-static infinite universe without any Hubble expansion. In such a universe, objects move with random finite velocity only. Some interesting features of this universe will emerge later.

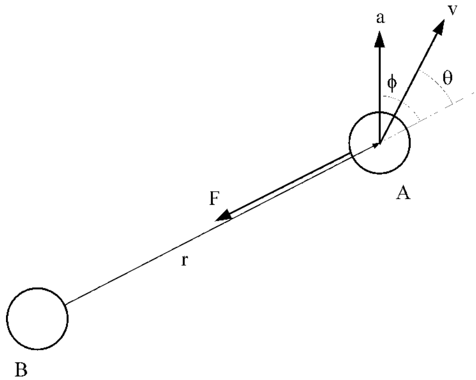


Figure 3: Interaction of two particles

4 Derivation of Force Law from Extended Mach's Principle

The basic idea behind the Extended Mach's Principle, which includes velocity dependent inertial induction among material objects in relative motion, was presented in the previous section. However, it would be desirable if the basic form of dynamic gravitational interaction (called here inertial induction) between two particles could also take care of parameters such as the directions of kinematic quantities. Fig. 3 shows particles A and B, while equation (3) shows the total force [4]

$$\overline{F} = -G\frac{m_A m_B}{r^2}\hat{u}_r - G\frac{m_A m_B}{c^2 r^2}v^2 f(\theta)\hat{u}_r - G\frac{m_A m_B}{c^2 r}af(\phi)\hat{u}_r \tag{3}$$

where m_A and m_B are the gravitational masses of A and B, respectively, v and a are the magnitudes of the relative velocity and acceleration of A with respect to B, $\hat{u}_r$ is the vector along r, $f(\theta)$ and $f(\phi)$ represent the inclination effects (with θ and ϕ being the angles of v and a with respect to $\hat{u}_r$ respectively), and c is the speed of light. We use $f(\theta) = \cos\theta|\cos\theta|$ and $f(\phi) = \cos\phi|\cos\phi|$. To derive the law of motion for a particle with gravitational mass m we treat the universe as infinite and quasi-static with an average mass density ρ, and find the total inertial induction of the particle due to the matter present in the rest of the universe. The particle is considered to be moving with velocity v and acceleration a with respect to the mean-rest-frame of the universe. We obtain the total force as follows [4]:

$$F = -\hat{u}_v\left(mv^2c\right)\int_0^\infty\left(\frac{\chi G\rho}{c}\right)dr - \hat{u}_a\left(\frac{ma}{c^2}\right)\int_0^\infty \chi G r_1 \rho dr_1 \tag{4}$$

where

$$\chi = 4\pi\int_0^{\pi/2}\sin\theta\cos\theta f(\theta)d\theta = 4\pi\int_0^{\pi/2}\sin\phi\cos\phi f(\phi)d\phi$$

The first term of the R.H.S. of (4) represents a cosmic drag force on the particle because of its velocity with respect to the mean rest frame of the universe and the second term represents the force on *m* because of its acceleration. Since the drag acts on everything, we may assume that even gravitons (the agents that carry the gravitational interaction) are subjected to this drag, implying that *G* will decrease with distance. If we assume

$$\int_0^\infty \left(\frac{\chi G \rho}{c}\right) dr = k \,, \tag{5}$$

then (4) takes the following form:

$$F = -\hat{u}_v k \left(\frac{mv^2}{c}\right) - \hat{u}_a \left(\frac{ma}{c^2}\right) \int_0^\infty \chi G r_1 \rho \, dr_1 \quad . \tag{6}$$

If gravitation is assumed to propagate with the speed *c* and possess an energy *E* (*i.e.*, an equivalent mass of E/c^2), the magnitude of the drag on the graviton will be kE/c. If dE is the change in energy while the graviton travels a distance dr then

$$dE = -\left(\frac{kE}{c}\right) dr \,.$$

Thus,

$$E = E_0 \exp\left(-\frac{kr}{c}\right)$$

where E_0 is the initial energy of the graviton. It is reasonable to assume that the intensity of the gravitational interaction represented by *G* will also vary as

$$G = G_0 \exp\left(-\frac{kr}{c}\right) \tag{7}$$

where G_0 is the local value given by 6.67×10^{-11} m^3 kg^{-1} s^{-2}. Using this in (5) we get

$$\left(\frac{\chi G_0 \rho}{c}\right) \int_0^\infty \exp\left(-\frac{kr}{c}\right) dr = k \,,$$

or

$$\left(\frac{\chi G_0 \rho}{c}\right) \cdot \left(\frac{c}{k}\right) = k \,,$$

and finally

$$k = \left(\chi G_o \rho\right)^{1/2} \tag{8}$$

Using (7) in (6), we find that

$$F = -\hat{u}_v \, k \left(\frac{mv^2}{c}\right) - \frac{\hat{u}_a m a \chi G_0 \rho}{k^2} \,.$$

Substituting from equation (8), we find the force law

$$F = -\hat{u}_v k \left(\frac{mv^2}{c} \right) - ma \, . \tag{9}$$

Thus the acceleration dependent term becomes exactly $-ma$, not requiring any fine-tuning. This exact equivalence of inertial and gravitational masses does not depend on the nature of $f(\theta)$ and $f(\phi)$. Only the value of k depends on χ, which can be determined when the form of $f(\theta)$ is known. With the form $\cos\theta|\cos\theta|$ we have $\chi = \pi$, and

$$k = \left(\pi G_0 \rho\right)^{1/2} \tag{10}$$

Thus, the feed-back mechanism introduced by the proposed velocity-dependent inertial induction (as an extension of Mach's Principle) can attenuate gravity, resulting in Newton's second law of motion with an extra cosmic drag term.

It can easily be shown [4] that this cosmic drag term results in the observed cosmological redshift, with k equivalent to the Hubble constant. Thus, no expansion of the universe is required to explain the cosmological redshift.

5 Concluding Remarks

It is seen that an extension of Mach's Principle can result in exact equivalence of gravitational and inertial masses without fine-tuning, as required in the case of Mach's Principle. The Extended Mach's Principle also explains the observed cosmological redshift quantitatively without the need for universe expansion. It has also been shown that the objection arising from the need for instantaneous action at a distance is not tenable. In deriving the force law there is no need to bring in the phenomenon of instantaneous action at a distance.

With this dynamic gravitational model, the parameter G decreases with distance, as originally proposed by Laplace. The rate at which G decays has also been determined. The Extended Mach's Principle model has been applied to a number of phenomena of a local nature, and in all these cases the predicted extra effects have been found to exist from observations [4].

It is desirable that a suitable terrestrial experiment be planned to verify the presence of the velocity dependent term in equation (1). An attempt should also be made to detect if a secular retardation of the spin rotation of Mars exists, as this model predicts a secular retardation of 1.25×10^{-22} rad s^{-2} for Mars. If the retardation is detected to be present, this will constitute further support for the proposed principle, as it cannot be explained by a tidal phenomenon.

References

Assis, A.K.T., *Relational Mechanics*, Apeiron, Montreal, 1999.

Mach, E., *The Science of Mechanics – A Critical Historical Account of its Development*, Open Court, La Salle, 1960.

Sciama, D.W., On the Origin of Inertia, *Monthly Notices of the Royal Astronomical Society,* Vol. 113, 1953, p.34.

Ghosh A., *Origin of Inertia-Extended Mach's Principle and Cosmological Consequences*, Apeiron, Montreal 2000, Affiliated East West Press, New Delhi, 2002.

The Technical End of Mach's Principle

James F. Woodward[*]

The theoretical motivation for the prediction of mass fluctuations in accelerated objects based on Mach's principle is reviewed. It is pointed out that one of the two predicted fluctuations, normally hopelessly below the level of detectability, can be made quite large in "just so" circumstances. Since this fluctuation is always negative, when driven as a periodic fluctuation, its time-average is non-zero and negative. As such, this effect holds out the possibility of inertia manipulation at a scale with practical consequences. Results of recent experiments with lead-zirconium-titanate (PZT) devices where evidence for the predicted mass fluctuation was sought as a weight shift are reported, together with a description of the check protocols used to eliminate spurious sources of the signals seen. Those results, if not conclusive, are at least promising.

Key Words: Mach's principle, mass fluctuations, wormhole

Introduction

Although many issues of scientific interest surrounding Mach's principle are yet to be resolved, I argue here that core features of the principle are sufficiently well understood to justify their exploration with an eye to possible technological applications. What might those technical applications be? Well, should inertia turn out to be manipulable by some means, one might be able to do things presently regarded as the stuff of science fiction and science fantasy. In particular, one might be able to facilitate rapid spacetime transport in several ways, the most extreme involving the induction of large amounts of "exotic" matter. Although not yet advanced to the stage of straight-forward technical implementation, I relate results of some experiments now in progress, following strict scientific protocols, that suggest such manipulation may one day prove feasible. It goes without saying, of course, that all this depends on the ultimate outcome of those experiments. And they may eventually reveal this to be nothing more than wishful thinking—a foolish pipedream.

What is it that we know about Mach's principle and the origin of inertia that makes experiments with a technological orientation possible? Perhaps the most important and fundamental thing we know is that in a universe like ours—with essentially isotropic matter distribution (at cosmological scale anyway) characterized by a Friedman, Robertson, Walker (FRW) cosmological model—inertial reaction forces are a consequence of the *gravitational* action of chiefly distant matter on local objects when they are accelerated. There are those who will try to tell you that inertial reaction forces are not so caused, that they are caused by the action, say, of the electromagnetic quantum vacuum zero point field (EZPF). Aside from the fact that the EZPF conjecture on the origin of inertial reaction forces is deeply flawed, the fact of the matter is that it

[*] Departments of History and Physics, California State University, Fullerton, CA 92634. e-mail: jwoodward@fullerton.edu

has been known since the work of Derek Raine in the mid-1970s that inertial reaction forces in FRW cosmologies are caused by gravity (Raine, 1975). Equally important, owing to the work of Dennis Sciama, dating back to the early 1950s, we know that inertial reaction forces are the result of a *radiative* interaction, as one would expect from the analogous case in electrodynamics where *accelerations* of electric charges produce propagating electromagnetic waves that carry the chief interaction with distant electric charges (Sciama, 1953; see also J.-F. Pascual-Sanchez, 2000).

The radiative nature of the gravitational forces that yield everyday inertial reaction forces poses something of a problem: Inertial reaction forces are instantaneous. When you push on something, it pushes back immediately. How can that be if the interaction that produces the reaction force is communicated to the thing you push from chiefly the most distant matter in the universe at the speed of light in vacuum? Some have tried to deal with this by claiming that inertial reaction forces can be attributed to "constraint" equations on "initial data" that, being elliptic rather than hyperbolic, act instantaneously across arbitrarily large distances. The only plausible alternative to this scheme is to regard inertial reaction forces as forces of radiative reaction. As is well-known in the case of electrodynamics, since the work of Wheeler and Feynman in the 1940s on "action-at-a-distance" electrodynamics, radiation reaction forces can be accounted for by invoking advanced as well as retarded propagating wave solutions of the field equations where an accelerated local charge interacts with a large, isotropic, distant absorber (Wheeler and Feynman, 1945). This, of course, is precisely the sort of behaviour that one should expect for the gravitational interaction that produces inertial reaction forces if Mach's principle is correct. (Appreciation of the importance of Wheeler-Feynman style action-at-a-distance electrodynamics in physical phenomena, including gravitation, is not new. It was noted long ago by Hoyle and Narlikar, 1974.)

Now, one may think: There is a problem with the radiation reaction picture of inertial reaction forces if they are gravitational in origin. The gravitational radiation given off by any accelerated local object of reasonable mass is incredibly minute. Newton's third law then suggests that the reaction force produced by launching this radiation should be correspondingly minute. Inertial reaction forces are, by comparison, however, gigantic—many, many orders of magnitude larger. Since inertial reaction forces are gravitational forces in general relativity theory, Sciama's analysis reveals them to be radiative, and their instantaneity requires that they be seemingly minuscule radiation reaction forces, we seem to be faced with an insuperable paradox. The problem here, I think, lies not in the consistency of the mathematical formalism. Rather, it is a problem of visualization.

If we take the customary view of the distant matter in the universe as being at rest (ignoring cosmological expansion) and local objects as accelerated with respect to the distant matter, then it is easy to believe that inertial reaction forces cannot be gravitational radiation reaction forces because the gravitational disturbance launched by any acceleration is so small. If, instead, we adopt the view made explicit in Sciama's 1953 paper on Mach's principle and the origin of inertia, where the accelerated local object is regarded as (instantaneously) at rest and the external force on it causes the distant matter in the universe, as viewed from the local object, to appear to accelerate rigidly, then it is quite reasonable to assume that the gravitational action of all of that distant stuff accelerating will produce a force on our local object of the order of magnitude of typical inertial reaction forces. In any event, I am going to assume that inertial reaction forces are forces of gravitational radiation reaction communicated by a gravitational field interaction that displays the action-at-a-distance character of Wheeler-Feynman absorber theory. Why? Because in absorber theory there are phase lags and small time delays that, at least in principle, hold out the hope of getting some purchase on inertia. That purchase may allow us to test Mach's principle. And if we are truly fortunate, it may open the way to technological applications of some interest.

A Bit of Theory

Radiation reaction in electrodynamics is not automatically included in Maxwell's equations. Considerations of conservation of energy and momentum, however, require that it be dealt with since the electromagnetic waves predicted by the theory carry energy and momentum. The launching of such waves by accelerating electric charges, therefore, must produce a reaction force on the charges that preserve the conservation principles. In electrodynamics this problem has been dealt with in several ways: the deformable extended electron model of Lorentz where the reaction force arises from propagation delays as parts of the electron communicate with each other during the acceleration; the point electron of Dirac where the reaction force is computed by demanding that the conservation laws obtain; and the already mentioned action-at-a-distance absorber theory of Wheeler and Feynman with its phase shifts and time-delays. The counterpart of the time-delays in the treatments of Lorentz, and Wheeler and Feynman for Dirac's theory is "pre-acceleration"—electrons begin to accelerate some small time before the force that produces the acceleration acts.

As a soon to be superannuated experimentalist, I do not propose to tackle inertial reaction forces in an elaborate, formal way. So, rather than try to solve the field equations of general relativity theory to recover inertial forces as forces of radiation reaction, I will approach things differently. What we are interested in, given our presumed technological bent, is: Can we affect the masses of things by local operations on them? Ultimately, the only thing we can do locally is to apply forces on objects. Thus, the question to be answered can be restated as: Does the application of forces on objects cause their inertial masses to change? If the forces cause accelerations, inertial reaction forces arise. The inertial reaction forces are caused by the action of the gravitational field of chiefly distant matter, so our question becomes: Does the action of the gravitational field, in generating inertial reaction forces, cause the inertial masses of accelerated objects to change? The simple, if a bit unorthodox way to answer this question is to write the field that produces the inertial reaction force as that inertial reaction force per unit local charge (inertial/passive gravitational mass) and take its divergence to get the local source charge density (active gravitational mass density). If the local source charge density changes as a result of the applied force that produces the acceleration, then the Equivalence Principle insures that the local inertial mass density changes too.

We must do this in a relativistically correct way, so we use the four-vector force to obtain the field strength and take the four-divergence to get the local source charge density. We can simplify the expression produced in this way by noting that the field that causes the inertial reaction forces that arise in response to elementary accelerations is irrotational. One cannot "wind up" the effects. Consequently, the three-vector part of the field can be written as the gradient of a scalar potential. The result of these operations (which—up to a typographical sign error in Equation (3.10)—can be found spelled out in "Twists of Fate: Can We Make Traversable Wormholes in Spacetime" available at <http://chaos.fullerton.edu/~jimw/Twists.pdf>) is the following equation:

$$\nabla^2\phi - \frac{1}{\rho_o c^2}\frac{\partial^2 E_o}{\partial t^2} + \left(\frac{1}{\rho_o c^2}\right)^2\left(\frac{\partial E}{\partial t}\right)^2 = 4\pi G\rho_o \tag{1}$$

In this equation ϕ is the scalar potential of the field, ρ_0 the rest-mass density of the material in the volume element of the accelerated field source and E_0 the corresponding rest-energy density. G is Newton's universal constant of gravity. It must be included on the right hand source side to scale *passive gravitational* mass into *active gravitational* mass. I should also mention that this equation is only exactly true in the instantaneous frame of rest of the local matter that is being accelerated where all of the Lorentz contraction factors are equal to one, and thus disappear. This is not an approximation. But it does mean that if you want to see what the field equation looks like in any other reference frame, these factors must be restored explicitly.

Equation (1) almost looks like a normal inhomogeneous field equation where the d'Alembertian of some quantity, usually a potential, is equated to a local source charge density (or a source current in the case of a vector potential). So it is natural to ask: Is there some way to express the local rest-energy density E_0 that will allow us to put Equation (1) into this customary form? Well, if we invoke the strong form of Mach's principle—which says that the inertial masses of things *per se* arise from their gravitational interaction with chiefly the most distant matter in the universe—then we can do this. Note that it is the strong form of Mach's principle that is used here, not the weak form that only requires that inertial reaction forces arise from gravitational interactions. The strong form of Mach's principle allows us to assert that the local energy density E_0 is just the total *gravitational* potential energy of local matter:

$$E_0 = \rho_0\phi. \tag{2}$$

Is there any reason to take such an assertion seriously? After all, one normally learns that the gravitational potential ϕ is arbitrary up to an additive constant. Clearly, E_o cannot have this property. And in FRW cosmologies it does not. ϕ turns out to be a *locally measured* invariant in those cosmologies, just like the vacuum speed of light c. Indeed, not only does ϕ have the same invariance property as c, it must be equal to c^2 if inertial reaction forces are to be accounted for as gravitational forces (as Sciama [1953] showed now nearly a half-century ago). In consequence, we can re-express Equation (2) in the form:

$$E_o = \rho_o c^2 \tag{3}$$

which we immediately recognize as the well-known relationship between mass and energy that is a consequence of *special* relativity theory.

Using Mach's principle as embodied in Equation (2) we can separate the variables in Equation (1) by substituting $\rho_o\phi$ for E_o. When the time derivatives are computed and we take account of the fact that we may take $\phi = c^2$, after a modest amount of algebra we find that:

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi G\rho_o + \frac{\phi}{\rho_o c^2}\frac{\partial^2\rho_o}{\partial t^2} - \left(\frac{\phi}{\rho_o c^2}\right)^2\left(\frac{\partial\rho_o}{\partial t}\right)^2 - c^{-4}\left(\frac{\partial\phi}{\partial t}\right)^2. \tag{4}$$

Now we have the d'Alembertian of ϕ equal to a source charge density. But the source charge density has several time-dependent terms in it. It is the time-dependent terms that are the ones of interest. Before discussing them, however, we simplify Equation (4) by taking note of the fact that $\phi = c^2$ in the coefficients of the time-dependent terms and setting $c^{-4}(\partial\phi/\partial t)^2 \approx 0$ since, with a c^{-4} coefficient, it will always be minuscule. Equation (4) becomes:

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 4\pi G\rho_o + \frac{1}{\rho_o}\frac{\partial^2\rho_o}{\partial t^2} - \left(\frac{1}{\rho_o}\right)^2\left(\frac{\partial\rho_o}{\partial t}\right)^2. \tag{5}$$

Note that if ρ_o is a constant (as well as an invariant), then the time-dependent terms on the right hand side of Equation (5) vanish and we are left with a classical wave equation for ϕ of standard form; and if we assume that all time derivatives vanish, we recover Poisson's equation for ϕ. So, it would seem, we haven't done anything seriously foolish so far.

Since it is the local, instantaneous passive gravitational/inertial matter density that we are interested in, we note that the right hand side of Equation (5) can be written as $4\pi G\rho(t)$, where $\rho(t)$ is that matter density. From Equation (5) we have:

$$\rho(t) = \rho_o + \frac{1}{4\pi G}\left[\frac{1}{\rho_o}\frac{\partial^2\rho_o}{\partial t^2} - \left(\frac{1}{\rho_o}\right)^2\left(\frac{\partial\rho_o}{\partial t}\right)^2\right]. \tag{6}$$

For convenience of calculation we'll use $\rho_o = E_o/c^2$ to write Equation (6) as:

$$\rho(t) = \rho_o + \frac{1}{4\pi G}\left[\frac{1}{\rho_o c^2}\frac{\partial^2 E_o}{\partial t^2} - \left(\frac{1}{\rho_o c^2}\right)^2\left(\frac{\partial E_o}{\partial t}\right)^2\right]. \tag{7}$$

While the mathematics that has led us to the time-dependent terms in Equations (6) and (7) may be fairly straight-forward, one may wonder if there is any plausible physical basis for them. In view of the small time-delays/phase lags that typically occur in radiation reaction processes, I would argue that these time-dependent terms might reasonably be expected. They must be quite small in all normally encountered circumstances in order not to conflict with experience. Substitution of realistic values for ρ_o and E_o into these equations bears out this expectation.

Experimental and Practical Matters

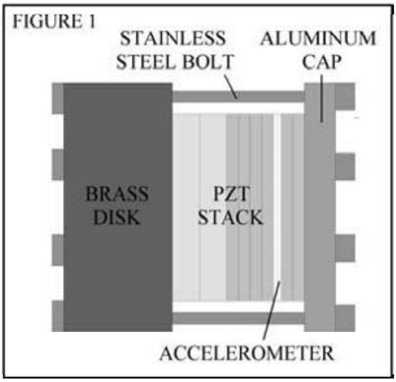
Just because the time-dependent terms in Equations (6) and (7) are normally negligibly small does not mean that they cannot be engineered to be quite large. For example, consider a capacitor made with a highly polarizable dielectric like barium titanate. In titanate materials, the ions in the lattice undergo large excursions when they are subjected to strong electric fields. So, if we apply an AC voltage signal with an amplitude on the order of hundreds of volts at a moderately high frequency to such a capacitor, the lattice ions will undergo large accelerations. And, from the point of view of the predicted effects, the accelerations will be accompanied by large fluctuations in the stored internal energy in the capacitor. To estimate the magnitude of the expected effects in our capacitor we need merely note that $\partial E_o/\partial t$ in Equation (7), when integrated over the volume of the capacitor, is just the instantaneous power P being delivered to the capacitor. (P is just the product of the voltage V and current i in the circuit containing the capacitor.) If we assume P to be a simple sinusoidal signal with an amplitude P_o of, say, a hundred watts and a frequency of 10 kHz, the first transient term (with the second time derivative of E_o) turns out to yield a mass fluctuation with an amplitude of a few milligrams. At higher frequencies and powers the amplitude of the mass fluctuation becomes larger still. Indeed, running in the 60 to 80 kHz range with a power amplitude of several hundred watts, the amplitude of the mass fluctuation rises toward the gram range.

Although there may be a way to take advantage of the large mass fluctuations expected on the basis of the first transient term in Equation (7), for experimental and simple practical purposes it has an intrinsic drawback: Its time-average is zero. This means that in order to detect an effect based on this term you must be able to "weigh" a device in which it is driven as quickly as the mass fluctuates. An experiment of this sort, the work of John Cramer and one of his graduate students, is now in progress at the University of Washington. They have found that the "weighing" speed constraint limits them to less than one kHz operating frequency. Since the first transient term effect scales with the square of the operating frequency, the amplitude of the mass fluctuation they will be able to generate will be quite small, making detection of the effect difficult.

The zero time-average of the first transient term effect doesn't apply to the second transient term, for, being quadratic in P, it is positive definite. From the practical point of view, this is the term of greater interest too, because, being positive definite, it is always negative. That means, if we can excite an effect based on this term to practical levels, since its time-average is not zero we might be able to drive a *stationary* state of reduced, perhaps even negative mass in our capacitor with an AC applied voltage signal. The mundane, technical problem here is that the magnitude of effects expected on the basis of the second transient term in Equation (7) are usually many orders of magnitude smaller than those predicted with the first transient term. For example, if we assume that ρ_o in the denominator of the coefficient of the time derivative can be treated as a constant of order unity—that is, we assume that any mass fluctuation is only a very small perturbation of the total mass—then we will discover that the second transient term effect is down by 20 orders of magnitude or so from the magnitude of the first transient term effect. Need I say that this does not appear to be very promising?

We have ignored an important property of Equation (7). It is non-linear. And if we do not make the small perturbation approximation assumption, we find that the second transient term effect—normally negligibly small—can rival, indeed outstrip, the first transient term effect. You will find this remarkable property of Equation (7) already mentioned in "Twists of Fate" [1997] and its precursor "Making the Universe Safe for Historians" [hereafter, MUSH, 1995, pp. 19-20, which also suffers from the same typographical sign error in the derivation of the effect as Twists]. As Ronald Crowley and Stephen Goode pointed out (in a thesis defense) a couple of years ago, the significance of the second transient term is easily shown. One simply substitutes the *ansatz* $\rho_o = \rho \cos(\omega t)$ into the transient terms in Equation (6) and computes their derivatives. This *ansatz*, of course, is not an exact solution of Equation (6). But this computation shows that when the ρ_0's in the denominators of the coefficients of the transient terms are not treated as constants, the two terms turn out to be of about the same magnitude. Since the first transient term is linear in the applied power and the second is *quadratic*, when enough power is applied to make the amplitude of the first term transient effect a non-negligible fraction of the total unperturbed matter density *we may reasonably expect the second transient term effect to manifest itself.* And, as discussed in MUSH and *Twists*, in extreme circumstances, if the bare masses of elementary particles are negative and hideously large (as they likely are), it may be possible even to drive wormhole formation with this effect. For that reason, I will call this the "wormhole" term/effect, notwithstanding that some will doubtless find this hopelessly romantic, or perhaps even delusionary.

Evidently, in order to try to detect the predicted effects we need a capacitative device that stores large amounts of internal energy and produces large, bulk accelerations in its dielectric material. The obvious material that answers these requirements is lead-zirconium-titanate, so-called PZT, that is routinely used to make electromechanical actuators and ultrasonic devices. Depending on the sign of the applied voltage to a PZT crystal, it will either expand or contract. So if it is subjected to an AC voltage signal, it will oscillate at the applied

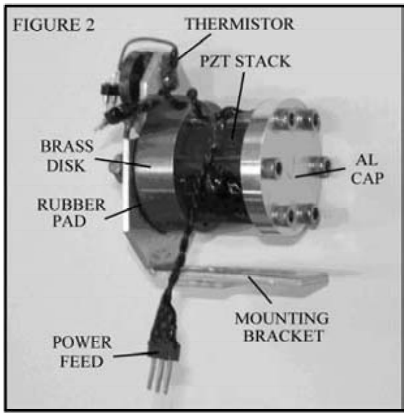


voltage frequency. To optimize the operation of a device made of PZT crystals, we will want to put them together, interleaved with electrodes, into a stack and mount the stack on a reaction mass so that the oscillation induced by the applied voltage signal produces the largest possible acceleration of the stack where the largest stored energy fluctuation is taking place. That is, we will want to build a device that looks, schematically, like that shown in longitudinal section in Figure 1.

The Test Device

The device shown in Figure 1 is a stack of PZT disks about 2 cm. in diameter (actually, 0.750 inch) glued together (with special epoxies) fitted with electrodes and then clamped between a thin aluminum cap and a brass disk about one cm. thick with six (4-40) machine screws. (Such clamping to produce a preload is needed to improve the performance of the mechanical oscillation and to keep the stack from tearing itself apart when operated at high power.) Since the largest accelerations will occur at end of the PZT stack next to the aluminum cap, the PZT crystals there are thin (about 1 mm. thick) so that most of the energy stored in the stack will be stored there. The PZT crystals next to the brass disk are thicker (about 3 mm. thick) since they only serve to carry the acoustic wave generated in the active end of the stack to the reaction mass (and thus should be of the same material as the active end of the stack to avoid an acoustic impedance mismatch that might degrade the performance of the device). Note that a pair of very thin PZT crystals (about 0.25 mm thick) are included in the end of the stack where the largest effect is expected. They are used as a passive accelerometer to monitor the accelerations in this part of the stack. (A more elaborately instrumented device would include more such accelerometers.)

The device must be mounted on the stage of a weigh system. One might be inclined to think that how this is done will not make much difference; and were we chiefly concerned to observe the first transient term effect, that might actually be the case. Experience, however, teaches that if one seeks the second transient term effect *everything* matters, for to bring it to detectable levels *everything* must be *just so*. In the case of mounting the PZT stack/reaction mass assembly I used the aluminum bracket shown in Figure 2, for when first built the device was to be suspended from the beam of a torsion pendulum. In an attempt to reduce the vibration generated in the stack communicated to the beam, I inserted a thin rubber pad between the bracket and the reaction mass. The communicated vibration was reduced, as expected. But an unexpected consequence of the rubber pad was dramatically improved performance of the



device. The performance of the device even depends on the presence of the 1 cm square by 1 mm thick plastic pad glued to the arm of the bracket (visible in Figure 2 below the bracket just to the right of where the power feed crosses it) and the torque on the nut on the mounting stud in the weigh system stage that passes through it (not shown in Figure 2). (The arm of the bracket was slotted at the outset to allow for adjustment. When the attachment point was fixed, the plastic pad with a centering hole was added to make mounting on the weigh stage easily repeatable.) Even the type of mounting stage on the weigh sensor matters. In retrospect it is clear that all of these things affect the propagation of acoustic waves in the device, and such quirkiness should be expected. But before the fact, this was not obvious.

The behaviour of the PZT crystals in the stack depend quite sensitively on their operating temperature. In part, this is a consequence of the fact that the thermal expansion properties of the crystals and the materials in the stack preload clamp are not the same. The brass reaction mass, stainless steel bolts, and aluminum cap all have higher coefficients of thermal expansion than the PZT crystals. So, as the device heats up during operation, the preload changes, and with it the mechanical behaviour changes too. In order to get reproducible behaviour the device must be run only for short intervals in a fairly narrow operating temperature range. So the device must be equipped with a thermometer. Originally, when the preload dependence sensitivity was not fully appreciated, a thermometer was included only to insure that the device was always operated well below the Curie temperature of the PZT crystals so that they would not be depoled when run. The first thermometer used is the spiral bimetallic strip thermometer mounted on an aluminum ear bolted to the brass disk (visible in Figure 2 to the left of the thermistor). When it became clear that more precise temperature monitoring was required, a thermistor was glued to the aluminum ear to monitor the device temperature.

The Weigh System

The core measurement in an experiment to determine whether the mass fluctuation effects predicted by Mach's principle are present is the measurement of the weight of the device in which the effects are driven. Since a fast response, high resolution weigh system is needed, commercial devices, with their long integration times, are unsuitable. A proprietary weigh system was therefore built using a Unimeasure U-80 position sensor fitted with a stainless steel diaphragm spring to transform it into a force sensor. It works on the basis of a magneto-resistive effect in Hall probes that move in a magnetic magneto-resistive effect in Hall probes that move in a magnetic field of known configuration. It is shown with its protective case removed in Figure 3.

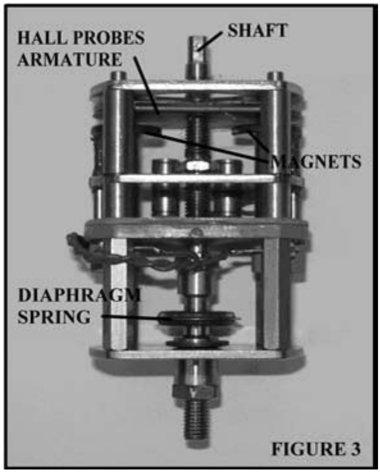
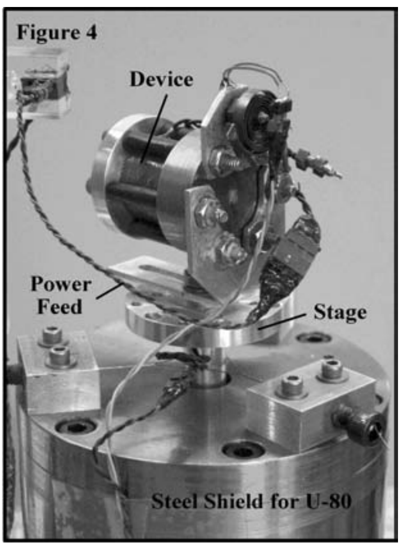


FIGURE 3

Since the U-80, especially operated at high sensitivity, is susceptible to electromagnetic interference, great care must be taken in shielding the device and its associated circuitry. The U-80 used in the experiment described below was encased in a steel shielding container one centimeter thick, shown in Figure 4. The leads to the electronics a few tens of centimeters away were a shielded, twisted pair of conductors (Trompeter Twinax cable with gold-flashed Trompeter connectors). And the “remote” electronics were enclosed in cast aluminum circuit boxes, all of them then being located in a double wall steel box. While these measures sufficed to eliminate all routine electromagnetic interference (tests were done to insure this), the system remained sensitive to 100 mHz radio transmissions. Fortunately, the distinctive nature of this intermittent interference made it easy to identify and suppress either by data correction, or by data elimination. Another important feature of the weigh system can be seen in Figure 4: the three tensioned fine steel guy wires that stabilize the central shaft against lateral motion. This ensures that only motion along the axis of the central shaft can take place. Thus, lateral forces of spurious origin are not mistaken for a weight signal.

Other Apparatus

Figure 4



The rest of the apparatus needed to do the experiment is straight-forward. A sinusoidal signal generator equipped with voltage controlled frequency modulation for automatic frequency sweeps produces the signal that is amplified by a power amplifier. (A Carvin DCM-1000 run in bridged output mode, capable of driving nearly a kilowatt into a 4 Ohm load up to nearly 100 kHz, is a well-suited power amplifier.) Since the peak output swing voltage of inexpensive commercial power amplifiers is about 60 to 70 volts, a stepup transformer is required to bring the peak voltage up to the several hundred volts range. (The transformer used was

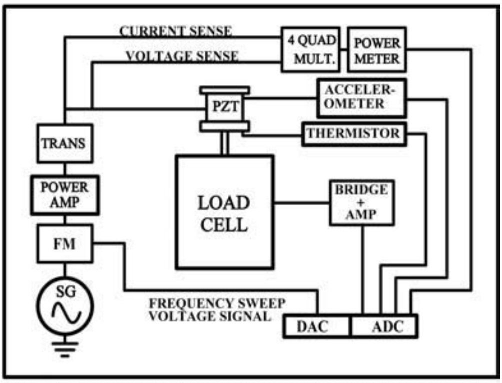


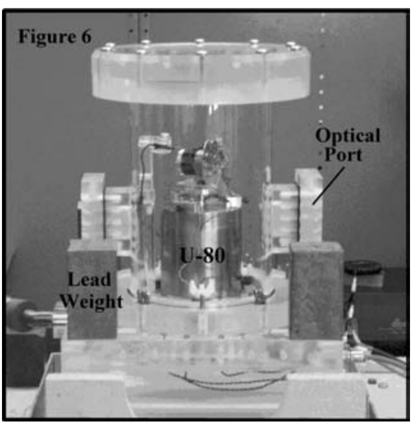
Figure 5: A schematic diagram of the chief electronic circuits used in this experiment. The DAC and ADC are computer controlled.

wound on an Amidon T-300 powdered iron [mixture 26] torus with a 6 to 1 turns ratio.) The secondary circuit, that includes the driven device, is wired with sense resistors to monitor the voltage and current. These signals are then multiplied with a four-quadrant multiplier chip (an Analog Devices 633 chip) and its output is rectified and filtered (with an Analog Devices 630 synchronous demodulator chip) to give a voltage signal that tracks the real-time amplitude of the power wave in the driven device.

Switching and frequency modulation (when used) of the power circuit is accomplished with the digital-to-analog part of a PC based data acquisition and control board, which also acquires data from several channels during data acquisition cycles. The weigh signal, power, and thermistor on the device were routinely monitored. And a fourth channel was used to monitor either the accelerometer embedded in the PZT stack, or an accelerometer attached to the weigh stage (to monitor vibration communicated to the weigh system during operation of the device). A schematic diagram of these circuits is displayed in Figure 5. Each channel was equipped with an anti-aliasing filter. The standard data acquisition protocol was to take data at a rate of 50 Hz during a 14 second interval. The first and last several seconds of each data cycle were quiescent conditions, the device only being operated for a few seconds in the middle of the cycle. Data acquisition and subsequent data reduction was done with proprietary software specially created for the tasks.

The only other things worth mentioning are vibration isolation and the vacuum system. The U-80 with the device mounted on it was placed in a plexiglass vacuum chamber. (See Figure 6.) A rotary vane vacuum pump was used to achieve vacua of a few tens of milli-Torr or better in the chamber. While this is not a hard vacuum, it is sufficient, when compared to operation at atmospheric pressure, to insure that any effect seen with the weigh system is not due to acoustic and/or discharge effects. Vibration produced by the vacuum pump and other ambient seismic noise, given the sensitivity of the weight measurement attempted, mandates that significant vi-

Figure 6



bration isolation be used. This was done with inner tubes and massive lead weights, Barry Stablevel devices and more lead weights, and finally layers of Sorbothane and yet more lead weights.

Some Recent Results

Although it is tempting to present here secure results that have "aged," perhaps it is more interesting and instructive to relate what I have been getting with this system in the recent past. The objective here is simple: To drive large accelerations accompanied by large, rapid changes in internal energy in a titanate material, the aim being to get everything "just so" so that a stationary wormhole term effect manifests itself at a detectable level. The problem is not to produce apparent weight changes in the system described here. Indeed, given that a small device mounted on a very sensitive differential weigh system is being driven at high power, exciting strong ultrasonic vibrations in the vicinity of a mechanical resonance of the system (to get the large accelerations), it would be quite surprising if apparent weight fluctuations were not produced. The real problem is showing, once such weight fluctuations have been detected, that they are the effect sought, not just some spurious signal attributable to mundane origins.

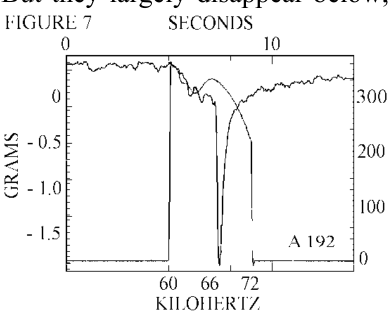
As one proceeds in this sort of investigation, one develops an arsenal of tests that allow one to detect spurious signals, and a variety of protocols that eliminate or stabilize behaviours that can compromise the quality of the data obtained. Tempting though it is to load up with illustrative data related to the tests and protocols I have developed in the course of the past year and a half (and more), I will merely mention several of them.

Since the weigh system is based on the amplification of a rather small electrical signal and the device being tested is run at a power typically of several hundred Watts, perhaps the most obvious source of spurious signals is electromagnetic pickup of the power signal in the weigh system. A simple test can be done to make sure that the shielding of this weigh system circuit is adequate, and the weigh signals are not so contaminated. One puts a small shorting loop of wire at the device sitting on the weigh system and places a nearly identical device elsewhere in the high voltage circuit. Run in this configuration, the electromagnetic fields normally present are mimicked, but no sought effect is produced. Other electromagnetic tests that can be done include placing strong permanent magnets near the system to check for coupling to the Earth's magnetic field. Strong electric fields that can be expected in the system can induce dipoles in nearby dielectrics, and if the fields have strong gradients, forces can arise. The electric fields, of course, will be alternating given the ultrasound frequency of the applied voltage signal. But the induced dipoles will oscillate too, so stationary forces may be produced. The magnitude of such forces can be checked by the simple test of applying a static voltage to the device. None of these tests revealed the presence of spurious signals that might account for signals reported here.

What sort of signals are we talking about? Ones like that displayed in Figure 7. There three traces are displayed. The noisy trace that starts in the upper left hand corner of the figure is that for the weight sensor; the other trace tracks the power applied to the device. As mentioned above, data are taken during a 14 second interval where power is applied during the center four seconds while the frequency of the voltage signal is swept, in this case, through a range of 12 kHz centered on 66 kHz. The weight and power scales are shown on the left and right hand sides of the figure respectively. The full-scale range of the weigh-stage accelerometer signal (not shown) is given in ADC counts in the lower right hand corner of the figure. (No attempt at absolute calibration of this accelerometer was made.)

Very obviously, something interesting happens at about 67 kHz. The weight of the device appears to decrease by in excess of a gram. Considering that the active PZT material in the stack has a mass of about 10 to 15 grams, this weight fluctuation, *if real*, is enormous. It approaches 10 percent of the active mass. We must, therefore, try to show that the effect in Figure 7 is attributable to mundane causes. Perhaps the vibration induced in the weigh system at the mechanical resonance of the device might cause flash heating of the spring, and its transient expansion might generate a brief apparent weight reduction. In fact, thermal effects of this sort are present. Their time-constants, however, are far too long to produce the prompt part of the sort of effects seen.

If the weight spike in Figure 7 (and other results like it) can't be written off entirely to electromagnetic or transient thermal causes, only a few other candidate causes of spurious signals remain. Those most easily dealt with are corona and "sonic wind." Given the presence of several hundred volts in and around the device during operation, one may expect that coronal discharge might take place. Careful construction and insulation should be sufficient to suppress corona. To make sure that it is absent the device can be operated in total darkness and observed with a night-vision scope. And the system can be operated at various levels of vacuum, which should change any coronal effects present. Sonic wind is commonplace in ultrasonic systems. It is a consequence of the fact that ambient gas cannot follow the motion of solid devices operating at high frequency since their excursions exceed the speed of sound in the gas. Sonic wind effects can be quite pronounced all the way down to about a Torr. But they largely disappear below, roughly, 100 milliTorr. So operation in the range of less than a few tens of milliTorr is sufficient to eliminate sonic wind effects. And as a further check, runs can be done at atmospheric pressure to compare with those in the 10 to 30 milliTorr range.

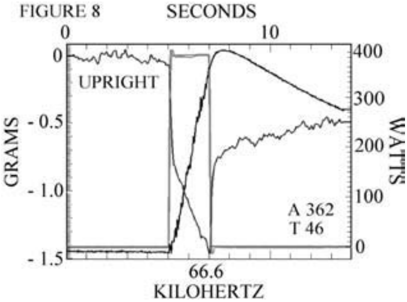


The most troublesome source of spurious signals is mechanical vibration in the weigh system. Accordingly, the obvious thing to do is to try to attenuate the vibrations generated by the operation of the device before they reach the weigh system. And vibration isolation devices were implemented to accomplish just this end. Using them it is possible to reduce vibration in the weigh system to levels undetectable with the accelerometers normally employed. Do weight fluctuation signals like that shown in Figure 7 disappear when this is done? No. They can be sharply reduced in amplitude; but signals on the order of a few tens of milligrams persist. You may be wondering why I haven't shown you smaller signals of this sort instead of that in Figure 7. Because the situation is more complicated than one might like. The conditions needed to produce large wormhole term effects, as I have said, are "just so." And there is reason to believe that some significant part of the weight signals like that in Figure 7 *may* be due to a real Machian effect that depends on "just so" conditions that require the participation of vibratory motion in the weigh sensor.

If you are deeply skeptical of what I have just said, you should be. Until recently I simply dismissed this possibility. But not long ago, quite fortuitously, I encountered behaviour that forced me to reconsider my belief that vibration in the weigh system could only produce spurious signals. In particular, after extended operation of the system, I succeeded in slowly driving down the amplitude of a weight fluctuation signal I was studying to an undetectable level. As a check, I decided to operate the system with the device bolted directly to the weigh stage, that is, with the vibration isolation device removed from the system. As expected, even directly bolted to the weigh stage, no signal was present, notwithstanding that the device continued to operate "normally." (Both the thermistor and the accelerometer embedded in the PZT stack showed that the device continued to produce mechanical excursions of normal operation.) This result was obtained late on a Friday afternoon. The following Monday I set out to pursue this apparent null result. To my amazement, however, the weight signal that I extinguished the previous Friday had returned. It persisted even when the system was "warmed up" to the operating conditions of the previous Friday. Relaxation of the system over the weekend had restored "just so" conditions that I had compromised by extended operation the previous week. Knowing that the sort of system relaxation that could take place in the space of 60 hours or so could not substantially change the vibration present in the weigh system, I was forced to consider the possibility that a real effect might be present in the results. (After the fact, though, an engineer friend tracked down work on PZT devices where precisely the sort of relaxation effect with a time-scale of tens of hours, evidently present, had been systematically studied.)

How does one discriminate a real from a spurious signal in these circumstances? One way is to check for promptness of the effect seen. A real wormhole term effect, given operation at a fixed frequency for an interval short enough so that heating doesn't appreciably change the conditions of operation, should simply switch on and off with the applied power. Vibration induced effects, especially thermal effects induced by vibration, one might expect not to

FIGURE 8

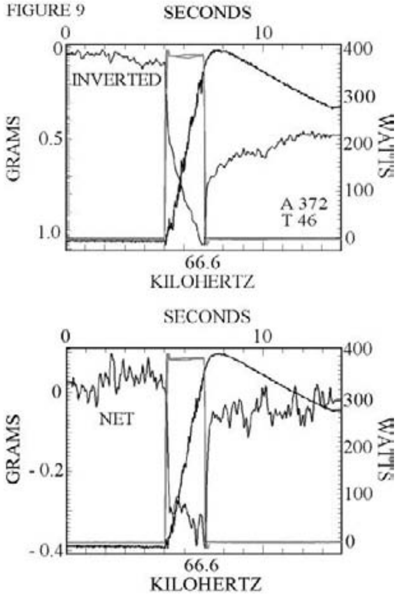


show such prompt switching behaviour. Rather, steady secular evolution should characterize effects of this type.

Since programming for a short fixed frequency pulse had been done long before, changing to this simple switching protocol was trivial. Results of this sort are presented in Figure 8. The peak-to-peak thermistor and mounting stage accelerometer readings in ADC counts are given in the lower right hand corner of the figure. "Upright" refers to the orientation of the weigh system and vacuum case. The weight trace (noisy), as expected, shows both a prompt change at switch-on and switch-off of the power and a secular drift during the powered interval that only very slowly decays after the power is shut off. The thermistor trace (that rises steadily in the powered interval) records the steady input of energy to the system. But the heating it records is in the device assembly, whereas the thermal drift in the weight trace arises from the heating of the diaphragm spring caused by vibration.

While it is tempting to interpret the prompt part of the weight signal in Figure 8 as the Machian effect sought, it would be premature to do so. The reason why is that ultrasonic vibration in the diaphragm spring might change its properties in such a way as to mimic the effect sought. There are two ways in which this might happen. One is that the vibration might induce a stationary distortion in the spring. Given the sign of the effect in Figure 8, such a distortion would have to be an induced distension of the spring. The other is that the vibration might cause the static spring constant to change, creating, as it were, a "dynamic" spring constant. (My engineer friend tracked down a study done within the last few years purporting to show precisely this sort of effect. See: Slotwinski, J.A. and Blessing, G.V. [1999].) I choose to call this the "meringue effect." Can these effects be checked for? Yes. The distortion effect can be isolated by simply inverting the entire system and doing another data run. Inversion, when allowance is made for the possibility that the static spring constant may change in the inverted configuration, will not change the direction of the distortion effect. A real weight effect, however, will reverse direction under inversion (accommodated by reversing the signs of the weight scale relative to zero in the displays).

FIGURE 9



The results for inversion corresponding to Figure 8 are displayed in the top panel of Figure 9. Since the direction of the effect is the same as in Figure 8, evidently we are dealing with a spring distortion effect. The amplitudes of the effects, nonetheless, are different, signaling the presence of a contributory effect that does reverse direction when the system is inverted, as a real Machian effect would. That part of the signal is easily isolated by subtracting the inverted weight trace from the upright weight trace, yielding the net weight trace displayed in the lower panel of Figure 9. The weight fluctuation that presumably contributes equally to the upright and inverted signals should have an amplitude half of that for the net signal in Figure 9, that is, on the order of 125 milligrams. Although this signal is down by about an order of magnitude from the naïve interpretation of the signal in Figure 7, having passed the weigh sensor inversion test, it is a much more reliable result.

Before proceeding, I should say a word about errors in the data presented here. The precision of the data can be estimated directly from the results (which are typically the average of a half-dozen to a dozen cycles). The signals of interest, very obviously, are much larger than the quiescent variability of the traces on any time-scale. The accuracy of the data is another matter. It depends on the accuracy of the absolute calibration of the weigh sensor and voltage and current measurements in the driving circuitry. I won't subject you to a long discussion of the calibration methods employed. Suffice it to say that the accuracy of these measurements were better than ten percent of the full-scale signals recorded, which is more than adequate for our present purposes.

Getting Lucky

Far and away the most common activity of any experimentalist is repeated reconfirmation of Murphy's Law (and its various corollaries). But every once in a while dumb luck strikes and everything works far better than one has any right to expect. After the data in Figures 8 and 9 were obtained, through a "foolish mishap," the system was seriously degraded and had to be opened up and minor tweaking done in an attempt to restore reasonable behaviour. Nothing major. Just checking that the nut on the stud on the mounting stage was secure; that the leads were properly dressed; that the tension of the steel guy wires that stabilize the mounting stage hadn't been compromised; that sort of thing. Since interesting behaviour was most easily detected when the system was run in the inverted orientation, that is the way the system was put it back together. The first cycle taken after reassembly was completed looked very much like the average of inverted cycles shown in the center panel of Figure 10. At first the significance of the structure of the weight trace didn't register. I thought I had further fouled up the system. Only slowly did it dawn on me that I was looking at *quintessentially* "just so" data. Although the secular thermal effect present in Figures 8 and 9 is still prominent, a prompt weight *reduction* that outstrips any spring distortion effect in the opposite sense is plainly in evidence.

After taking several more inverted cycles (to ensure that I wasn't just looking at flukey data), I restored upright orientation and held my breath while the first cycles were acquired. They looked like the average of several cycles in the top panel of Figure 10. I had not loused up the system. Quite the opposite. Subtraction of the inverted results from the upright results yields the net weight signal displayed in the bottom panel of Figure 10. Comparison on the amplitude of the mounting stage accelerometer signal for this and the prior system configuration (approximately 315 ADC counts versus roughly 365 ADC counts) reveals that it is exceedingly unlikely that the change in the amplitude of the prompt weight signal—by a factor of about five—can be attributed to a change in the vibration in the weight sensor—the only plausible spurious source of anomalous prompt weight signals. Perhaps one day the discipline of "gravinertial engineering" will grace the curriculum of this and other institutions after all.

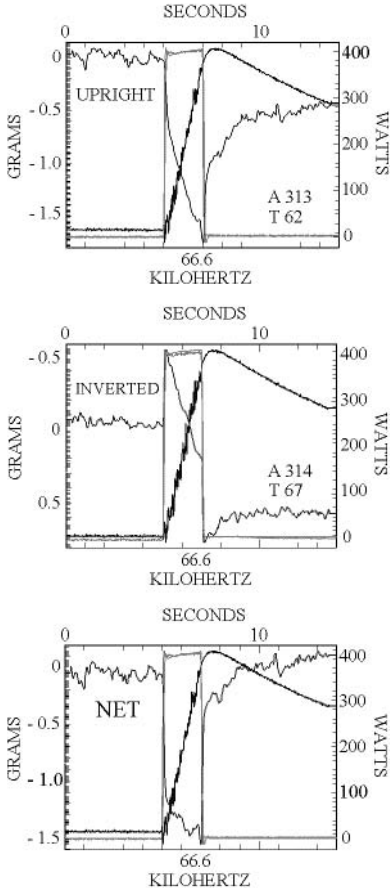


Figure 10

Closing Comments

You may, at this point, be thinking that this is all just ridiculous, far, far too "good" to be true. After all, the principle of the preference for the most prosaic result would seem to suggest that the "just so" conditions that produced the data displayed in Figures 7 through 10 must be "just so" conditions of mechanical vibration generating a subtle meringue effect in the weigh sensor spring, notwithstanding that the magnitude of the effect is essentially uncorrelated to the amplitude of the vibration present in the weigh system—as indicated by the mounting stage accelerometer readings for Figure 10 compared with those for Figures 7 through 9. Taking the contrary point of view, one might argue that the most prosaic result is not some weird meringue effect in the spring; rather, it is the Machian effect sought. Although the Machian effect is surprising and unexpected, no "new physics" beyond acceptance of the relativity of inertia—Mach's principle—is needed to produce its prediction. Accordingly, one might argue that we should be amazed were the predicted effect *not* to be found when sought. But perhaps that is just wishful thinking. In any event, I think it fair to say that the experimental results presented here merit an at least modest further investigation. Gravinertial engineering will never happen if we don't ask questions and take risks.

I think it fitting that my last remarks be acknowledgement of the contributions of others. For many years I have enjoyed the tacit, and occasionally overt, support of my colleagues at CSU Fullerton. Given the "speculative" nature of this research, I do not think I can overstate how important that support has been. I have also enjoyed modest support from a major American corporation. (When I asked if they wanted to be identified, they told me to say that were I to tell you who they were, I'd have to kill you. They were, of course, joking. I think that they were really concerned about the reaction of their stockholders were their support of such "speculative" work to be made public.) Of my colleagues I owe a special debt to Ronald Crowley and Stephen Goode, both of whom went through the derivation of the effect with considerable care on more than one occasion. Indeed, it was Ron's insistence that I do a particular calculation exactly, instead of making a simplifying approximation, that brought the wormhole term to light. And both of them, in Thomas Mahood's masters thesis defense, went out of their way to call attention to the fact that the wormhole term could have consequences as large as the other time-dependent term in laboratory conditions—an eventuality I had not seriously considered hitherto. I have also profited significantly from many conversations with John Cramer and Keith Wanser. Both are masters of theory and experiment, and I know that they will see the imprint of their comments in the work reported here.

I would be remiss were I not to mention the contributions of Thomas Mahood and Paul March. Tom worked with me from the spring of 1997 through the end of 1999. Always ready with good, often inspired ideas about how to deal with the problems one encounters in doing experiments, he also went out of his way to insure that needed tests actually got done. He also kept pushing to forward innovations in the work. Paul, at a distance (he lives in Texas), has also kept prodding to keep the work moving forward. Recently, Tom and Paul have been joined by Kirk Goodall. Should gravinertial engineering ever come to be, I expect it will be in no small part due to the interest and efforts of all these people (and those not known to me who also may be pursuing the effect).

References

Hoyle, F. and Narlikar, J.V. (1974), *Action at a Distance in Physics and Cosmology*, W.H. Freeman and Co., San Francisco.

Pascual-Sanchez, J.-F. (2000), "The Harmonic Gauge Condition in the Gravitomagnetic Equations," available at: arXiv:gr-qc/0010075.

Raine, D..J. (1975), "Mach's Principle in General Relativity," *Mon. Not. R. astr. Soc.* **171**, 507-528.

Sciama, D.W. (1953), "On the Origin of Inertia," *Mon. Not. R. astr. Soc.* **113**, 34-42.

Slotwinski, J.A. and Blessing, G.V. (1999), "Ultrasonic Measurement of the Dynamic Elastic Moduli of Small Metal Samples," *J. Testing and Eval.* **27**, 164-166.

Wheeler, J.A. and Feynman, R.P. (1945), "Interaction with the Absorber as the Mechanism of Radiation," *Rev. Mod. Phys.* **17**, 157-181.

Woodward, J.F. (1995), "Making the Universe Safe for Historians: Time Travel and the Laws of Physics," *Found. Phys. Lett.* **8**, 1-39.

Woodward, J.F. (1997), "Twists of Fate: Can We Make Traversable Wormholes in Spacetime?" *Found. Phys. Lett.* **10**, 153-181.

The Relationship between Mach's Principle and the Principle of Physical Proportions

A.K.T. Assis*

Mach's principle is compared with the principle of physical proportions. Laws that are compatible and others not compatible with the latter principle are discussed. Avenues for the implementation of this principle are also outlined.

Keywords: relative and absolute magnitudes, Mach's principle, principle of physical proportions, relational mechanics.

PACS**:** 01.55.+b (General physics), 01.70.+w (Philosophy of science).

1 Newtonian Mechanics and Mach's Principle

In his book *Mathematical Principles of Natural Philosophy* (1687) Newton laid the foundations of classical mechanics [1]. In the *Scholium* after the Definitions in the beginning of this book Newton defined absolute time, absolute space and absolute motion, the concepts to be employed in his laws. According to Newton, absolute time flows equably without relation to anything external, while relative time is some sensible and external measure of duration by means of the motion of bodies; absolute space remains always similar and immovable without relation to anything external, while relative space is some movable dimension or measure of the absolute spaces which our senses determine by its position to bodies; and absolute (relative) motion is the translation of a body from one absolute (relative) place to another. We can thus say that relative time is a measure of duration by means of motion of material bodies (like the angle of rotation of the earth relative to the fixed stars), relative space is a measure of dimension by means material bodies (as the distance between two bodies measured by a material rule; or the relative order of three bodies).

In order to distinguish absolute from relative motion, Newton performed the famous bucket experiment, also presented in this *Scholium*: when the bucket and the water are at rest relative to the earth, the surface of the water remains flat and horizontal; when the bucket and the water rotate together relative to the earth with a constant angular velocity, the water rises up the sides of the vessel, forming a concave figure. Newton attributed this real and observed curvature to the absolute rotation of the water relative to absolute space, not to the rotation of the water relative to ambient bodies (earth and distant stars).

Leibniz, Berkeley and Mach rejected these concepts, proposing that only relative time, relative space and relative motion could be perceived by the senses and produce observed effects. Accordingly, only these relative concepts

* Institut für Geschichte der Naturwissenschaften, Universität Hamburg, Bundesstr. 55, D-20146 Hamburg, Germany. Permanent address: Instituto de Física 'Gleb Wataghin', Universidade Estadual de Campinas – Unicamp, 13083-970 Campinas, São Paulo, Brasil, Email: assis@ifi.unicamp.br, homepage: http://www.ifi.unicamp.br/~assis

should appear in the laws of physics. For references and discussion see the author's monograph, *Relational Mechanics* [2, Chapters 5 and 6].

Mach expressed these ideas clearly in 1883 in his book *The Science of Mechanics* [3]. In place of Newton's absolute space, Mach proposed the frame of distant stars, that is, the frame in which the distant stars are seen to be at rest [3, pp. 285-6 and 336-7]. In place of Newton's absolute, time Mach proposed the angle of rotation of the earth relative to the fixed stars [3, pp. 273, 287 and 295]. According to Mach the curvature of the water in Newton's bucket experiment was due only to its rotation relative to the distant stars, not to its rotation relative to absolute and empty space [3, pp. 279 and 283-4]. Two key statements by Mach in this connection are as follows [3, pp. 279 and 284]: "Try to fix Newton's bucket and rotate the heaven of fixed stars and then prove the absence of centrifugal forces;" and "The principles of mechanics can, indeed, be so conceived, that even for relative rotations centrifugal forces arise."

The ideas expressed by Mach became generally known by the name "Mach's principle." Formulations of this principle by different authors are presented in *Relational Mechanics* [2, Section 6.8]. The main idea is that only motions of bodies relative to one another should enter in the laws of physics. No effects should arise due to specific motions of bodies relative to empty space.

2 The Principle of Physical Proportions

We concur with Leibniz, Berkeley and Mach on this problem, and as a generalization of their ideas [4] we propose the principle of physical proportions (PPP). Mach advocated doing away with all absolute quantities of motion (reducing local, absolute quantities to global, relational quantities). Here we advocate the abolition of all absolute quantities, whatsoever. In classical physics, space and time are absolute, as well as mass, electrical charge, *etc*. We propose that none of these absolute quantities should appear in the laws of physics, but only ratios of these quantities.

We formulate the principle as follows: (1) All laws of physics must depend only on the ratio of known quantities of the same type. This principle can also be understood in four further ways in order to clarify its meaning: (2) In the laws of physics, no absolute concepts should appear, only ratios of known magnitudes of the same type should be present; (3) Dimensional constants should not appear in the laws of physics; (4) The universal constants (such as G, c, h, k_B, ...) must depend on cosmological or microscopic properties of the universe; (5) All laws of physics and all measurable effects must be invariant under scale transformations of any kind (length, time, mass, charge, *etc*.).

This principle shows similarities with the principle of homogeneity, which was introduced by the Greeks. The idea of dimension had its origins in ancient Greek geometry. It was considered then that lines had one dimension, surfaces had two dimensions and solids had three dimensions [5, Vol. 1, pp. 158-9 and Vol. 3, pp. 262-3] and [6]. These dimensions were related to the rule or principle of homogeneity, according to which only magnitudes of the same kind could be added or equated, and only such magnitudes could have a numeric ratio (it is not possible to divide a volume by a length, for instance) [6]. Heath has called this principle the principle of similitude, and has also spoken of the theory of proportions [5, Vol. 1, pp. 137 and 351; Vol. 2, pp. 112-113 and 187]. The geometrical notion of dimension was extended by Fourier to include physical dimensions [7, §§160-161].

The principle of physical proportions presented here is thus related to the principle of homogeneity introduced by the ancient Greeks in geometry. It should therefore be extended to physics in a new way, a way not implemented by Newton, Fourier, *etc*.

Perhaps the PPP will not be feasible in all laws of physics; but it can at least be utilized as a guiding principle in order to explore more deeply the known laws and see their possible limitations. It seems plausible that whenever a law can be put in this form, with known terms and ratios, a better understanding of the physical principles involved will be achieved.

3 Laws that Satisfy the PPP

There are laws of physics that satisfy the PPP. The law of the lever is a prime example. It can be written as follows: two weights P_1 and P_2 at distances d_1 and d_2 from a fulcrum remain in horizontal static equilibrium (relative to the surface of the earth) when $P_1 / P_2 = d_2 / d_1$. Only ratios of local weights and local distances are relevant here. No fundamental constants appear in this law. Doubling all lengths or all weights (or gravitational masses) in the universe does not affect the equilibrium of the lever.

The law of the inclined plane also satisfies this principle. Consider a frictionless triangle ABC in a vertical plane with its side AC parallel to the horizon and the two bodies above hanging on sides AB and BC, respectively, connected by a string. They will be in equilibrium relative to the surface of the earth when $P_1 / P_2 = AB / BC$. Once more, only ratios of weights and of known lengths are involved here.

Another example is the law of floating bodies discovered by Archimedes. Consider a homogeneous solid body of density ρ_S lower than the density of the fluid ρ_F in which its floats. The condition of equilibrium (no motion relative to the fluid) is obtained when

$$\frac{V_B}{V_T} = \frac{\rho_S}{\rho_F} . \tag{1}$$

Here V_B is the submersed volume of the body (below the surface of the fluid) and V_T its total volume. Only ratios of known volumes and known densities appear in the law. No fundamental constants are involved in this law. Doubling all densities in the universe will not affect the ratio V_B / V_T.

Another example involves communicating vessels filled with liquids. If the cross-sectional area of vessel 1 (2) is A_1 (A_2) and if the forces P_1 (P_2), respectively, are applied on the vessels' free surfaces, equilibrium (no motion relative to the surface of the earth) will result if $P_1 / P_2 = A_1 / A_2$.

There are also dynamical laws which satisfy this principle. One example is Kepler's second law of planetary motion: Areas swept out by the radius vector from the sun to the planet in equal times are equal [8, p. 135]. In other words, the area is proportional to the time. In algebraic terms if one planet describes an area A_1 in time t_1 and area A_2 in time t_2 then $A_1 / A_2 = t_1 / t_2$.

Another example is Newton's second law of motion coupled with his third law. Consider two bodies of inertial masses m_{i1} and m_{i2} interacting with one another along a straight line. If they are subjected to accelerations a_1 and a_2 relative to an inertial system of reference, from Newton's laws we obtain (assuming constant inertial masses): $m_{i1} / m_{i2} = -a_2 / a_1$.

4 Laws that do Not Satisfy the PPP

The majority of physical laws do not comply with the PPP. A number of examples were discussed in earlier work [4]. Here we briefly present some of them.

The free fall acceleration *a* near the surface of the earth according to classical mechanics is given by $a = GM_{ie} / R_e^2$, where $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2\ kg^{-2}}$ is the constant of gravitation, $M_{ie} = 5.98 \times 10^{24}\,\mathrm{kg}$ is the earth's inertial mass and $R_e = 6.37 \times 10^6\,\mathrm{m}$ its average radius. This acceleration is known to be independent of the mass of the falling body. Hence there is no ratio of masses in this law, and the acceleration of free fall would then be a measure of the absolute value of the earth's mass: doubling this mass would double the acceleration of free fall, independent of what happens to the mass of the test body, to the mass of stars and galaxies, *etc*. This shows that not only space and time are absolute in classical mechanics, but inertial mass is as well.

The flattening of the earth due to its diurnal rotation is also an example of this absolute aspect of mass in classical mechanics [4].

The law of elastic force is a further example of a law that does not comply with the PPP. Consider a spring of relaxed length ℓ_o and elastic constant *k*. A body of weight *P* can be suspended in static equilibrium when this spring is fixed vertically, provided that its final length ℓ satisfies the relation $P = k(\ell - \ell_o)$. There are no ratios of weights here. This law is correct in the sense that it describes the behaviour of springs. (It is valid as long as the lengthening of the spring is not so great as to become irreversible.) But because it does not satisfy the PPP, it must be regarded as incomplete.

The great majority of laws of physics do not comply with the principle of physical proportions. Whenever we encounter physical laws expressed in terms of equalities in which there appear local constants (such as the spring constant *k*, the dielectric constant ε of the material, *etc*.) or universal constants (such as *G*, ε_o, k_B, *h*, *etc*.), they must be incomplete, although correct. Examples include: the law of ideal gases, $PV = k_B NT = RnT$ (*P* being the pressure, *V* the volume, $k_B = 1.38 \times 10^{-23}\,\mathrm{JK}^{-1}$ Boltzmann's constant, N the number of atoms or molecules, T the temperature, $R = 8.3\,\mathrm{JK}^{-1}\mathrm{mol}^{-1}$ the universal gas constant, and n the number of moles); the velocity of sound, $v_s = \sqrt{B/\rho}$ (B being the bulk modulus of the fluid with density ρ); Ohm's law, $V = RI$ (where V is the voltage or potential difference between two points A and B of a conductor of resistance R in which the constant current I flows), *etc*.

5 Implementation of the PPP

We now discuss a method for implementing this principle in order to make natural laws complete. We first consider hydrostatics and Archimedes's principle. Although Eq. (1) satisfies the principle, we will discuss an incomplete form of this law.

It is easy to imagine how people unaware of Archimedes's results might arrive at a correct but incomplete law when experimenting with floating bodies. They might set a piece of ice, cork, wood, *etc*. afloat only in water, and observe that the ratio of the submersed to the total volume was proportional to the density of the material, namely

$$\frac{V_B}{V_T} = A\rho_S\,, \tag{2}$$

where A would be a constant of proportionality with dimensions of the inverse of density. This constant would be the same for all solid bodies specified above. This equation is correct dimensionally and is invariant under unit transformation (the numerical value of A will depend on the system of units employed, for instance $A = 10^3\,\mathrm{kg/m}^3$ or $A = 2.2 \times 10^3\ \mathrm{lb/m}^3$, but the form of the equation will be the same in all systems of units).

Although this law correctly describes the behaviour of bodies floating in water, it is incomplete. In order to transform this law into one that is compatible with the PPP, it would be necessary to discover if A was of cosmological, local or microscopic origin. Specifically, it would be necessary to discover if $1/A$ was proportional to the mean density of mass in the universe, to the density of the local fluid in which the solid was floating, or to the density of the molecules composing the fluid, for instance. By floating the same solids in different fluids like liquid mercury, gasoline and alcohol it would be possible to arrive at $A = 1/\rho_F$. The situation would then be described by Eq. (1) and the law could be considered complete.

Relational mechanics completely satisfies Mach's principle and the more general PPP [2, 4]. It is based on Weber's law for gravitation and electromagnetism [9]. Weber's force depends only on the relative distance between the interacting bodies, on their relative radial velocity and on their relative radial acceleration, so that it is completely relational. Relational mechanics is also based on the principle of dynamical equilibrium [10, 2 Section 8.1]: The sum of all forces of any nature (gravitational, electric, magnetic, elastic, nuclear, *etc*.) acting on any body is always zero in all frames of reference. As the sum of all forces is zero, only ratios of forces will be detectable or measurable. The system of units (MKSA, cgs, *etc.*) to be employed is not relevant. Moreover, the unit or dimension of the forces can be arbitrarily chosen.

According to the theory of relational mechanics [2, Sections 8.4, 8.5 and 9.2], the acceleration of free fall a_{me} of a test body of gravitational mass m_g toward the earth is given by

$$\frac{a_{me}}{a_o} = \left(\frac{2}{2^n \xi}\right)\left(\frac{R_o}{r}\right)^2 \left(\frac{M_{ge}}{M_{go}}\right) .$$

Here M_{ge} is the gravitational mass of the earth, r is the distance of the test body from the center of the earth, $R_o = c / H_o$ is the radius of the known universe (c is the value of light velocity in vacuum and H_o is Hubble's constant), M_{go} is the gravitational mass of the known universe (mass inside a sphere of radius R_o) and $a_o = R_o H_o^2$ is a fundamental acceleration characteristic of the universe. In this expression there are only ratios of accelerations, distances and gravitational masses. Doubling all distances or all masses in the universe, for example, will not affect the ratio a_{me} / a_o. According to this expression, the acceleration of free fall is independent of the mass of the test body, as known since Galileo. On the other hand, it shows that this acceleration is not only directly proportional to the mass of the earth, as known by Newton, but also inversely proportional to the mass of the distant galaxies. We can double a_{me} either by doubling the mass of the earth (compared to any standard, without simultaneously affecting the masses of the distant galaxies according to this standard), or by halving the masses of the distant galaxies (compared to any standard, without simultaneously affecting the mass of the earth according to this standard). If the distance of the test body from the earth is doubled, the acceleration of free fall decreases by a factor of 4. According to the expression above, the same should happen even if the distance between the earth and the test body is not changed, but the size of the known universe is shrunk by a factor 2. The meaning of a_o is not yet clear, but it must be the acceleration of some material object. Perhaps it is the average acceleration of all bodies in the universe, or some other as yet unknown acceleration. In the future, it may prove interesting to investigate the relationship between this acceleration and the acceleration introduced by Jaakkola in his research on cosmology and gravitation [11]. In any event, the important aspect of the result above is that only ratios of gravitational masses, of distances and of accelerations are relevant here.

The implementation of the PPP as regards the flattening of the earth has already been discussed in a recent essay [4], where we also discussed applications of the PPP to electromagnetism and the equation of ideal gases. There it was shown that these laws in their present form do not comply with this principle, indicating that they may be incomplete. Possible ideas and avenues for completing them were also outlined.

6 Discussion

In closing, it may be appropriate to quote a very pertinent passage from the last chapter of Amitabha Ghosh's book *Origin of Inertia* [11]:

> When I find school students nowadays solving mechanics problems involving pulleys, inclined planes, rockets, cars, I cannot but help think of the early summer of 1956. I had just completed my high school in a remote village of Bengal and was waiting for my admission to the district college for the Intermediate Science programme. My father thought that the time might be better utilized if I were to get some prior exposure to science. In those days, up to Class-10 there were hardly any science topics in the programme, and we had absolutely no introduction to mechanics. Even the terms like velocity, acceleration, momentum, *etc*. were totally unfamiliar to us in the high school. One of my cousins had finished his Intermediate Science and was a trainee in a steel plant. He came to spend a few weeks at our home, and first introduced me to the names of Newton and Galileo. He gave me my first ever lesson in elementary kinematics, the parallelogram laws of the addition of forces and motion parameters. Soon afterwards, I was introduced to the laws of motion by another young postgraduate in Mathematics from the village. By then he had left Mathematics and was studying Law, but had returned to spend his summer vacation at home.
>
> I remember the tremendous mental block I had in conceiving of the basic concepts. By that time, I was familiar with multiplying physical quantities by numbers. Somehow, ideas of velocity and acceleration, which involved length and time, I could grasp. What was very difficult for me at that time was to conceive of the idea of one physical quantity being multiplied by another physical quantity. For me the stumbling block was the concept of momentum—the product of mass and velocity. I can still remember the utter exasperation of the young law student who had already completed a Master of Science in Mathematics from Calcutta University. He was completely baffled by my difficulty. It took a long time for me to accept the concept of momentum.

As we saw above, Prof. Ghosh's difficulty in conceiving the idea of linear momentum reflects a deeper problem in the laws of physics themselves. According to the PPP we should only have ratios of quantities of the same type. When we examine the problem more closely, we see that it makes no sense to multiply a mass by a velocity. These are two completely different physical concepts, with different units and operational definitions for their measurements. The most we can say is that, by definition, the linear momentum μ of a body 1 is to the linear momentum of a body 2 as the ratio of their masses m multiplied by the ratio of their velocities v, namely:

$$\frac{\mu_1}{\mu_2} = \frac{m_1}{m_2}\frac{v_1}{v_2} . \tag{3}$$

According to the principle of homogeneity of the ancient Greeks, only magnitudes of the same dimension should be added or equated. The same must be valid for physical magnitudes, as postulated here by the PPP. How should the concept of velocity be handled? Instead of defining it as the ratio of a length by a time interval, as is usually done, the same procedure as above should be utilized, as indicated by Mach [3, p. 273]: "A motion is termed uniform in which equal increments of space described correspond to equal increments of space described by some motion with which we form a comparison, as the rotation of the earth. A motion may, with respect to another motion, be uniform. But the question whether a motion is *in itself* uniform, is senseless." Accordingly, the ratio of velocities should be defined operationally as $v_1 / v_2 = (s_1 / s_2)(t_2 / t_1)$, where *v* means velocity and *s* space described in time *t*. When the ratio v_1 / v_2 is a constant in time, we can say by definition that the motion of body 1 is uniform in comparison with the motion of body 2. The same should be applied to other magnitudes. For instance, instead of defining density as the ratio of mass to volume, only ratios of densities should be defined. That is, the ratio of density of two bodies 1 and 2 should be defined as the ratio of their masses multiplied by the inverse ratio of their volumes, namely: $\rho_1 / \rho_2 = (m_1 / m_2)(V_2 / V_1)$.

Because not all laws of physics are written in terms of ratios of known quantities of the same type, they must be incomplete. The ideas presented in this work may help to indicate possible ways to complete these laws.

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References

[1] I. Newton, *Mathematical Principles of Natural Philosophy*, University of Chicago Press, Berkeley (1934). Cajori edition.

[2] A.K.T. Assis, *Relational Mechanics*, Apeiron, Montreal (1999).

[3] E. Mach, *The Science of Mechanics - A Critical and Historical Account of Its Development*, Open Court, La Salle (1960).

[4] A.K.T. Assis, "Applications of the principle of physical proportions to gravitation," in K. Rudnicki, editor, *Gravitation, Electromagnetism and Cosmology - Toward a New Synthesis*, pp. 1-7, Apeiron, Montreal (2001).

[5] Euclid. *The Thirteen Books of The Elements*, volume 1-3, Books I-XIII, Dover, New York (1956). Translated with introduction and commentary by Sir Thomas L. Heath.

[6] R. de A. Martins, The origin of dimensional analysis, *Journal of the Franklin Institute* **311** (1981) 331-337.

[7] J. B. J. Fourier, "Analytical Theory of Heat," in *Great Books of the Western World*, Vol. 45, pp. 161-251, Encyclopaedia Britannica, Chicago (1952).

[8] K. R. Symon, *Mechanics*, Addison-Wesley, Reading, third edition (1971).

[9] W. Weber. *Werke*, W. Voigt, E. Riecke, H. Weber, F. Merkel and O. Fischer (editors), volumes 1 to 6, Springer, Berlin (1892-1894).

[10] A.K.T. Assis, On Mach's principle, *Foundations of Physics Letters* **2** (1989) 301-318.

[11] T. Jaakkola, Action-at-a-distance and local action in gravitation: discussion and possible solution of the dilemma, *Apeiron* 3, No. 3-4 (1996) 61-76.

[12] A. Ghosh, *Origin of Inertia: Extended Mach's Principle and Cosmological Consequences*, Apeiron, Montreal (2000).

Inertial Mass of the Electron

W. Farrell Edwards*

The inertial mass of the electron is derived through a consideration of its retarded electromagnetic radiative effects on all other electrons throughout the universe and their subsequent advanced electromagnetic effects back on the originating electron. Dirac's Large Number Hypothesis appears as a corollary of the derivation. The possibility that negative inertial mass might arise becomes evident, leading to the conjecture that, under certain unusual conditions, like charges attract.
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I. Introduction

Based on the collection of articles resulting from the 1993 Conference on Mach's Principle [1], Bondi and Samuel [2] generated a list of interpretations of Mach's principle. Sixth in their list (Mach6) is the following: "Inertial mass is affected by the global distribution of matter." Misner, Thorne, and Wheeler [3] succinctly express this interpretation as "Matter there governs inertia here." Guided by Mach6, several attempts to account for inertia have been made with strong connections to electromagnetic theories.

In 1953, by applying the formalism of Maxwell's equations to gravity, Sciama accounted for inertia in a few special cases as a gravitational effect due to all other bodies in the universe [4]. Sciama maintained that his theory was "intended only as a model" and others point out several defects [5].

In 1974 Edwards applied a one-parameter class of velocity and acceleration-dependent potentials to gravity as well as electromagnetism [6]. By selecting a particular value of the parameter the potential becomes equivalent to Weber's electrodynamic potential; another value yields Riemann's electromagnetic theory. Edwards' approach, which assumes that the kinetic energy term in the action is zero, leads not only to inertial mass but as well to the principle of equivalence and negative inertia (hence binding forces for like charges at small scale lengths). On the other hand, it apparently fails to properly account for electromagnetic radiation and suffers from the use of action-at-a-distance equations with infinite signal speeds.

Assis rederived much of Edwards' theory of inertia [7] and extended it in a series of interesting papers [8]. He maintains that a Weber-type formulation of electromagnetism can account for radiation.

Following is a derivation of the inertial mass of the electron based on conventional electromagnetic interactions. It follows Mach6 and is entirely classical. In the present form the theory utilizes an oversimplified view of the universe similar to that used by Sciama [4]; consequently, it suffers from several limitations. Despite these limitations confirming results arise from the

* Utah State University, Logan, UT 84322-4415

derivation, including the inertial mass of the electron, Dirac's Large Number Hypothesis [9], and the possibility of negative mass. The approach does not fully account for the inertia of particles other than electrons.

The presentation is maintained at the simplest level that will preserve and illustrate the basic ideas. While it is true that a mature formulation must be consistent with more sophisticated physics, *i.e.*, relativity, quantum field theory, *etc.*, the conclusions that are drawn, even at this preliminary stage, lead one to suppose that the theory might hold the germ of new understanding at a fundamental level.

II. Advanced Fields and Radiation Reaction Theory

In their 1945 paper [10], "Interaction with the Absorber as the Mechanism of Radiation," Wheeler and Feynman attempt to account for radiation reaction, *i.e.*, energy loss by accelerating—hence radiating—charged particles. The paper proceeds through four stages or "derivations". All involve retarded electric fields radiating from the primary source charge to secondary absorbers causing them to accelerate and radiate, and advanced fields from these secondary charges traveling back to the source, arriving at the instant the process began.

Wheeler and Feynman demonstrate that advanced signals, thus employed, do no violence to causality; *i.e.*, in observations, cause would still precede effect. Misner, Thorne, and Wheeler [3] review the causality debate in connection with Mach's principle and draw the same conclusion, "No violation of causality, despite appearances." An extensive literature has followed this use of advanced electromagnetic effects with important contributions by Hoyle and Narlikar [11], Hogarth [12], and others.

Wheeler and Feynman reject the results of their first derivation of radiation reaction. They write:

> The force gives an account of the phenomenon of radiative reaction which is not in accord with experience: (1) The force acts on the source in phase with its acceleration; or in other words, it is proportional to the acceleration itself rather than to the time rate of change of acceleration. (2) The reaction depends upon the nature of the absorbing particles. (3) The force appears at first sight to grow without limit as the number of particles or the thickness of the absorber is indefinitely increased.

Though unacceptable as an explanation of radiation reaction, this force is suggestive of inertia which (1) *is* proportional to acceleration, (2) might well depend upon the nature of the absorbing particles, and (3) would be expected to grow as the thickness of the absorber is increased, if Mach6 is operating.

The present work follows Wheeler and Feynman's first derivation except for the following modifications: (1) the theory is restricted to electrons rather than arbitrary charged particles, (2) charge conjugation is applied to the electrons that produce the advanced fields, and (3) the effects are summed over all electrons in the universe.

III. Self-Consistent Derivation of the Electron's Inertial Mass

Assume that a force **F** acts on an electron (called the *source* or *primary* electron), resulting in its acceleration **a**. This acceleration, however, is not given simply by **F**/*m*, as, presumably, it would be if **F** were the only force acting. Instead, a secondary force **f** acts simultaneously on the source electron. It arises because of the radiation fields produced by the accelerating electron itself. These retarded fields travel outward, encountering other electrons which themselves radiate. The advanced component of this secondary radiation acts back on the source electron. Force **f** is the net effect due to all radiating secondary electrons, hence the net force acting on the source electron is **F** + **f** and **a** represents the acceleration resulting from this sum. As we shall shortly see, **f** has the characteristics of inertia.

The radiation electric field traveling outward from the source electron to a field electron k at retarded position **r** with respect to the source electron is

$$\mathbf{E}_k = \frac{e}{4\pi\varepsilon_o c^2 r^3}\mathbf{r}\times(\mathbf{r}\times\mathbf{a})\,. \tag{1}$$

SI units are used. The expression is valid for low electron speeds.

The field electron feels a force, $e\mathbf{E}_k$ and itself accelerates according to

$$\mathbf{a}_k = \frac{e\mathbf{E}_k}{m} = \frac{e^2 a}{4\pi\varepsilon_o m r c^2}(\sin\theta)\mathbf{e}_\theta\,, \tag{2}$$

where θ is the angle between **r** and **a**, and $\mathbf{e}_\theta$ is the direction given by the double cross product. We assume that the field electron has inertial mass m as a result of its own radiative interactions with electrons throughout the universe and that this value is the same as that for the source particle's inertial mass, which we are attempting to derive. Later we will solve for the mass of source and field electrons in a self-consistent manner. We need consider only electrons as field particles because the accelerations of more "massive" charged particles are much smaller; consequently, their contributions to the inertia of the source electron may be neglected in the present approximation.

Because of its acceleration, $\mathbf{a}_k$, the field particle also radiates. As did Wheeler and Feynman, we focus on the advanced component of the resulting electric field, $\mathbf{E}_s$, which travels in negative time and acts on the originating source particle at just the instant the process began.

In Wheeler and Feynman's first derivation, $\mathbf{E}_s$, though acceleration dependent, is in the wrong direction to account for inertia. This problem can be resolved as follows: Maxwell's equations are invariant under three transformations: time reversal (T), charge conjugation (C), and parity (P). In all of their derivations Wheeler and Feynman applied T which yields advanced radiation fields. Added here by postulate is conjugation of the charge of the field electron in its production of the advanced field. As a consequence the direction of $\mathbf{E}_s$, when summed over all electrons in the universe, becomes just that required for inertia. Therefore, as assumptions of the theory, we apply both C as well as T to the production of $\mathbf{E}_s$.

$\mathbf{E}_s$ is obtained through a second application of Eq. (1). It produces a secondary force on the source electron, $\mathbf{f}_s$ (= $e\mathbf{E}_s$) and is given by

$$\mathbf{f}_s = \frac{e^4}{(4\pi\varepsilon_o)^2 mr^2c^4} a\sin(\theta)\mathbf{e}_\theta \,. \tag{3}$$

To obtain the net force $\mathbf{f}$ (=Σ $\mathbf{f}_s$) due to all field electrons, imagine a model universe consisting of a static sphere of radius R containing a uniform distribution of electrons.* Integrating the effects due to all these electrons results in

$$\mathbf{f} = -\frac{2Ne^4}{(4\pi\varepsilon_o)^2 \; mR^2c^4}\mathbf{a} \,, \tag{4}$$

where N is the total number of electrons.

Force $\mathbf{f}$ opposes the acceleration of the source electron and is proportional to its magnitude; consequently it is a candidate for inertia. For $\mathbf{f}$ to properly represent inertia, the proportionality factor must equal the mass of the source electron. The factor itself contains the mass of each secondary electron, which is also m. Therefore we set the proportionality factor equal to m, solve the resulting equation for m^2 and take the square root. This gives

$$m = \left[\frac{e^2}{(4\pi\varepsilon_o)Rc^2}\right](2N)^{1/2} \,. \tag{5}$$

To obtain a numerical value, we assume that our model universe consists of 75% hydrogen, 24% helium, and 1% all other elements; this is a current estimate of the composition of the universe. Assuming that the total numbers of electrons and protons are the same, it follows that $N = 0.88\, M/m_p$ where M is the mass of the universe and m_p the mass of the proton. The coefficient is less than 1 because of the presence of neutrons.

The mass and radius of the universe are not well known. We approximate M as lying between 10^{52} kg (an estimate from consideration of visible mass in stars and galaxies) and 10^{54} kg (which includes a factor to account for dark matter). Using $R = 10^{10}$ ly†, the resulting electron mass falls within the range 10^{-31} to 10^{-30} kg. For so simple a model, this agreement seems satisfactory.

The foregoing showed that $\mathbf{f} = -m\mathbf{a}$ within a reasonable uncertainty. Newton's Second Law states that the net force equals the intrinsic inertial mass times the acceleration; that is to say,

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* Wheeler and Feynman [10] also assumed a static model of the universe in their derivations. The idea that the universe has a "radius" is, of course, highly oversimplified and problematical. At the next level of sophistication we would regard R as the greatest distance that maintains a causal connection with the source electron and then use an expanding universe for the model. Preliminary calculations using such a model suggest that the resulting numerical changes are less than the uncertainties introduced elsewhere. In this paper the value for R is taken to be the distance in Hubble's law that results in a recessional speed equal to c.

† Including the contribution of the universe does not substantially change the results but makes the calculations more cumbersome.

$$\mathbf{F} + \mathbf{f} = \mathbf{F} - m\mathbf{a} = m_{intrinsic}\mathbf{a}. \tag{6}$$

Because the derived inertial mass of the electron, m, approximates well what we have heretofore ascribed to the electron as intrinsic mass, we may consider the possibility that the electron has no intrinsic inertial mass, *i.e.*, $m_{intrinsic} = 0$. Under this assumption the fundamental form of Newton's Second Law would be $\mathbf{F}_{net} = 0$ and using the usual form, $\mathbf{F} = m\mathbf{a}$, merely simplifies calculations as long as the entire universe is the dominant source of inertia.

IV. Dirac's Large Number Hypothesis

In 1938 Dirac published a conjecture known as the Large Number Hypothesis (LNH) [9]. He noticed that the ratio of the electrical to the gravitational force between an electron and a proton is approximately 10^{40} and dimensionless, and that the ratio of the age of the universe T to time t it would take light to traverse a classical electron is also near 10^{40} and dimensionless. Dirac said, "such a coincidence we may presume is due to some deep connection in Nature between cosmology and atomic theory." He therefore hypothesized that these two expressions should be proportional with a coefficient of order unity. He even suggested that this, together with other similar relationships, could become "new assumptions on which to build up a theory of cosmology." The reason for the coincidence and "deep connection" Dirac sought has remained a mystery.

From Eq. (5) we obtain the LNH if we use the Schwarzschild condition, $R = 2GM/c^2$—which holds to a reasonable approximation for the universe itself. Square both sides of Eq. (5), use $N = 0.88\ M/m_p$ as we did earlier. Replace M using the expression for the Schwarzschild radius. Express the radius of the universe R as cT and the factor $e^2/(4\pi\varepsilon_o mc^2)$, which represents the classical radius of the electron, r_o, as ct. Rearrange the parameters and obtain

$$\frac{F_{electrical}}{F_{gravitational}} = \frac{e^2/4\pi\varepsilon_o}{Gm_p m} = 1.14\frac{T}{t} \ . \tag{7}$$

This is a statement of Dirac's Large Number Hypothesis.

Another, very concise, large number condition, follows easily from Eq. (5) without relying on the Schwarzschild relation. It is

$$R/r_o = (2N)^{1/2} \ ; \tag{8}$$

that is to say, the ratio of the radius of the universe to that of the classical electron equals the square-root of twice the number of electrons in the universe.

The appearance of Dirac's LNH is an unexpected confirmation of the present theory. The proposed mechanism for producing inertia in electrons might be the secret Dirac suspected was lurking within his LNH.

V. Inertia from Nearby Charges

If electron inertia is produced by the presence of other electrons, one can imagine situations in which nearby rather than distant matter would dominate. Equation (3) gives the contribution to the inertia of an electron from just one other electron. From this we can estimate how close two electrons must be in order that the magnitude of the inertia produced by one on the other approaches or exceeds that from the rest of the universe. Let $\sin\theta = 1$ and set the coefficient of **a** equal to *m,* with *m* representing the traditional mass of the electron, *i.e.*, the value which is obtained when all the electrons in the universe are assumed to contribute. Then solve for *r*; this yields

$$r = \frac{e^2}{4\pi\varepsilon_o mc^2}, \tag{9}$$

which is the classical radius of the electron, r_o. Thus, if two electronic charges have a separation less than r_o, the inertial influence of the universe becomes less important in producing inertia than that of the nearby charge.

This result, $r \leq r_o$, takes us outside the classical domain assumed for this paper and reminds us of old issues concerning the stability of the electron and the distribution of charge within it including dilemmas about enormous repulsive forces and huge self energy. Presumably these problems have been resolved using quantum electrodynamics with the conclusion that the electron consists of a "bare" point charge surrounded by a "cloud" of virtual particles that shield the core. This leads to the further conclusion that the electromagnetic coupling constant increases as the core is approached. Probing the electron at a center-of-mass energy of 58 GeV, Levine, *et al.* [12] obtained results consistent with this picture.

The present work adds a new element to this discussion. As shown in the next section, within the context of the present theory circumstances might arise under which like charges attract. Whether or not such a possibility can help explain the internal stability of the electron or be made consistent with recent electron structure measurements must await the results of attempts to incorporate this approach into quantum theory. Nevertheless, preliminary ideas follow.

VI. Negative Mass

In solving for the inertial mass *m* imparted to an electron by all other electrons, Eq. (5), we took the square root of m^2. Formally, at that point we should have considered the negative as well as the positive solution. The negative solution would constitute negative inertial mass.

Although in familiar microscopic and macroscopic phenomena the inertial mass is demonstrably positive, we can't be as confident in the choice of the sign in extreme situations such as the one just considered ($r < r_o$). In such cases we have no compelling reasons, either observational or theoretical, for ruling out the possibility of negative mass. Furthermore, the negative solution might solve the self-destructive force problem when considering an internal structure for the electron because, under those circumstances, like charges would attract rather than repel. A simple exercise reveals this feature.

Imagine three point charges *q* arranged at the vertices of an equilateral triangle having sides α. The particles are acted upon by a number of forces, including: electrostatic, magnetic, inertial forces from the universe, and inertial radiation forces from the nearby charges. The charges are taken to be so close that universe-produced inertia is negligible and magnetic forces sum to zero if the system has no angular velocity.

The inertial term is obtained from Eq. (3) and contains the mass of each contributing nearby particle, m_i. After adding the effects on the source particle we set the coefficient of the acceleration to m_i, solve for m_i^2, and take the square root, obtaining

$$m_i = \pm \frac{\kappa q^2}{4\pi\varepsilon_o r c^2} \tag{10}$$

where $\kappa = (3)^{1/2}/12$, and r ($= \alpha/(3)^{1/2}$) is the distance from each charge to the center of the triangle. The inertial mass, m_i, is not constant but a function of r.

If we assume that the particles have no intrinsic inertial mass and therefore set the net force equal to zero we get,

$$\frac{q^2}{(3)^{1/2}(4\pi\varepsilon_o)r^2} \mp \frac{\kappa q^2}{(4\pi\varepsilon_o)rc^2} a = 0 . \tag{11}$$

Note that both the charge and the electrical force constant divide out.

Eq. (11) is a differential equation governing the motion of the particles. The positive solution for the inertial mass in Eq. (10) leads to the negative sign in (11) so the acceleration would be positive, the configuration unstable and the system would blow up.

On the other hand, the negative mass solution leads to negative acceleration and the charges oscillate through the origin. In this case the solution yields $v > c$. Perhaps this is an artifact of the oversimplified model; or arises because we are outside the theory's region of applicability, or because we have used nonrelativistic equations. However this might be, the general result—negative mass and binding forces between like charges at short distances—is suggestive and could serve as a guide for future research. Details of the model should not be taken too seriously before the model is refined and forces and fields are treated using relativistic quantum mechanics properly adapted to this situation. On the other hand, the possibility of negative mass solutions under extreme conditions would be an interesting addition to physical theory.

VII. Possibilities, Questions and Problems

If the fundamental expression of Newton's Second Law for electrons is $\mathbf{F}_{net} = 0$ [see also refs. 6 and 7] then momentum, kinetic energy, and similar quantities requiring inertial mass in their definitions have their present meanings only within the context of the present universe. The Lagrangian would be fundamentally defined as $-V$, and the kinetic energy term would only be used as a convenience. The idea of *inertial reference frame* would be clarified.

On the other hand, there are problems: As presented here the theory fully applies only to electrons; that is to say, superficial applications to hadrons and muons fail to account for their entire masses. Perhaps these particles have intrinsic inertial mass as well as an electromagnetic contribution from the universe. There are other possibilities.

Other questions arise: Might galaxies, quasars, or black holes cause measurable anisotropies in the inertia of nearby electrons? What about gravity and the principle of equivalence? How might one reconcile these ideas with quantum theory? What modifications in fundamental physical theory would be required if all inertial mass were derived rather than intrinsic? These questions are the subjects of ongoing research.

VIII. Conclusions

Despite theoretical and experimental reasons for questioning the theory of inertia presented here, it merits our attention because of (1) its internal consistency, (2) agreement between predicted and measured values of the electron mass, (3) the appearance of Dirac's LNH, (4) the circumstance where inertia is dominated by nearby charges, and (5) the possibility of negative mass. Determining the merit of the ideas, whether they turn out to be lasting or ephemeral, will require examining them against the body of physical facts and theories. This would have been easier a half century ago when Wheeler and Feynman's paper appeared; physics was not so well developed at that time. On the other hand, the present theory might help answer questions remaining even today.

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References

[1] J. Barbour and H. Pfister (Eds.) *Mach's Principle - From Newton's Bucket to Quantum Gravity*, (Birkhauser, 1995).
[2] H. Bondi and J. Samuel, *Phys. Lett.* A, **228**, 121 (1997).
[3] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, (Freeman, 1973), p. 543.
[4] D. W. Sciama, *Monthly Notices of the Royal Astronomical Society* **113**, 34 (1953).
[5] Max Jammer, *Concepts of Mass in Contemporary Physics and Philosophy,* (Princeton University Press, Princeton, N.J., 2000), pp. 151-156.
[6] W. F. Edwards, *Proceedings of the Utah Academy of Sciences, Arts, and Letters* **51**, 1 (1974).
[7] A.K.T. Assis, *Foundations of Physics Letters* **2**, 319 (1989).
[8] A.K.T. Assis*, Canadian Journal of Physics* **70**, 330 (1992); *Journal of the Physical Society of Japan* **62**, 1418 (1993), *Pramana Journal of Physics* **55**, 393 (2000), and many others.
[9] P.A.M. Dirac, *Proc. Roy. Soc.* A, **165**, 199 (1938); V. Gordon Lind first drew attention to the connection between the present work and Dirac's LNH.
[10] J. A. Wheeler and R. P. Feynman, *Rev. Mod. Phys.* **17**, 157 (1945); See also J. A. Wheeler and R. P. Feynman, *Rev. Mod. Phys.* **21**, 425 (1949).
[11] See, for instance, F. Hoyle and J. V. Narlikar, *Proc. Roy. Soc.* A, **277**, 1 (1964) and *Annals of Physics* **54**, 207 (1969).
[12] J. E. Hogarth, *Proc. Roy. Soc.* A, **267**, 365 (1962).
[13] I. Levine *et al*., *Phys. Rev. Lett.* **78**, 424 (1997). See also K. Miyabayashi *et al*., *Phys. Lett.* B **347**, 171-178 (1995).

Mach's Principle and Inertial Forces in General Relativity

A.R. Prasanna*

Introduction

One of the most debated topics of classical physics is the concept of "inertia," which according to Newton is an inherent attribute, devoid of any external influence. Newton, in spite of realising the problems inherent in the concept of absolute space, had assumed its existence, to describe motion, relative to it. In spite of the successes of Newtonian mechanics, Leibniz and later Bishop Berkeley were among the vociferous critics of the notion of absolute space, claiming it to be metaphysical.

However, it was Ernst Mach, who boldly rejected the concept of 'absolute space' and instead introduced that all motions are described relative to a 'fixed frame' as defined by the Universe (matter distribution) at large, which he put in terms of defining 'inertia'. According to Mach's point of view, 'Inertia arises due to interaction of a body with the rest of the Universe, with the so-called fixed stars background given by the average motion of the cosmic sources, which provides the inertial fame'. Einstein apparently expressed these ideas of Mach in terms of what is presently called 'Mach's principle'—"the whole inertia of any material point is an effect of the presence of all other masses, depending on a kind of interaction with them." (Carl Hoefer, 1995) There are indeed several interpretations as to what exactly Mach's principle is (Tübingen Conference, 1993) and whether the most successful theory of gravity, *viz*., general relativity is Machian or not.

If one looks at the spirit of Machian idea it simply says that what is observable is only the relative motion of a body with respect to other bodies and that the inertial motion of a body is influenced by all other masses of the Universe (Goenner, 1995). Einsteinian idea of geometrising physics to describe gravity through field equations, which relate space-time with matter distribution is indeed non-local and any description of inertial within this structure should be in consonance with Mach's principle. As Brill (1995) proposes: "Of all the predictions that follow from or have been read out of Mach's principle, the dragging of inertial frames by rotating bodies is certainly the most definite and least controversial. If one measures this dragging by Coriolis forces, then the answer of general relativity is unambiguous."

Rigorous mathematical treatment of the problem of finding exact solution for a rotating shell of matter by Pfister and coworkers (1995) on the lines following the works of Thirring (1918), Brill and Cohen (1966), Pfister *et al.*

* Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India

(1989, 1990, 1992), and Meinel and Kleinwachter (1995) has shown several important features regarding the inertial forces—centrifugal and Coriolis within the framework of general relativity, depicting Machian idea of inertia.

We would like to follow a different approach, in first reformulating the equations of motion of general relativity in Newtonian language of (space+time) and then looking at the different parts of the total acceleration, which indeed can be identified as 'inertial accelerations', including the effects due to space-time curvature.

Formalism

As the idea is to bring in Newtonian language into a geometric theory of space time, one needs to slice the 4-space into a (3 space + time) structure and look at the features on the absolute 3 space so obtained. In fact, such a 3+1 split of space time is nothing new in general relativity, as, long ago Arnowitt, Deser and Misner (1962) introduced such a scheme while looking for a method to give a Hamiltonian description of general relativity. As Misner, Thorne and Wheeler (1972) mention:

> The slicing of space time into a one parameter family of space-like hypersurfaces is called for, not only by the analysis of the dynamics along the way, but also by the boundary conditions as they pose themselves in any action principle of the form—give the 3-geometries on the two faces of a sandwich of space time and adjust the 4-geometry in between to extremize the action.

Instead of a fully dynamical system, suppose one has a stationary system wherein a time-like Killing vector exists then one can get a lower dimensional quotient space through an isometry group action and one can study certain dynamical features within a given geometrical background. Abramowicz, Carter and Lasota (1988, hereafter referred to as ACL) used such a prescription with a conformal rescaling factor and showed that one can indeed obtain a 3+1 splitting wherein the 3-space is the quotient space obtained from the action of the time-like Killing vector and the metric conformal to the spatial geometry of the original four-space. As they realised the most significant feature of a conformal reslicing was that the normally geometrical geodesic equation for a test particle would separate into language of Newtonian forces wherein one can directly interpret terms as gravitational, centrifugal and Coriolis accelerations.

Abramowicz, Nurowski and Wex (1993, hereafter referred to as ANW) later gave a covariant approach to this formulation which does not depend upon any particular symmetry and is as follows:

In the given space time manifold M with the metric

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu \tag{1}$$

introduce a congruence of world lines which is globally orthogonal to $t = \text{const.}$, hypersurface which ensures that the vorticity of the congruence to be zero. In fact, Bardeen (1972) adopted such a congruence in axisymmetric, stationary space times defining what are called locally non-rotating observers or zero angular momentum observers (ZAMO). The advantage of having such a congruence is that these local observers 'rotate with the geometry' and the connecting vectors between two such observers with adjacent trajectories do not precess with respect to Fermi-Walker transport. Denoting such a vector field by n^μ ($n_\mu n^\mu = -1$ time-like) it can be verified that the corresponding four-acceleration is proportional to the gradient of a scalar potential.

$$n^\nu \nabla_\nu n_\mu = \nabla_\mu \phi; \; n^\mu \nabla_\mu \phi = 0 \tag{2}$$

Though the vector field n^μ is not uniquely determined by (2), locally each particular choice of n^μ uniquely defines a foliation of the space time into slices each of which represents space at a particular instant of time, whose geometry is given by

$$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu; \; h^\mu_\alpha = \delta^\mu_\alpha + n^\mu n_\alpha \tag{3}$$

(2) also ensures that the special observers n^μ see no change in the potential as their proper time passes by and thus have fixed positions that help them in distinguishing between different 'inertial forces'.

Consider a particle of rest mass m_o and four-velocity U^μ, which can be expressed as

$$U^{\mu =} \gamma(n^\mu + v\tau^\mu) \tag{4}$$

wherein τ^μ is the unit tangent vector (space-like) orthogonal to n^μ and parallel to the 3-velocity v of the particle in the 3-space (Lorentz speed) and γ the Lorentz factor $\left(=1/\sqrt{1-v^2}\right)$. The four-acceleration of the particle a^μ may now be obtained through direct computation (Abramowicz, 1993)

$$\begin{aligned} a_\alpha = u^\mu \nabla_\mu u_\alpha &= -\gamma^2 \nabla_\alpha \phi + \gamma^2 v \nabla \left(n^\mu \nabla_\mu \tau_\alpha + \tau^\mu \nabla_\mu n_\alpha \right) \\ &+ \gamma^2 v^2 \tau^\mu \nabla_\mu \tau_\alpha + (v\gamma) \cdot \tau_\alpha + \dot{\gamma} n_\alpha \end{aligned} \tag{5}$$

Using now the ACL approach of conformal rescaling of the 3-metric, one can define the projected metric and the vectors

$$\tilde{h}_{\mu\nu} = e^{2\phi} h_{\mu\nu}; \; \tilde{\tau}^\mu = e^\phi \tau^\mu \tag{6}$$

such that the acceleration may now be written as

$$a_\alpha := -\nabla_\alpha \phi + \gamma^2 V \left(n^\mu \nabla_\mu \tau_\alpha + \tau^\mu \nabla_\mu n_\alpha \right) + (\gamma V)^2 \tilde{\tau}^\mu \tilde{\nabla}_\mu \tilde{\tau}_\alpha \tag{7}$$

with the last two terms in (5) becoming zero for a particle with constant speed, and conserved energy. $\tilde{\nabla}$ in (7) refers to the covariant derivative with respect to the metric $\tilde{h}_{\mu\nu}$. As may be seen, the acceleration is made up of three distinct terms, (*i*) gradient of a scalar potential, (*ii*) a term proportional to V, and (*iii*) one proportional to V^2. The first and the third terms may be immediately recognised as the gravitational and centrifugal accelerations. Further, as n^μ and τ^μ are parallel to the Killing vector η^μ and ζ^μ respectively using the definition of Lie derivative, one can obtain the second term in (7) to be

$$\gamma^2 V\left[n^\mu\left(\nabla_\mu \tau_\alpha - \nabla_\alpha \tau_\mu\right)\right] \tag{8}$$

which represents the Lense-Thirring effect of the inertial drag and thus identified as the generalization of the Coriolis acceleration.

If the general axisymmetric and stationary space time is represented by the metric

$$ds^2 = g_{tt}\, dt^2 + 2g_{t\phi} dt\, d\phi + g_{\phi\phi} d\phi^2 + g_{rr}\, dr^2 + g_{\theta\theta} d\theta^2 \tag{9}$$

with $g_{\mu\nu}$ being functions of r and θ only then the total force acting on a particle in circular orbit with constant speed Ω may be expressed as (Prasanna, 1997)

$$F_\mu := (Gr)_\mu + (Co)_\mu + (Cf)_\mu \tag{10}$$

wherein

$$\begin{aligned}(Gr)_\mu &:= -\nabla_\mu \phi = \frac{1}{2}\partial_\mu \left\{\left[\left(g_{t\phi}^2 - g_{tt} g_{\phi\phi}\right) / g_{\phi\phi}\right]\right\} \\ (Co)_\mu &:= \gamma^2 V n^\alpha \left(\nabla_\alpha \tau_\mu - \nabla_\mu \tau_\alpha\right) \\ &= -a^2 (\Omega - \omega)\sqrt{g_{\phi\phi}}\left\{\partial_\mu \left(\frac{g_{\tau\phi}}{\sqrt{g_{\phi\phi}}}\right) + \omega \partial_\mu \sqrt{g_{\phi\phi}}\right\}\end{aligned} \tag{11}$$

and

$$\begin{aligned}(Cf)_\mu &= (\gamma V)^2 \tilde{\tau}^\alpha \tilde{\nabla}_\alpha \tilde{\tau}_\mu \\ &= -A^2 (\Omega - \omega)^2 \frac{g_{\phi\phi}}{2} \partial_\mu \left\{\ln \frac{g_{\phi\phi}^2}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}\right\}\end{aligned} \tag{12}$$

with

$$\Phi = -\frac{1}{2}\ell\left[-g_{tt} - 2\omega g_{t\phi} - \omega^2 g_{\phi\phi}\right];\ \omega = -g_{t\phi} / g_{\phi\phi}$$

and

$$A^2 = -\left[g_{tt} + 2\Omega g_{t\phi} + \Omega^2 g_{\phi\phi}\right]^{-1} \tag{13}$$

If instead of a single particle we need to consider the forces acting on a fluid element, then it is useful to consider the 3+1 ADM splitting

$$ds^2 = -\left(\alpha^2 - \beta^2\right) dt^2 + 2\beta_i dx^i dt + \gamma_{ij}\, dx^i\, dx^j \tag{14}$$

with the lapse function, β_i the shift vector and γ_{ij} the 3-space metric

$$\gamma_{ij} = g_{ij} + n_i\, n_j \tag{15}$$

and the spatial 3-velocity

$$V^i = \frac{\left(\frac{U^i}{U^t} + \beta^i\right)}{\alpha} \tag{16}$$

One can now relate these two splittings of the axisymmetric, stationary space-time through the definition

$$n^{\mu} = \left(\frac{1}{\alpha}, -\frac{\beta^i}{\alpha}\right)$$

$$n_{\mu} = (-\alpha, 0) \tag{17}$$

$$\tau^{\mu} = \left(0, \frac{V^i}{V}\right)$$

$$\tau_{\mu} = \left(\frac{\beta^i V^i}{V}, \frac{V_i}{V}\right)$$

Using (16) and (17) one can express the components of acceleration vector a_{μ} in terms of α, β_i, γ_{ij} and the 3-velocity components v^i.

The centrifugal acceleration acting on a fluid element $\tilde{V}^2 \tilde{\tau}^{\nu} \tilde{\nabla}_{\mu} \tilde{\tau}_{\mu}$ is given by

$$(Cf)_i = \gamma^2 \left[VV^j \partial_j \left(\frac{V_i}{V}\right) + \left(V_i V^j \partial_j - V^2 \partial_i\right)(\Phi) - \frac{1}{2} V^j V^k \partial_i \gamma_{jk} \right] \tag{18}$$

while the Coriolis type (Lense-Thirring) is given by

$$(Co)_i = -\frac{\gamma^2}{\alpha} \left[V \beta^j \partial_j \left(\frac{V_i}{V}\right) - V^j \partial_i g_{oj} + \beta^k V^j \partial_i \gamma_{kj} \right] \tag{19}$$

Thus for any given background geometry one can evaluate the specific acceleration components, if one has the 3-velocity field of the fluid on that geometry evaluated through the equations of motion.

Specific Applications

We start from the simplest application of the methodology outlined above to study the particle kinematics in the static space times, taking the Schwarzschild geometry as the first example (Abramowicz and Prasanna, 1990; hereafter referred to as AP).

The metric as expressed in the usual coordinates,

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{20}$$

would yield for the gravitational and centrifugal accelerations, acting on a test particle in circular orbit, the expressions:

$$(Gr)_r = \frac{m}{r^2}\left(1 - \frac{2m}{r}\right)^{-1} \tag{21}$$

and

$$(Cf)_r = -\Omega^2 r\left(1-\frac{2m}{r}\right)^{-1}\left(1-\frac{2m}{r}-\Omega^2 r^2\right)^{-1}\left(1-\frac{3m}{r}\right) \tag{22}$$

From (22) it is clear that while (*Gr*) and (*Cf*) are in opposite directions up to $r = 3m$ from infinity for $r < 3m$ they both have the same direction. As gravitational acceleration is always undirectional, it is clear that the centrifugal acceleration reverses its sign *at* $r = 3m$.

In fact, if one looks at the photon effective potential in the Schwarzschild geometry, one finds that $r = 3m$ is the location of the maximum and thus corresponds to unstable circular orbit. What has been noticed now is that this null line is the straight line path for the photon in the quotient space and thus corresponds to a location at which the centrifugal acceleration is zero, and on either sides the centrifugal force acts in opposite directions.

As shown in AP, this feature has important kinematical implications in the study of accretion flows near ultra compact objects (black holes) like the Rayleigh criterion for stability of flow turns out to be

$$\frac{2m\left(r^3-2mr^2\right)}{\left(r^3-\ell^2 r+2m\ell^2\right)^2}\left(1-\frac{3m}{r}\right)\frac{d\ell^2}{dr} > 0 \tag{23}$$

This means $d\ell^2/dr > 0$ for $r > 3m$ and < 0 for $r < 3m$, indicating that for $r < 3m$ the angular momentum has to be advected inwards for stability. In fact, this result clearly explained the findings of Anderson and Lemos (1988), who had obtained inward advection of angular momentum very close to black holes.

On the other hand, instead of circular orbit, the particle is on a non-circular orbit ($U^r \neq 0$) then one can get the expression for V^r and V^ϕ the components of 3-velocity, using the constants of motion

$$\left(1-\frac{2m}{r}\right)U^t = -E, \quad r^2 U^\phi = \ell \tag{24}$$

and the radial velocity

$$U^r = \sqrt{E^2-\left(1-\frac{2m}{r}\right)\left(1+\frac{\ell^2}{r^2}\right)} \tag{25}$$

as given by

$$\left(V^r\right)^2 = \left(1-\frac{2m}{r}\right)\left\{1-\frac{1}{E^2}\left(1-\frac{2m}{r}\right)\left(1+\frac{\ell^2}{r^2}\right)\right\} \tag{26}$$

$$\left(V^\phi\right)^2 = \frac{\ell^2}{E^2 r^4}\left(1-\frac{2m}{r}\right)$$

Using these in the expression (24) for the centrifugal force, one finds

$$(Cf)_r = \frac{-m\ell^2}{r^2\left[2m-\left(1-E^2\right)r\right]} \tag{27}$$

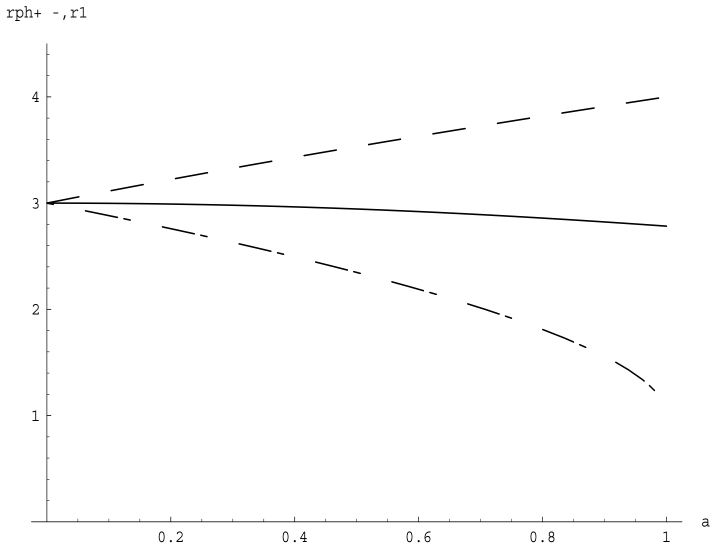

Fig. 1. Location of the point where *Cfg* = 0, (--------), retrograde photon orbit (- - - -) and of the prograde photon orbit (-. -. -.) for different values of *a*.

indicating no reversal, unlike in the case of circular orbits.

As mentioned in the introduction, rotation plays an important role in the discussion of space time kinematics and thus it should be more interesting to study the behaviour of inertial forces in stationary space times rather than the static ones. We shall now consider the space time exterior to a rotating black hole, as given by the Kerr metric

$$ds^2 = \left(1 - \frac{2mr}{\Sigma}\right)dt^2 - \frac{4amr}{\Sigma}\sin^2\theta dt d\phi + \frac{\Sigma}{d\theta^2} + \frac{B}{\sigma}\sin^2\theta d\phi^2 \tag{28}$$

with

$$\begin{aligned} \Delta &= r^2 - 2mr + a^2 \\ \Sigma &= r^2 + a^2\cos^2\theta \\ B &= \left(r^2 + a^2\right) - \Delta a^2 \sin^2\theta \end{aligned}$$

For a particle in circular orbit one can then get the components of forces in the radial direction as given by

$$(GR)_r = \frac{m\left(a^2\Delta + r^4 + a^2r^2 - 2ma^2r\right)}{r\Delta\left(r^3 + a^2r + 2ma^2\right)} \tag{29}$$

$$(Cf)_r = \frac{-(\Omega-\omega)^2\left[r^5 - 3mr^4 + a^2\left(r^3 - 3mr^2 + 6m^2r - 2ma^2\right)\right]}{r^2\Delta\left[1 - \Omega^2\left(r^2 + a^2\right) - \left(\frac{2m}{r}\right)(1 - \Omega a)^2\right]} \tag{30}$$

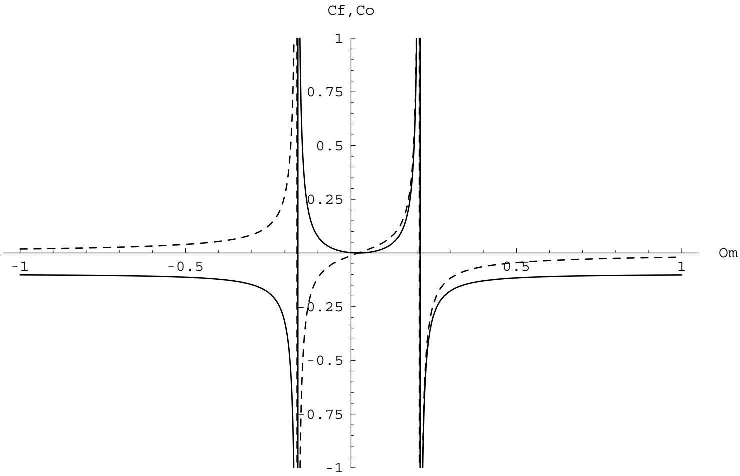

Fig. 2. Centrifugal (-------) and Coriolis (- - -) force plots for a = 0.5, along the retrograde photon orbit for $-1 < \Omega < 1$.

$$(Co)_r = \frac{2ma(\Omega-\omega)\left(3r^2+a^2\right)}{r\left(r^3+a^2r+2ma^2\right)\left[1-\Omega^2\left(r^2+a^2\right)-\left(\frac{2m}{r}\right)(1-\Omega a)^2\right]} \tag{31}$$

One can immediately see the difference in the nature of centrifugal and Coriolis forces, whereas the Coriolis depends on the coupling of the angular momentum of the central source with that of the particle $a(\Omega-\omega)$ the centrifugal can go to zero at different locations solely depending upon 'a' due to the zeros of the quintic equation

$$r^5 - 3mr^4 + a^2\left(r^3 - 3mr^2 + 6m^2r - 2ma^2\right) = 0 \tag{32}$$

It may be easily seen that for $0 < a < 1$ the equation (32) at best can have only three real roots of which, one is always definitely outside the event horizon (Iyer and Prasanna, 1993) as depicted in Fig. 1. However, as also shown in this figure this location does not coincide with the location of the unstable photon orbit, prograde or retrograde. The centrifugal force vanishes at a location between the two photon orbits and for the case $a = 0$, they all coincide at $r = 3m$. Unfortunately, the direct link between the unstable photon orbit and the centrifugal force reversal, depicted in static space times do not find a parallel in stationary space time. Rotation does indeed bring in some new features of which the frame dragging is the most important one.

Figs. 2 and 3 show the nature of the centrifugal and Coriolis forces at the location of retrograde and direct photon orbits as a function of Ω for $a = 0.5$. The first impression that one gets is that these forces change sign for different values of Ω across the asymptotes. However, one has to check that the asymptotes are caused by the infinity of the redshift factor A^2 at the roots of the equation

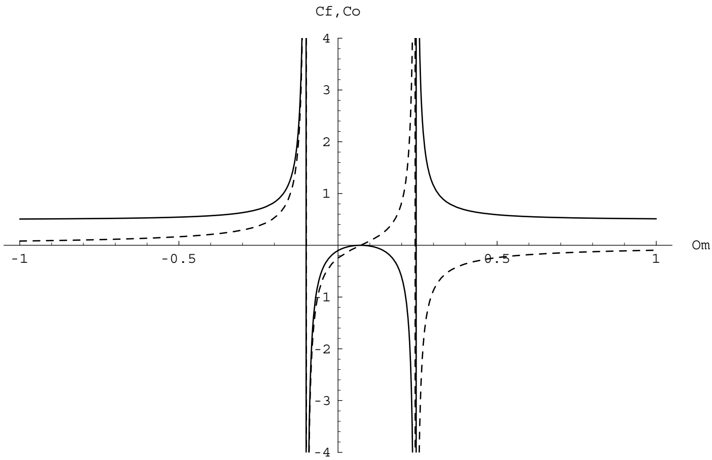

Fig. 3. Same as (2) at the prograde photon orbit.

$$\Omega^2 g_{\phi\phi} + 2\Omega g_{t\phi} + g_{tt} = 0 \tag{33}$$

$$\Omega_{\pm} = \omega \pm \sqrt{\omega^2 - g_{tt} / g_{\phi\phi}} \tag{34}$$

Hence the only portion of the plots which is meaningful is the region corresponding to the values of $\Omega; \Omega_- < \Omega < \Omega_+$. As may be seen in this region the centrifugal is positive along the retrograde photon orbit (*rph*+) and negative along the direct photon orbit (*rph*−) as it should be according to Fig. 1. On the other hand, the Coriolis changes sign in both cases at the point $\Omega = \omega$ wherein centrifugal also vanishes because of the fact that the angular velocity is just equal to the dragging of inertial frames, by the space time due to the rotation of the central object.

On the other hand, we have seen that there are locations in the given space time, wherein the centrifugal force is zero but $\Omega \neq \omega$ and thus the Coriolis is non-zero. In view of this, Prasanna (1997) defined an index of reference called the 'Cumulative Drag Index' as defined by the ratio

$$C = \frac{(Co - Gr)}{(Co + Gr)}$$

which could characterize purely the rotational feature of a space time through its influence on a particle in circular orbit at the location where the centrifugal force is zero.

Fig. 4 shows the plot of C as a function of Ω for a fixed a and R. As may be seen, there are two zeros and two infinities for the function. As $a > 0$, $\Omega > 0$ represents the co-rotating particles and $\Omega < 0$ represents the counter-rotating particles.

As the orbit chosen is the one where the centrifugal force is identically zero the infinities of C refer to the trajectories along which the total force acting on the particle is zero keeping it in equilibrium. Fig. 5 shows the index C

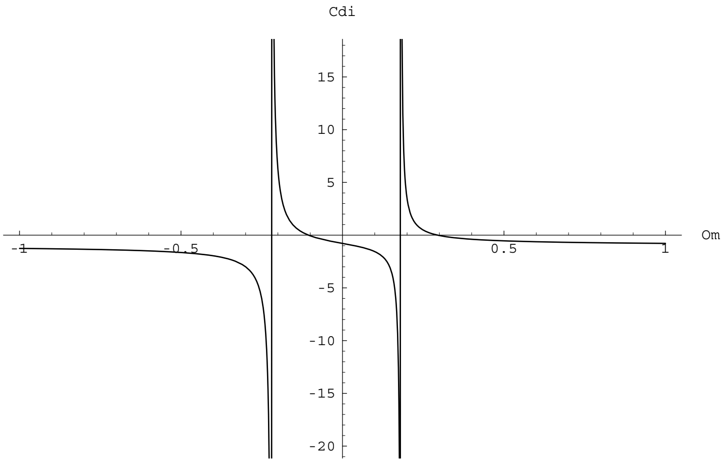

Fig. 4. Cumulative drag index C for a = 0.5, R = 2.9445 as a function of (Ω – 1 < Ω < 1).

for three distinct cases of a = 0.1, 0.5 and 1 at the respective locations where $Cf = 0$.

It is interesting to note that as a increases, the co-rotating particles have to decrease their angular velocity Ω very little to stay in equilibrium, whereas the counter-rotating ones have to increase their Ω much more, as depicted explicitly in Table 1.

It may indeed be seen that, the excess adjustment on the part of counter-rotating particles is essentially due to the dragging of inertial frames ω which is always in the direction of rotation of the central source. Table 1 gives the values of ω for given a and R, which matches quite closely with the difference in the corresponding Ω for given a and R, between co- and counter-rotating particles.

Table 1

a	R	Ω_+	Ω_-	$\vert\Omega_- - \Omega_+)\vert$	ω
0.1	2.9978	0.189022	–0.1964	0.0074	0.0074
0.5	2.9445	0.180094	–0.21965	0.0395	0.0374
1.0	2.7830	0.17722	–0.27452	0.097	0.076

Discussion

The reversal of centrifugal force acting on a test particle in circular orbit at the last circular unstable photon orbit, in static space time has drawn some attention in recent times in varying context ranging from X-ray sources to infinitely multiple image forming. Actually one can clearly see that these features are closely associated with the photon behaviour in static space times. Though test particle trajectories do give some understanding of the geometry on which they are moving, for astrophysical applications it is fluid flows that are important.

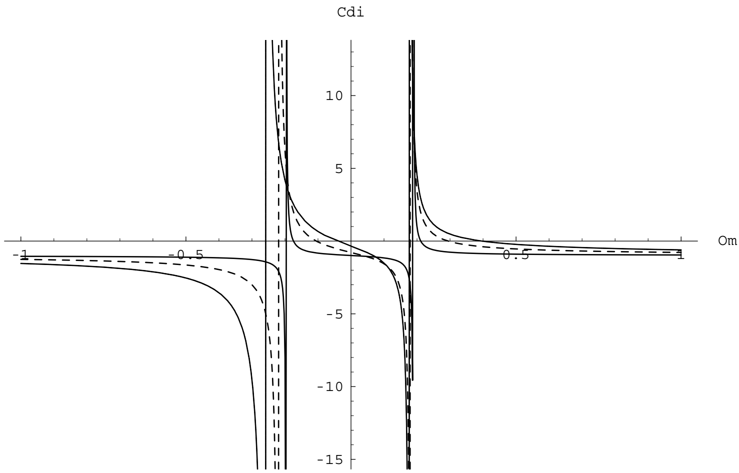

Fig. 5. C for three different values of a showing difference in Ω_{-} values for equilibrium orbit ($Co = -Gr$).

Having obtained the expressions for the inertial accelerations in terms of the fluid velocity components and the metric components one can now write down the individual forces on any background space time, provided one has the solution for the velocity components on the given background for the flow. For a dusty fluid ($p = 0$) as may be expected for a purely radial flow $\left(V^{\theta} = 0, V^{\phi} = 0\right)$ the centrifugal acceleration is zero, whereas for a purely azimuthal flow $\left(V^{r} = 0, V^{\theta} = 0, V^{\phi} \neq 0\right)$, the centrifugal acceleration is just as for a test particle, both in Schwarzschild and the Kerr background. The fluid in this case is a collection of test particles in circular orbits and thus the result is as known earlier.

However, it is important to understand the behaviour of forces for a more general motion of particles, when the trajectory is a non-circular geodesic. For

Table 2

Location of the last root of $Cfr = 0$ and the second root of $(F2-F3)=0$, along with the location of event horizon (EH) for different values of a.

a	**EH**	**(*Cfr*)=0**	**(*F2-F3*)=0**
0.1	1.00499	-	2.0615
0.2	1.9798	-	2.10945
0.3	1.95394	-	2.14586
0.4	1.91652	1.28011	2.17201
0.5	1.86603	1.48094	2.18871
0.6	1.8	1.54586	2.19647
0.7	1.71414	1.78834	2.19554
0.8	1.6	1.91397	2.18595
0.9	1.43589	2.02589	2.16752
1.0	1.0	2.12612	2.13987

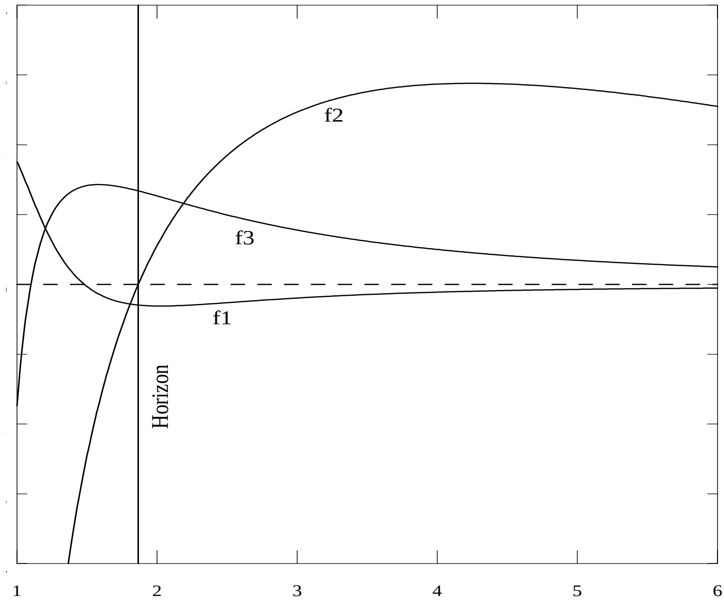

Fig. 6: Plots of *Cfr* (*f*1), U^r/U^t (*f*2) and U^ϕ/U^t (*f*3) for the parameter values E = 0.9, ℓ = 1 and a = 0.5. The vertical line shows the location of the horizon.

such a case, it is easy to find the expressions for three velocity components from the components of the four velocity, which are directly integrable from the equations of motion using the symmetries of the given space time. This would bring in the physical characteristics of the particle, the energy E and angular momentum ℓ and restricting the discussion to particles on the equatorial plane $\left(\theta = \Pi/2,\ U^\theta = 0\right)$, one can easily obtain the components V^r and V^ϕ for the particle.

From the expression for $(Cf)_r$ in Kerr space time, it is clear that its behaviour depends upon the particle parameters E and ℓ and the rotation parameter 'a'. Direct evaluation for fixed E and ℓ shows that when $E \leq 1$, the reversal radius occurs outside the event horizon only for very high value of a (Table 2). As may be seen, the table also gives for the same set of parameters the radius at which $\left(F_3 = U^\phi/U^t\right)$ crosses over $\left(F_2 = U^r/U^t\right)$ and it is clear that the centrifugal reversal occurs only after the angular velocity supersedes the radial velocity. Figures 6 to 8 show the behaviour of the curves, centrifugal (*f*1), radial velocity (*f*2) and the azimuthal velocity (*f*3) for different values of a, for same E and ℓ.

Indeed one can see that the centrifugal reversal which was inherent for circular geodesics in both static and stationary spacetimes, does not follow automatically for general non-circular geodesics. Whereas it does not occur at all in static spacetime, in stationary spacetime, the occurrence depends upon the energy, angular momentum of the particle and the rotation parameter a, which needs to be sufficiently high for the reversal to occur outside the event

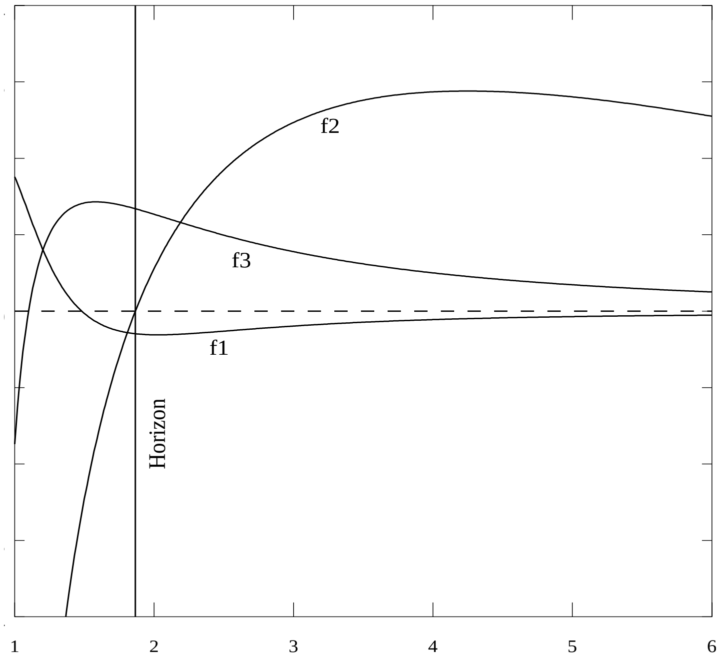

Fig. 7: Same as Fig. 6, but for *a* = 0.7.

horizon. The difference in the behaviour for static and stationary spacetimes is essentially due to the fact that the rotation induces 'frame dragging', which adds to the azimuthal velocity and makes it larger than the radial velocity after which the behaviour resembles that of a circular orbit. This interpretation gets further support from the fact that for particles in retrograde motion ($a > 0$, $\ell < 0$), there are no positive real roots for the equation $(Cf)_r = 0$.

Thus we find that for particles on non-circular geodesics, in static spacetime, there is no reversal of centrifugal force, and in Kerr spacetime, the reversal occurs only for particles in prograde motion, which get the additional input to their angular velocity from the effects of frame dragging. For particles in retrograde motion, the frame dragging contribution would not suffice to overcome the effects of radial velocity and thus like in static case shows no reversal.

It is well known that the Kerr parameter 'a' in the stationary case is interpreted through the asymptotic boundary condition, as defining the rotation of the source with respect to distant fixed frame. The concepts of inertial forces, the centrifugal and Coriolis, as known, do depend on the existence of a far away inertial frame in a fixed background. The inertial frame dragging as envisaged in the Kerr geometry is indeed considered as a true Machian effect (Brill, 1994). We have seen above how this effect influences the particle in curved geometry. As the 3-velocity of the test particle does depend upon the local physics, one can clearly see from the above example the inherent aspects of Mach's principle in general relativity.

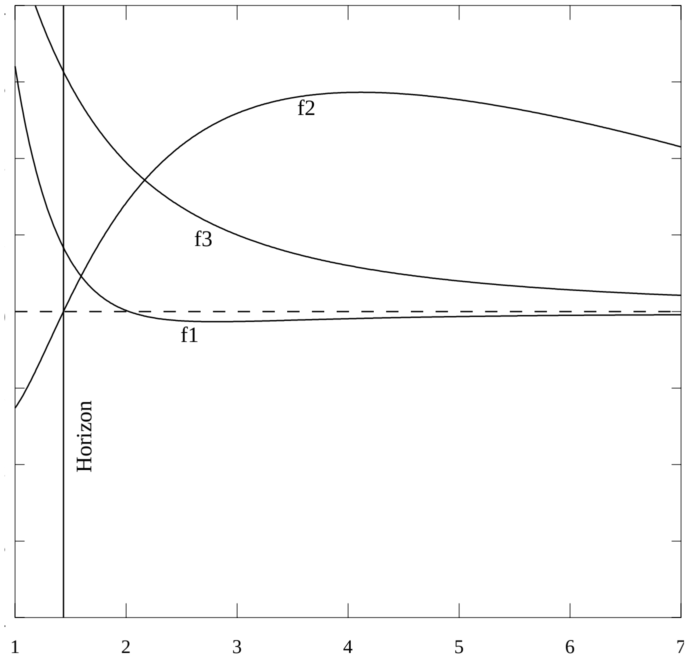

Fig. 8: Same as Fig. 6, but for a = 0.9.

References

Abramowicz M.A., Carter B. and Lasota, J.P. (1988), *Gen. Rel. & Grav.* **20**, 1173.

Abramowcicz M.A. and Prasanna A.R. (1990) *Mon. Not. R. Astr. Soc.* **245**, 720.

Abramowicz M.A. (1993) *Sci. Am.* **268**, 74.

Abramowicz M.A., Nurowski and Wex (1993) *Class. Quant. Gr.* **10**, L183.

Abramowicz M.A., Nurowski and Wex (1995) *Class. Quant. Gr.,* **12**, p. 1467.

Anderson M.R. and Lemos J.P.S. (1988) *M.N.R.A.S.,* **233**, p. 489.

Arnowitt Deser S. and Misner C.W. (1962), in: *Gravitation - An Introduction to Current Research*, Ed. L. Witten, p. 227-265.

Bardeen J.M., Press W.H. and Teukolsky S.A. (1972) *Astrophys. J.*, **178**, p. 347.

Brill D.R. (1995), in: *Mach's Principle from Newton's Bucket to Quantum Gravity* (Tübingen Conference Proceedings (1993), Eds. J. Barbour and H. Pfister, p. 333.

Chandrasekhar (1983) *Mathematical Theory of Black Holes,* p. 328.

Goenner H.F.M. (1995), in: *Mach's Principle from Newton's Bucket to Quantum Gravity* (Tübingen Conference Proceedings (1993), Eds. J. Barbour and H. Pfister, p. 442.

Hasse W. and Perlick V. (2001) *gr-qc 0108002*.

Heyl J.S. (2000) *Ap. J. (Letters)* **542**, L45.

Hoefer C. (1995), in: *Mach's Principle from Newton's Bucket to Quantum Gravity* (Tübingen Conference Proceedings (1993), Eds. J. Barbour and H. Pfister.

Iyer S. and Prasanna A.R. (1993) *Class. & Qu. Gr.* **10**, L13.

Misner C.V., Thorne K.S. and Wheeler J.A. (1973) *Gravitation*, Freeman & Co.

Pfister H. (1995), in: *Mach's Principle from Newton's Bucket to Quantum Gravity* (Tübingen Conference Proceedings (1993), Eds. J. Barbour and H. Pfister, p. 315 (and the references therein).

Prasanna A.R. (1997) *Class. Quan. Grav.*, **14**, p. 227.

Prasanna A.R. (2001) *GR-QC 0109020*.

York J.W. Jr. (1983), in: *Gravitational Radiation*, ed. N. Deruelle and T. Piran (Amsterdam, New York: North Holland) 175.

Inertial Mass in a Machian Framework

Jorge Guala Valverde* and Raúl A. Rapacioli†

We review the law of proportionality between gravitational mass and inertial mass within a framework consistent with the Principle of Mach as recently implemented by Assis.

Keywords: Relational Mechanics. Gravitational mass. Inertial mass. Mach's Principle. Dimensional Analysis.
PACS: 04.50.+ h (Unified field theories and other theories of gravitation), 12.25. +e (Models for gravitational interactions).

Relativity in Newtonian and Post Newtonian Mechanics

The law of force in Newton's theory of gravitation, written in obvious notation,

$$\vec{F}_{21}^{N} = -\left(\frac{m_{g1}m_{g2}}{r^2}\right)\hat{r} \tag{1}$$

is the first relativistic universal law which appeared in the development of science, since only the *gravitational masses*, m_{gk} involved enter in its formulation, and its instantaneous mutual distance, $r = r_{12} = \left|\vec{r}_1 - \vec{r}_2\right| \vec{F}_{21}$ is the force exerted by the point mass 2 on the point mass 1. Note that the masses involved in equ. (1) have nothing to do, a *priori*, with the *inertial mass* appearing in Newton' s second law, $f = m_i \boldsymbol{a}$. The force law, plus the proportionality law,[1,2] valid for any material particle *k*,

$$m_{gk} = \sqrt{G} m_{ik} \tag{2}$$

between gravitational mass and inertial mass, m_{ik}, suffice to explain most of the observed gravitational facts. Here $G = 6.67\ 10^{-11}\ \mathrm{m^3kg^{-1}s^{-2}}$ means the universal constant of gravitation. The mutual gravitational energy (potential energy) coherent with equ. (1) is

$$U^{N} = -\left(\frac{m_{g1}m_{g2}}{r}\right), \tag{3}$$

from which equ. (1) follows when the usual procedure $\vec{F} = -\nabla U$ is performed.

Despite being one of the best verified laws of physics, with a relative uncertainty below 10^{-11}, equ. (2) only appears as a fortuitous coincidence in classical mechanics (CM). This fact intrigued Mach throughout his life, and he thus envisaged the idea that distant matter should regulate, inertially, local interactions. Referring to the Newton's well-known bucket experiment, he said:[3]

> Try to fix Newton's bucket and rotate the heaven of fixed stars and then prove the absence of centrifugal forces.

* Fundación Julio Palacios, Neuquén, Argentina; Confluencia Tech University Neuquén, Santa Fe 449 – 8300 – Neuquén – Argentina. Email: gual@ieee.org, fundacionjuliopalacios@usa.net
† Universidad Nacional del Comahue, Neuquén, Argentina. E. Mail: rrapacio@uncoma.edu.ar

In 1925 Schrödinger sought the origin of inertia by modifying equ. (3) in a suitable manner.[4,5] Guided by heuristic arguments he wrote, for two interacting point masses:

$$U^S = U^N\left(1-\frac{\varepsilon \dot{r}^2}{c^2}\right) \tag{4}$$

where $\dot{r} \equiv dr/dt$; c is the velocity of light in a vacuum; and ε is a dimensionless parameter that becomes 3 in order to fit the observed planetary precession.

Schrödinger emphasized the fact that any interaction energy should depend only on the separation and relative velocity between the particles, in order to follow Mach's views.

With the aid of his modified energy, Schrödinger calculated the energy of interaction for a spherical shell (gravitational mass M_g, radius R) interacting with an internal point mass m_g, moving in the neighbourhood of its center. Thus, he obtained:

$$U = -\left(\frac{M_g m_g}{R}\right)\left(1-\frac{v^2}{c^2}\right) \tag{5}$$

Schrödinger identified the component of this potential energy which depends on the velocity with the kinetic energy of the particle, $K = m_i v^2/2$. That is, $M_g m_g v^2/Rc^2 = m_i v^2/2$. It then follows:

$$m_i = \left(\frac{2M_g}{Rc^2}\right)m_g = \left(\frac{8\pi\sigma R}{c^2}\right)mg \tag{6}$$

wherein σ labels the (assumed constant) surface density of gravitational mass and v is the velocity of the moving particle, referred to the rest frame centered in the sphere. Later on, Schrödinger integrated the result of the spherical shell for a "world" of radius R_o, supposing a constant mass density. He concluded that taking the radius and the mass density of our own galaxy, we would obtain a value of G some 10^{11} times smaller than what is really measured. Therefore, the inertia of particles in the solar system must be mainly due to matter farther away from our galaxy.

Relational Mechanics: A Recent Implementation of Mach's Principle

The pioneer work of Schrödinger was recently improved by Assis,[6,7,8,9] who was able to implement Mach's ideas in a rigorous, entirely general, way. Following Schrödinger, the starting point of Assis formulation is a Weber-like law of force, which reads, in obvious notation,

$$\vec{F}_{21} = H_g \vec{F}^N_{21}\left(1-\frac{\xi \dot{r}^2}{2c^2}+\frac{\xi r\ddot{r}}{c^2}\right) \tag{7}$$

where $\ddot{r} \equiv d^2r/dt^2$; and H_g and ξ are constants. The outstanding mathematical property of equ. (7) is that it is *invariant* (frame independent), which means that each term in the Weber-Assis's force has the same value to *all* observers, even for non inertial ones.[8,9].

With the aid of equ. (7) Assis was able to explain the origin of inertia and the *reality* of the so called *fictitious* forces of inertia ($-m\vec{a}$, centrifugal, Coriolis, *etc*.). The above forces are due, in a Machian scenario, to the gravitational interaction between any accelerated particle and the whole universe.[10,11,12,13,14]

In short, Assis was able to develop a *true* relativistic mechanics which, besides comply with Mach's requirements, can be considered as a genuine extension of CM. Assis coined the name *Relational Mechanics* (RM) when referring to his model.

Epistemological and Dimensional Considerations

Recently we have revisited Assis's formulation of RM, stressing some dimensional ambiguities concerning the net distinction between gravitational mass and inertial mass.[1,2,13] In fact, we performed a critical revision of RM based upon the physical and dimensional *hierarchy* of the involved magnitudes.[1,2]

First of all, we consider the gravitational mass as being a *primary* magnitude,[1,2,13] similar in this sense to electric charge and to spin. A primary magnitude cannot be derived, *up to now*, from other previously known properties.

Some authors prefer to write equ. (1) with a multiplicative constant η, $F_{21}^N = \eta\left(m_{g1}m_{g2}\right)/r^2$. Now we will show that the above constant is *superfluous*.

In the first place, gravitational force does not depend upon the medium in which the particles are immersed. There is no *gravitational permittivity*. This is a very important difference from the Priestley-Coulomb law for material media. Thus, η being a number *independent of the medium*, for the *sake of symmetry*, it will affect in the same way each point mass.

Thereby, $F_{21}^N = \left[\left(\sqrt{\eta}m_{g1}\right)\left(\sqrt{\eta}m_{g2}\right)\right]/r^2$. In such a case, we *define* $m'_{gj} = \sqrt{\eta}\ m_{gj}$ as the gravitational mass of the point mass *j*. QED.

> *We must avoid including superfluous elements in the description of physical phenomena*. -Newton, *Principia*.

By inserting equ. (2) in equ. (1) we get the familiar force law, written in terms of *inertial masses*, m_{ik}. Equation (2) allows us to grasp the "size" of the standards of gravitational mass in terms of the most familiar standards of inertial mass. Thus, in the cgs system, a body having 1 *Unit* of gravitational mass has an inertial mass amounting to $1/\sqrt{G} \approx 4\cdot 10^3\,\text{g}$, *i.e.*, some 4 kg. From equ. (1) and $\left[F\right] = L^1M^1T^{-2}$ we deduce the *dimensional formula* for gravitational mass.[15]

$$\left[m_g\right] = L^{3/2}M^{1/2}T^{-1} \tag{8}$$

wherein, as shown by Palacios,[15] the bracket means the ratio of the standards employed to measure gravitational mass in two *coherent* systems of units (such as the cgs and the MKS),. Thus, $\left[m_g \right] = U'_{gm}/U_{gm}$ = *a real number*. The symbols $L \equiv U'_L/U_L$ for length, and M and T, for inertial mass and time, respectively, have the same meaning.[15]

Due to equ. (8) we get $U'_{gm} = U_{gm}(100/1)^{\frac{3}{2}}(1000/1)^{\frac{1}{2}}(1/1)^{-1} = 3.162\ 10^4$, U'_{gm} and U_{gm} being the MKS and cgs standards of gravitational mass, respectively. Thus, 1 *MKS unit* of gravitational mass is 31,620 times greater than the *cgs Unit* of gravitational mass, in spite of the fact that 1 kg = 1,000 g.

Completing Relational Mechanics

The customarily adopted, very strong, constraint $m_g = m_i$ precludes the rigorous implementation of Mach's Principle[1,2] since for this purpose it must be $m_i/m_g = f\left(\rho_g, H_0\right)$, ρ_{go} being the average matter density of the distant universe (galaxies) and H_0 the Hubble constant. For the above reasons, as a starting point for calculations, we adopt equ. (7) with $H_g = 1$, dimensionless, as done by Schrödinger in 1925.

The force exerted by the whole isotropic universe on an accelerated test particle k (gravitational mass m_{gk}) is[1,2] $-m_{gk}\Phi_G\vec{a}$. Here, $\Phi_G = \left(2\pi\xi\rho_g/3H_0^2\right)$, where ρ_g is the mean density of gravitational mass in the universe and H_0 represents the Hubble constant. If $\vec{f}$ is the local force responsible for the acceleration, then we get $\vec{f} = m_{gk}\Phi_G\vec{a} \equiv m_{ik}\vec{a}$, an equation in which we have *defined* the inertial mass, m_{ik}, of the test particle, in order to recover CM:

$$m_{ik} \equiv m_{gk}\ \Phi_G \tag{9}$$

A New Theorem of Relational Mechanics

Theorem

An increase in the number of galaxies contained in the universe also increases the density of inertial mass as the square of the density of gravitational mass:[16]

$$\rho_I = \Phi_G\rho_g \propto \rho_g^2 \tag{10}$$

Proof

Adding equ. (9) for the N particles contained in an arbitrary volume V we get $\sum_{k=1}^{N} m_{ik} = \Phi_{GV}\sum_{1}^{N} m_{gk} \propto \rho_g \sum_k m_{gk}$. If, the volume remaining unchanged, N changes to $N+dN$, it will be $d\left(\sum_k m_{ik}\right) \propto d\left(\rho_g\right)\sum_k m_{gk} + \rho_g d\left(\sum_k m_{gk}\right)$. Bearing in mind that $\rho_g \equiv (1/V)\sum_k m_{gk}$, the above relation becomes $d\left(\sum_k m_{ik}\right) \propto V\left(2\rho_g d\rho_g\right) = Vd\left(\rho_g^2\right)$ which means that $(1/V)\sum_k m_{ik}\rho_i \propto \rho_g^2$, in accordance with equ. (10). QED.

On account of eqs. (2, 9, 10) we find $G = \left(3H_0^2/2\pi\xi\rho_i\right)$ in agreement with the relation advanced as early as 1938 by Dirac.[17]

Equation (10) has a clear physical meaning: an increase in the density of inertial mass arises from two different causes:

- An increase in the number of galaxies in the universe also increases ρ_g, and consequently ρ_i ("cumulative effect" when equ. (2) is taken into account in CM).
- The increase in the density of gravitational mass also increases the individual inertial mass of each particle (here is the core of Mach's Principle).

The above theorem becomes ambiguous in Assis's formulation since in his algorithm he finds $\Phi_A \equiv \left(2\pi\xi\rho_g G/3H_0^2\right)$ and, in order to recover CM, we are compelled to take $\Phi_A \equiv 1$, dimensionless.[6,8,9]

Related Considerations

Our above considerations enhance the role of Dimensional Analysis in the formulation of straightforward algorisms able to describe physical facts without ambiguity. The algorithms concerned must preserve the neat distinction which really does exist between two related qualitatively different magnitudes.

Thermodynamics provides us another interesting example: After Carnot we know that $Q_1/T_1 = Q_2/T_2$, wherein Q_1 and Q_2 are, respectively, the input and output heat in an ideal cyclical machine working between the absolute temperatures T_1, T_2. The above ratios can be expressed in cal / abs. degree, J/ºK, *etc.*

As far as we know, no author has never adopted an *ad hoc* system of standards in order to get the meaningless equation $Q = T$. As is well known, the core of thermodynamics is anchored to the largely ignored distinction between heat and temperature. The above crucial differentiation only appears in the lasting works of Black, Davy, Rumford, Mayer, Joule, Thomson, Helmholtz, and others.

Statistical Mechanics provides us another interesting example, when we deal with the connection between mechanical energy per degree of freedom and absolute temperature.[1,2] Here, the link between the above two magnitudes is one-half of Boltzmann's constant, $\langle E\rangle = (k/2)T$; $k = 1.38\ 10^{-16}$ erg/K.

The equation (1) resembles the Priestley-Coulomb law, when expressed in terms of the *electrostatic unit of charge*. We cannot avoid quoting Maxwell in reference to the above force law:[18]

> We may now write the general law of electrical action in the simple form $F = ee'r^{-2}$ .... If [Q] is the concrete electrostatic unit of quantity itself, and *e,e'* the numerical values of particular quantities, if [L] is the unit of length,..., then the equation becomes
>
> $$[Q] = \left[L^{3/2}T^{-1}M^{1/2}\right] \qquad (11)$$
>
> Other units may be employed for practical purposes, and in other departments of electrical science, but in the equations of electrostatics, quantities of electricity are understood to be estimated in electrostatic units, just as in physical astronomy we employ a unit of mass which is founded on the phe-

nomena of gravitation, and which differs from the units of mass in common use.

As we have seen, the view advocated by Maxwell was adopted by Schrödinger when he dealt with gravitational mutual energy.

Palacios has developed a sound and rigorous vectorial theory of Dimensional Analysis based upon the ideas of Fourier.[15,18] In his theory, the squared brackets mean the *ratio* of two *coherent* units (*i.e.*, *real numbers*), instead of the units themselves, as claimed by Maxwell. As far as we know Maxwell, was the first to write squared brackets when referring to units.

The ideas of Maxwell concerning dimensional analysis, when properly updated, are entirely consistent with our present views. Translating equ. (11) to modern symbolism[19,20,21] we get, according to Maxwell (Ref.17, chapter 1): $U_Q = (U_L)^{3/2} (U_T)^{-1} (U_M)^{1/2}$, a *symbolic*, *operationally undefined*, relation between *coherent* units, *e.g.*, cgs.

Taking another coherent system of units, such as MKS, it will be: $U'_Q = (U'_L)^{3/2} (U'_T)^{-1} (U'_M)^{1/2}$. On account of the above two relations, we get $U'_Q/U_Q = (U'_L/U_L)^{3/2} (U'_T/U_T)^{-1} (U'_M/U_M)^{1/2}$, an *algebraic*, *operationally defined* equation,[15] nowadays written in the form $[Q] = L^{3/2}T^{-1}M^{1/2}$. It is worthwhile to compare the last equation with equation (8).

References

[1] J.Guala Valverde, *Apeiron*, **6**, 202 (1999).
[2] J. Guala Valverde, *Physics Essays*, **12**, N 4, 785 (1999).
[3] E. Mach, *The Science of Mechanics...*, Open Court, La Salle (1960).
[4] E. Schrödinger, *Annalen der Physik*, **77**, 3325 (1925).
[5] J.B. Barbour & H. Pfister, *Mach's Principle: From Newton's Bucket to Quantum Gravity*. Birkhauser, Boston (1995).
[6] A.K.T. Assis, *Foundations of Physics Letters*, **2**, 301 (1989).
[7] A.K.T. Assis, *Apeiron*, **13**, 3 (1992)
[8] A.K.T. Assis, *Weber's Electrodynamics*, Kluwer Acad. Pub. Dordrecht, Boston, London (1994).
[9] A.K.T. Assis, *Relational Mechanics*, Apeiron, Montreal (1999).
[10] A.K.T. Assis and P. Graneau, *Foundations of Physics*, **26**, No 2, 271 (1996).
[11] T.E. Phipps Jr, *Speculations in Science & Technology*, **1**, 499 (1978).
[12] A.K.T. Assis & P. Graneau, *Hadronic Journal*, **18**, 271 (1995).
[13] A.K.T. Assis & J. Guala Valverde, *Apeiron*, **7**, N 3-4, 131 (2000).
[14] P. Graneau & N. Graneau, *Einstein vs. Einstein*, *How matter Interacts with Matter*, Carlton Press Inc. New York, (1993).
[15] J. Palacios, *Análisis Dimensional*, Espasa Calpe, Madrid (1956-64); Dimensional Analysis, McMillan, London (1964); *Analyse Dimensionnelle,* Gauthier Villar, Paris (1960).
[16] J. Guala Valverde, *Apeiron*, **3**, (2001).
[17] P.A.M. Dirac, *Proceedings of the Royal Society of London A,* **165** (1938).
[18] J.C. Maxwell, *A Treatise on Electricity & Magnetism*, Dover Pub. Inc. New York (1954). First Publication by the Clarendon Press (1891).
[19] J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill Book Co., Inc. New York and London (1941).
[20] J. Rey Pastor, P. Pi Calleja y C. A. Trejo., *Análisis Matemático*.Vol.3, Kapelusz, Buenos Aires, Argentina (1965).
[21] R.A. Rapacioli, *Apeiron*, **4**, (2001).

Mach's Principle and the Dualism of Space-Time and Matter

Horst-Heino v. Borzeszkowski* and Renate Wahsner†

The epistemological status of physics is characterized by the dualism of space-time and dynamics. General relativity theory modified the pre-relativistic dualism by taking the gravitational interaction from the side of dynamics to the side of space-time by the geometrization of gravity. The Mach principle (which is not found in Mach but read into Mach by Einstein) founded an alternative approach to the theory of gravity. The origin of the Mach principle is discussed, and its pros and cons in cosmology and quantum gravity are illustrated.‡

1. Introduction

Since Newton founded classical mechanics, physics has been based on the dualism of space-time and matter. This is a "trick" which physics devised in order to grasp motion in such a way that it could be measured and calculated. One can even say that the epistemological status of physics is characterized by this duality principle. [11]

Of course, this dualism must be given a new definition by each successive physical theory. In particular, general relativity theory defines it differently from classical mechanics; it narrows the gap between the two sides of the dualism, space-time (given by geometry) and matter (described by the dynamical equations), by geometrizing gravity. It shifts the gravitational interaction from the side of dynamics to the side of space-time. This leads to the effect that, to borrow Einstein's words, in general relativity theory space-time is a mollusc. [23] This mollusc has a separate existence which is not uniquely governed by cosmic matter (*e.g.*, there are vacuum solutions of Einstein's equations). As an implication of this, general relativity theory does not satisfy what Einstein called "Mach's principle."

When Einstein founded general relativity theory he wanted not only to generalize special relativity theory, but also to incorporate this principle. Thus, for Einstein, the "non-Machian" nature of his general relativity theory was very disappointing. [2, 46]

Although Einstein's attitude toward Mach's ideas and his opinion on the physical content of the Mach principle changed in his later years, this principle—or better: several different versions of it§—plays a certain role even today.

* Technical University Berlin, Institute for Theoretical Physics, Hardenbergstr. 36, D-10623 Berlin; e-mail: borzeszk@itp.physik.tu-berlin.de

† Max Planck Institute for the History of Science, Wilhelmstr. 44, D-10117 Berlin; e-mail: wahsner@mpiwg-berlin.mpg.de

‡ This paper is mainly based on references [8, 10, 11, 46, 47].

§ For this see [2].

This is particularly true for the two versions that could be called "frame induction" and "inertia induction" principles. The first requires that, *via* gravitational interaction, cosmic matter determines the reference frame which replaces the inertial systems of Newtonian mechanics and special relativity theory. The second requires that inertial properties (including inertial mass) of particles and bodies are determined by their gravitational interaction with the totality of cosmic masses.

In the present paper, we will first review earlier arguments [10] showing that the principle inferred by Einstein from Mach's mechanics is not only not found in Mach, but even in conflict with Mach's ideas. In this context, from the epistemological point of view, the pros and cons of deductive cosmology are discussed. Finally, we shall illustrate the constructive character of the Mach principle in its frame-induction version by discussing so-called teleparallel theories of gravity.*

2. Mach's Philosophy

Ernst Mach's work is characterized by the interplay between physical and philosophical interests. This interplay culminates in his book *The Science of Mechanics: A Critical and Historical Account of Its Development*.

It describes the history of physics from Archimedes' statics to the post-Newtonian elaboration of mechanics by d'Alembert, Lagrange and Hamilton. Mach holds the idea that to comprehend physics fully, a knowledge of its history is necessary. And primarily, in Mach's opinion, historical representation was a way to remove metaphysics from physics. For: "We are accustomed to call concepts metaphysical, if we have forgotten how we reach them." But: "One can never lose one's footing, or come into collision with facts, if one always keeps in view the path by which one has come." [34, p. 17] Mach expressed this conviction in his lecture *The History and the Root of the Principle of the Conservation of Energy*, which was programmatic for his life's work. In his *Mechanics* he wrote:

> The historical investigation of the development of a science is most needful, lest the principles treasured up in it become a system of half-understood prescripts, or worse, a system of *prejudices*. Historical investigation not only promotes the understanding of that which now is, but also brings new possibilities before us, showing that which exists to be in great measure *conventional* and *accidental*. From the higher point of view at which different paths of thought converge we may look about it with freer vision and discover routes before unknown. [35, p. 316]

To rescue the origin of concepts from oblivion Mach investigates the sensory-physiological processes of the human organism and the development of physics. Here he relies on the "metaphysical" assumption that the biogenetic law can be extended to society.

* Einstein [24] introduced such a theory in order to unify gravity and electromagnetism. In Ref. 6 teleparallel theories are discussed as purely gravitational theories that realize the Mach principle.

Mach's discussion with prominent physicists of his time, for instance, with H. Hertz, Boltzmann, Planck, and Einstein, shows that his historical and critical representation of Newtonian mechanics was more a criticism of theoretical physics altogether than a criticism of classical physics only. The controversial debates were part of the discussion about the foundation of physics toward the end of the 19th and at the beginning of the 20th centuries. Mach wrote his *Mechanics* at a time, initiated by the foundation of new physical theories (statistical physics, thermodynamics, electrodynamics), of violent arguments about the status of mechanics and its position in the framework of physics. This debate was necessary to clarify the epistemological basis of physics. In its essence, it concerned the relationship between mathematics, physics, and reality. In this connection, the question arose: In modern physics, does matter vanish in mathematical equations?

In analyzing and criticizing the conceptual basis of mechanics, as given by Newton, Mach revealed fundamental problems in the epistemological foundation of physics. Accordingly, he attacked the *opinion* of the nature of Newtonian concepts that was dominant at his time. Mach's aim was to make his contemporaries aware of the fact that experience cannot be replaced by anything else, either by logical deduction or metaphysical reasoning, and that physics cannot be reduced to mathematics. However, he misunderstood the role of irreplaceable experience in such a way that he overlooked the meaning of the theory for the *possibility* to acquire experience, to gain scientific experience. Thus, he denied Kant's insight that experience is a kind of cognition requiring understanding,* that one needs principles which enable one to gain experience e(for it is not given by mere perception).

In Eddington's words, the necessary connection between theory and experience reads as follows:

> A scientist commonly professes to base his beliefs on observations, not theories… I have never come across anyone who carries this profession into practice… Observation is not sufficient… It is better to admit frankly that theory has, and is entitled to have, an important share in determining belief. For the reader resolved to eschew theory and admit only definite observational facts, all astronomical books are banned. There are no purely observational facts about the heavenly bodies. Astronomical measurements are, without exception, measurements of phenomena occurring in a terrestrial… station; it is only by theory that they are translated into knowledge of a universe outside.

And he continues:

> When an observer reports that he has discovered a new star in a certain position, he is probably unaware that he is going beyond the simple facts of observation. But he does not intend his announcement to be taken as a description of certain phenomena that have occurred in his observatory; he means that he has located a celestial body in a definite direction in interstellar space. He looks on the location as an observational fact—on a surer footing

* Kant wrote: "… experience itself is a kind of cognition requiring the understanding, whose rule I have to presuppose in myself before any object is given to me." [30, p. 111].

> therefore than theoretical inferences such as have been deduced from Einstein's theory. We must break it for him that this supposed 'fact', far from being purely observational, is actually an inference based on Einstein's theory—unless, indeed, he has based it on some earlier theory which is even more divorced from observational facts. [13, p. 17]

Mach recognized that physics is based on principles, which are founded on uncompleted experience, but he concluded from this fact that they are not objective truths. To his mind, they are only *means of cognition* that help to find an economical representation of physics. As was shown [11, 47], it is not erroneous to characterize theoretical presumptions as *means*, but it is not correct to consider the *means of cognition* as being completely unrelated to the *object of cognition*, and to subsume it conceptually under the *subject of cognition*. Mach's philosophy replaced the system (*i.e.*, the physical theory) with a catalogue of experimental data and their mutual relations. Mach wrote:

> If we wish to say that a complete theory is the final aim of research, the word 'theory' must not be used in the [usual] sense… Rather must we understand by this word a complete and systematic representation of the facts. So long, however, as this final aim is not yet attained, 'theory' in the former sense always signifies an approximation toward a 'theory' in the latter sense. [36, p. 415]

This standpoint shows that Mach did not realize how a physical theory determines its notions. He overemphasized the role of what he called the factual (*das Tatsächliche*) so that his purging of metaphysics from physics degenerated into a liquidation of the basic epistemological presumptions of physics. [*cf.* 46, pp. 595-597]

Incidentally, by incorporating experience in handicrafts into his general notion of experience, Mach exerted an influence over the Austrian and Russian social democracy. In the closing words of *Mechanics*, Mach wrote that "…the doctrines of mechanics have developed out of the collected experience of handicraft by an intellectual process of refinement." [35, p. 612]

To summarize this Section, one can state the following: Mach recognized that, to epistemologically assess what is grasped by physics, it is necessary to investigate the basis of human experience. He saw that this investigation cannot be performed by physics itself. But, because of his rejection of any metaphysics—metaphysics both in the sense of mechanicism and in the sense of epistemology and science of categories—he did not consider it a reasonable task of philosophy. He placed his hopes in sensory physiology. The rational core of Mach's conception is his insight into the limitations of physics (as a special science) and his attempt to overcome them.

Mach did not carry out his main promise to remove metaphysics from physics. He was unable to do this because metaphysics as mechanicism is not contained in physics. Otherwise, metaphysics in the sense of epistemology and the science of categories provides the preconditions without which the foundations of physics cannot be constructed. The positive aspect of Mach's *philosophical* influence is that Mach essentially helped to overcome the mechanistic thinking of many scientists at the time. He did this by making them aware of the following insights:

- Experience cannot be replaced by anything else.
- Physics cannot be reduced to mathematics.
- To gain knowledge physics needs specific means of cognition.
- To understand a science completely one has to study its history and recognize its origin from the satisfaction of economical requirements.
- Mechanics is a part of the history of culture.

3. Mach's Criticism of Newton

In his effort to eliminate those elements from mechanics which are not accessible to individual sensual experience, Mach criticized the Newtonian concepts of *mass*, *inertia* and *space*. It is necessary to mention that he considered mechanics to be correct—but for historical reasons, represented by Newton in a manner that contained many metaphysical elements.

In particular, Mach was dissatisfied with Newton's definition of mass and his representation of the axioms of mechanics. Therefore, he started by reformulating them.

First he replaced Newton's definition of masses, saying that the ratio of the masses m_1 and m_2 of two bodies is equal to the ratio of their weights, $m_1/m_2 = G_1/G_2$, by the definition that states: The ratio of the masses m_1 and m_2 of two bodies is equal to the negative and inverse ratio of the accelerations b_1 and b_2 caused by their mutual interaction, $m_1/m_2 = -b_2/b_1$.

From this point of view, Mach regarded Newton's second law as a convention and the third law as a consequence of his definition of mass. So for him, there remained only the task of explaining the first Newtonian law. Mach's explanation was: This law is a fact first perceived by Galileo, but it only has a definite meaning when one can identify the reference system one needs in order to determine the motion.

He argued as follows. When one says "a system or a body on which no forces act is either at rest or in uniform motion" one has to ask "uniform motion, relative to what?" Newton's answer was "to absolute space." But this is—according to Mach—a metaphysical element that can be replaced by the totality of the cosmic masses; or, more precisely, by the fixed stars that form a rigid reference system. In Mach's words: "Instead, now, of referring a moving body K to space, that is to say to a system of coordinates, let us view directly its relation to the bodies of the universe, by which alone such a system of coordinates can be determined." [35, p. 286]

To demonstrate how absolute space could be eliminated, Mach showed that the center of mass of a N-body system on which no external masses act forms a system to which the motion described by mechanics can be referred. Assuming, then, that this N-body system is the system of cosmic masses (on which *per definitionem* no external forces act) he had thus defined a cosmic reference system. For Mach this was a proof of that one could free mechanics of the metaphysical element "absolute space," replacing it by something that is nearer to experience.

Thus, Mach presupposes the validity of Newton's mechanics and discusses only one of its kinematic implications; he assumes this as the starting point of his representation of mechanics.* However, it would be misleading to think that Mach believed in a "deductive" approach to physics, where one had to start from cosmological principles and work down to local physics by some method of approximations.

When Mach showed that the law of inertia can also be referred to the cosmic masses, he added that this reading implies the same difficulties as Newton's. He said:

> We have attempted in the foregoing to give the law of inertia a different expression from that in ordinary use. This expression will, so long as a sufficient number of bodies are apparently fixed in space, accomplish the same as the ordinary one. It is as easily applied, and it encounters the same difficulties. In the one case we are unable to come at an absolute space, in the other a limited number of masses only is within the reach of our knowledge, and the summation indicated can consequently not be fully carried out… The most important result of our reflections is, however, *that precisely the apparently simplest mechanical principles are of a very complicated character, that these principles are founded on uncompleted experience, may on experiences that never can be fully completed, that practically, indeed, they are sufficiently secured, in view of the tolerable stability of our environment, to serve as the foundation of mathematical deduction, but that they can by no means themselves be regarded as mathematically established truths but only as principles that not only admit of constant control by experience but actually require it.* [35, p. 289 f.]

For Mach, these obstacles were so fundamental that he did not believe they could be overcome by modifications to physical theory. According to him, the universe as a whole is not tractable as physical system. Physical notions referring to the universe as a whole have no tangible sense, because they imply the application of notions of measurement to an object that is not accessible to measurement.

In his *Theory of Heat* he wrote:

> What I said about the expected 'death of heat' of the universe still maintain, not because I suppose all processes to be reversible, but because phrases about 'the energy of the *universe*', 'the entropy of the *universe*', and so on, have no meaning. For such phrases contain applications of metrical concepts to an object which cannot be measured. If we could actually determine the 'entropy of the universe', this would be the best absolute measure of time, and the tautology which lies in the phrase about heat-death would be cleared up. [36, p. 439]

To repeat: It would be misleading to think that Mach believed in a "deductive" approach to physics where one had to start from cosmological princi-

* Much more critical of Mach's approach is, *e.g.*, M. Bunge, see [12].

ples and work down to local physics by some method of approximations. The only thing he wanted to do was to make us aware of the fact that the law of inertia (and other physical laws) are based on experience, on experience that is never complete—even more, that can never be completed.

But it must also be said that Mach's intentions were connected with his understanding of mechanical notions. When Mach started he believed, for instance, that the space of Newtonian mechanics is a rigid background given once and for all, like a stage on which physical processes are played out. He did not see that so-called absolute space is the totality of all inertial systems; it is thus not a metaphysical ghost, but a constructive element, like the quantity *mass* and other notions that are determined by the entire system of classical mechanics.

As mentioned above, Mach's misunderstanding of the status of a physical theory is a consequence of his philosophical standpoint. Thus he did not grasp how a physical theory determines its notions and, in particular, he did not understand that Newton's axioms determine simultaneously the physical dynamics and the systems of reference to which this dynamics refers or has to be referred. Since he only conceived of a catalogue of single statements and facts, but not of a theory, he only asked whether a statement under consideration is a fact or not. In this scheme there is no room for the *space* notion of classical mechanics. Therefore, he could not think of another physical theory as an answer to his criticism of Newtonian mechanics. (Moreover, Einstein's theory of general relativity must have seemed to him more remote from experience than Newton's theory.) As Bunge said, "his criticism of Newtonian mechanics was more a criticism of theoretical physics than a criticism of classical physics." [12, p. 243]

4. The Necessity of Principles and the Dualistic Structure of Physics

Given that experience can never be complete, the foundation of physics (and any other experimental science) requires us *to posit* principles.

In particular, for physics to make motion measurable and calculable, this means that we need a body with respect to which motion is described. However, since the totality of cosmic masses is never given, the "cosmos" cannot be used as reference body. It must therefore be introduced by principles. This reference body must satisfy certain requirements. In Newtonian mechanics, these requirements are as follows: There must not be any interaction between moving body and reference body (which has to be a force-free body), and the reference body has to be of such a nature that rectilinear and uniform motion can be noticed in reference to it, that a Euclidean straight line can be defined by it. This reference object is just the homogeneous, infinitely expanded Euclidean space. This space can be considered a specifically prepared body which makes it possible to define rectilinear uniform motion as a standard for the measurement of motion. The reference body that satisfies these demands is given by Newton's definitions and axioms. In view of this situation, the *absolute space* of Newton is not, as mentioned above, a transcendental phantom, but a product of idealization that defines standard motion.* (This function of absolute space was not recognized for a long time. Even at the end of the 19th century, Carl Neumann, Ludwig Lange and others found it necessary to clarify this point. [32, 39])

A few words about *idealization*: Classical mechanics as the first natural science in its proper sense, defines the motion *etalon* or supposed "natural" motion by the principle of inertia. In his *Principia* Newton presented this principle as the first law in the following form: "Every body remains in its state of rest or in rectilinear uniform motion unless forced to change its state by acting forces."† The term "natural motion" used for historical reasons (as is well known, Aristotelian physics differentiated between natural and forced motions), should not suggest that this is motion can be seen in every-day experience. It is highly unlikely that rectilinear and uniform motion will ever be found in nature. To realize this motion, ideal conditions have to be created artificially.‡ The equivalence of this motion with rest is also not sensually perceptible, but a theoretical conclusion that was only possible when it was discovered that the rectilinear uniform motion can be uniquely defined by the quantity *velocity*.§ Compared with this conception, the Aristotelian "natural motion" was far closer to every-day experience. But this was precisely the reason why it was only possible to establish a phenomenological natural philosophy on this basis, and why these conceptual determinations could not be used to develop a physical theory. [see 47, pp. 242-256]

To create ideal conditions for producing a uniform rectilinear motion, the body has to be isolated from its physical environment so that it cannot be affected by any forces. Since we can suppose that it is possible to realize a uni-

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* In more detail, see [11, 48].

† "*Lex I.* Corpus omne perseverare in statu suo, quiescendi vel movendi uniformiter in directrum, nisi quatenus a viribus impressis cogitur statum illum mutare." [40, p. 15]

‡ In recognizing this necessity and the importance of this necessity for the formulation of physical concepts, the constructive philosophy of science in its protophysical form makes a difference between two processes of forming concepts—the *abstraction* and the *ideation*. According to this conception of philosophy of science, concept to be used in the theory of measurement can only be formulated by the latter: "In contrast to the Aristotelian abstraction … ideation also contains standards of action, for example for the production of planes and orthogonalities." [33, p. 63].

§ See also P. Jaeglé, who writes: "The velocity can become an essential feature and one of the dimensions of the motion only when it is presented *as constant, as invariable in the course of time and as preserved in the course of motion*. Place and moment are changing but they remain identical as to their intervals irrespective of the form of the latter and how small they are thought. Outside this identity which changes an interval of time in such one of space, there is no exact concept of velocity and there is no mechanics!" ("La vitesse ne prend corps comme caractéristique essentielle du mouvement—comme l'une de ses dimensions—que pour autant qu'elle puisse être conçue comme *constante, invariable au cours du temps, conservée au cours du mouvement*. Le lieu et l'instant changent mais restent identiques dans le rapport de leurs intervalles, quels que soient ces derniers, aussi petits qu'on puisse les imaginer. Hors de cette identité qui transforme un intervalle de temps en un autre d'espace, point de notion précise de vitesse, point de méchanique!" [29, p. 36].

form rectilinear motion, one has to assume the possibility of isolating physical bodies or systems. Conversely, the principle of inertia shows what is grasped as an isolated body or system in physics. The fact that the principle of inertia contains Euclidean space as a necessary presupposition clearly shows that (this) physics can conceptualize motion only as a correlation of bodies, even in the case where the mutual effects of bodies must be excluded. Thus the concept of unbodily motion does not occur in physics, not even in connection with the purely kinematic aspect. The object of reference is *quasi* taken as a body without behaviour, but it is defined as body in such a way that it implicitly contains reference to other bodies.

Thus, the "natural" motion presupposed by classical mechanics is defined *via* the principle of inertia; as *etalon* of motion: it is used to measure accelerated motions that deviate from it and are produced by forces. In this way the *absolute space* of Newtonian mechanics is a product of idealization.

This approach to physics established a fundamental physical dualism of space (space-time) and matter, *i.e.*, of geometry and dynamics. This dualism proved to be decisive for all physics. However this dualism must be given a new definition by each new physical theory. General relativity theory, in particular, shifted the gravitational interaction from the side of dynamics to the side of space-time through the geometrization of gravity. The so-called *a priori* measurement-theoretical part of general relativity theory was reduced as compared to that in classical mechanics.

This reduction nourished hope in the possibility of removing dualism completely, a hope that is closely related to the so-called Mach principle.

5. Einstein's Reading of Mach

As is well known, Einstein did not refer to Mach during the first period of development of the theory of general relativity. Only when he arrived at the conclusion that the principle of relativity should be extended to arbitrarily moving reference systems, and that gravity is to be described by the metric tensor of a curved space-time, did he begin to speak of the relativity of inertia (this was about in 1912).[*] This becomes especially clear when one reads Einstein's papers and notes to himself in this context. [15-19, 21][†] One sees that the *ansatzes* of 1913 and 1914 which, from the viewpoint of the final formulation of general relativity theory, appear as mathematically incorrect, also had a physical motivation which does not justify calling them mathematically false. The material generally accessible now makes it obvious that Einstein's doubts about the form of the relativistic gravitational equations were also due to the fact that the physical content of the principle of general relativity was not clear: For Einstein, because Mach's principle was an essential aspect of general rela-

[*] In a 1912 paper he writes: "Es legt dies die Vermutung nahe, dass die ganze Trägheit eines Massenpunktes eine Wirkung des Vorhandenseins aller übrigen Massen sei, auf einer Art Wechselwirkung mit den anderen beruhend." [14, p. 39].

[†] In the next three paragraphs we follow [8].

tivity at that time, he was even ready to accept restrictions on general covariance, and thus of general relativity, if this would make the gravitational theory "Machian."

Einstein's arguments show not only the mathematical difficulties Einstein had to contend with at this time, but also make it clear that Einstein was dissatisfied with the fact that in a theory with covariant gravitational equations, Mach's principle is not satisfied. In Einstein's reading of Mach, the totality of masses induces a g_{ik} field (the gravitational field) which itself governs all physical processes, including the propagation of light rays and the behaviour of length *etalons* and watches. [17] This means that matter should also determine reference systems definable by light, rods and watches. However, in a general relativity theory, one can everywhere chose arbitrary reference coordinates and reference tetrads. The matter described by T_{ik} does not uniquely fix either the reference systems or the gravitational field.

Therefore, in 1913 and 1914 Einstein (partly in joint papers with Grossmann) [15-20, 27] argued as follows. The special relativistic matter equations imply four equations, which state that the divergence of the symmetric energy-momentum tensor T_{ik} of matter is equal to zero. These four equations are the special-relativistic laws of energy-momentum conservation which, when a gravitational field is present and under the condition that the principle of equivalence is satisfied, must necessarily be written in a covariant form. Now if, in accordance with the equivalence principle, one assumes that the tensor T_{ik} is the source term of gravity, then one obtains dynamical equations which state that the covariant divergence of T_{ik} vanishes. The 1913/14 gravitational equations were just so formulated that they, together with the dynamical equations, determine the metrical tensor g_{ik} of a Riemannian space-time up to affine (*i.e.*, linear) transformations. Thus, as an implication of the motion of matter given by the dynamical equations, the reference systems are specified (up to a linear transformation). And in 1918 he even used the notion *Mach's principle*. [22]

Although Einstein's attitude toward Mach's ideas changed in his later years [41] he never denied Mach's great influence on his generation of physicists. In Einstein's eulogy to Mach, one reads the following:

> The fact is that Mach through his historical and critical writing in which he followed the development of the individual sciences with so much love and traced historical details into the inner sanctum of the brain [*Gehirnstübchen*] of pathbreaking scientists has had a great influence on our generation of natural scientists. I even believe that the people who consider themselves opponents of Mach, scarcely know how much of Mach's way of thinking they have absorbed, so to say, with their mother's milk. [26, pp. 154 f.]

With regard to the Mach principle, in the last edition of his work *The Meaning of Relativity* he expresses his belief that Mach's principle is satisfied in the inertia induction version of his general relativity theory, as the following effects can be shown to exist:

1. The inertia of a body must increase when ponderable masses are piled up in its neighbourhood.
2. A body must experience an accelerating force when neighbouring masses are accelerated, and, in fact, the force must be in the same direction as the acceleration.
3. A rotating hollow body must generate inside of itself a 'Coriolis field', which deflects moving bodies in the sense of the rotation, and a radial centrifugal field as well. [25, p. 100]

One can really find traces of such effects,* but they are too weak to speak of an induction of inertia in general relativity theory.

Mach's work and Einstein's reading of it played a stimulating and constructive role in physical and cosmological discussions over the years, as different authors worked with different formulations of this principle. In an analysis of this situation, it was stated [28] that this is due to the fact that Mach did not propose a definite *ansatz* for an induction of inertia by cosmic masses. Consequently, Mach's principle says more about Einstein's and other authors' readings of Mach, than it does about Mach's intentions.

As we showed in Sec. 1, Mach not only did not create a cosmic principle of the kind deduced by Einstein from Mach's *Mechanics*, but such a principle is even in contrast to Mach's ideas.

6. Mach's Principle and Deductive Cosmology

The Mach principle, created by Einstein and supported by the cosmological foundation of physics hypothesized by Eddington, paved the way for a cosmology that, following Bondi, could be called "deductive cosmology."† The deductive approach starts with certain cosmological principles and seeks to deduce local physics, from them so that this physics is compatible with these principles.

Whatever one thinks of the existing deductive theories formulated by Milne, Bondi, Hoyle & Narlikar, and others, from the epistemological point of view, they do not differ essentially from usual physics and extrapolating cosmology. The *deductive attitude* starts out from certain *a priori* postulates about space and time, and the universe as a whole, and deduces from these postulates, with minimal help from observational information, a model, or models, of our universe. The *extrapolating attitude* starts with the laws of nature that have been obtained on the terrestrial scale, and constructs model universes primarily on the basis of extrapolation, which confers on these laws of nature a validity beyond the scale on which they have been confirmed experimentally.

Peter Bergmann believes that it is fair to say that actual model construction has proceeded on an eclectic basis, by and large. The terrestrial laws of nature are assumed to be valid, but a selection from conceivable model universes is made by the adoption of more or less stringent *cosmological princi-*

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* See [45]; for more recent papers, see refs. in [2].

† See, *e.g.*, [5].

ples, that is to say, principles concerning the large-scale homogeneity and isotropy of the universe. Although cosmological principles are subject to observational confirmation, to be sure (recall the experiments regarding the degree of isotropy of the background radiation) these principles are not terrestrial-scale laws of nature. They have no application to physical structures on anything less than a cosmological scale. [*cf.* 3, p. 19]

In other words: First, the deductive cosmological theory that is our goal secures justification from compatibility with local physical laws and local experiment. Second, this theory starts from principles, *i.e.*, from constructions concerning the structure and/or behaviour of the cosmos. These constructions necessarily establish an *ideal* cosmos, since the *real* cosmos can never be an object of experience. Or, *dicut* Kant: The totality of all *possible* experience can never be the object of *actual* experience. [*cf.* 31, §§ 27, 75, 77]

Although there is a difference between the philosophical and physical notions of the whole, the philosophical totality and the physical cosmos, Kant's words reflect the very problem of cosmology. This problem is caused by the fact that the object of cosmology, the whole universe, exists only once. Hence, we are unable to determine the general of this object, to separate general features from particular aspects of "our" universe [3, 4]—Deductive cosmology cannot avoid this problem.

To repeat, observation by itself is not enough: on this basis alone not even the usual physics can be constructed. The point is not that the universe as a whole is not observable. But a physical theory can be true for clearly defined (that means, each time particular) regions only.

In stronger terms, the world as a whole can only be the object of philosophy. This is because the world without the recognizing subject is not a whole. But physics cannot deal with the subject *as* subject. It is so constituted that (as was shown by Schrödinger) [42, 43] it grasps the world object. It is not to be rejected as a failure. Rather it hints at the need of philosophy for physics. (Incidentally, the so-called Anthropic Principle does not solve this problem, since it rests upon a mixing of philosophy and physics.)

In the final analysis, one can say that cosmology lies on the borderline between physics and metaphysics. [*cf.* 5] Neither Mach's philosophy nor his criticism of Newton removes the cosmological problem; he did recognize, however, that *it is a problem*.

7. Mach's Principle and Quantum Gravity

General relativity theory conflicts not only with the Mach principle in the two versions discussed in this paper, but also with the quantum principle. The latter conflict is also due to the general covariance of the theory, which gives spacetime a mollusc-like nature. As a consequence of general covariance, there is no background with reference systems which are related by symmetry transformations, such as the Lorentz or Poincaré transformations. "Until one introduces a metric, a manifold does not consist of physical events, but just of mathematical points with *no* physical properties; it is the metric that imparts the physical character of events to the points of a manifold." [44] It is therefore difficult to construct a physically meaningful canonical formalism and to quantize gravity.*

As was shown in [6, 7], there are new perspectives in quantum gravity if one goes over from general relativity theory to an Einstein-Mayer-type gravitational theory satisfying the Mach principle in the frame-induction version.

This theory works in teleparallized Riemann spaces. To summarize the fundamentals of this theory, we follow the representation given in earlier work. [8]

The Einstein-Mayer-type (tetrad) equations formulated in a teleparallized Riemann space read [6]:

$$\sqrt{-g}E_{ik} + \hat{\theta}_{(ik)} = -\kappa\sqrt{-g}T_{ik} \quad \text{and} \quad \hat{\theta}_{[ik]} = 0\,, \tag{1}$$

where E_{ik} is the Einstein tensor and $\hat{\theta}_{(ik)}$ denotes an additional tensor formed from the tetrads and their derivatives and satisfying the conditions

$$\hat{\theta}^{k}_{l,k} = 0\,. \tag{2}$$

These equations are not Lorentz-covariant because $\hat{\theta}_{(ik)}$ is not a Lorentz-covariant tensor. Noether's theorem together with the Bianchi identities and the second equation of (1) provide the conservation law

$$\left(\kappa\sqrt{-g}T_i^k + \sqrt{-g}\,{}_M t_i^k\right)_{,k} = 0 \tag{3}$$

where ${}_M t_i^k$ is Møller's energy-momentum tensor of the gravitational field. $\hat{\theta}_{(ik)}$ represents "hidden" matter which contributes only to the Einstein curvature E_{ik} of the teleparallized Riemannian space-time. In regions where the usual and the hidden matter vanish, $T_{ik} = \hat{\theta}_{ik} = 0$, the remaining tensor ${}_M t_i^k$ is the energy-momentum density of the gravitational field. In contrast to the situation in general relativity theory, here ${}_M t_i^k$ is well defined because, up to global Lorentz transformations, the reference tetrads are fixed by the gravitational equations.

This theory satisfies the Mach principle in the frame-induction version insofar as the gravitational field equations fix the tetrads representing the reference frames. Or in other words, *via* gravity (described by the gravitational equations), the cosmic matter (given by T_{ik}) determines the reference frame.

Moreover, in this theory progress can be made toward a quantum gravity, for now Møller's statement [37, 38] is valid, *viz.*, ${}_M t_i^k$ is the localized energy-momentum density of gravity. According to the Einstein-Mayer theory of gravity, the energy-momentum density of gravitational fields is a measurable quantity; and, thus, the quantization of this theory should lead to a physically meaningful theory of quantum gravity. All this is due to the Machian properties of

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* For the conceptual problems of quantized general relativity theory see [1].

this theory, such that measurable quantum effects here appear as "Machian effects" with reference to the universe.

At first sight, it may be surprising that Mach's principle can "save" quantum gravity. But this fact is less surprising when one takes into account that, due to Mach's principle, a reference background is now recovered that replaces the mollusc of general relativity theory. Thus, one has a similar situation as in special relativity theory, with the difference that reference systems are now determined by the cosmic matter.

This brings us back to our earlier remarks on the dualism of space-time and matter: This dualism is weakened by general relativity theory, but possibly, to unify gravity and quantum physics, it should not be weakened too much. By realizing the frame-induction version of the Mach principle, the Einstein-Mayer-type theory, here discussed only briefly, partly re-establishes the original dualism.

References

1. A. Ashtekar and J. Stachel (eds.), *Conceptual Problems of Quantum Gravity*, (Birkhäuser, Boston-Basel-Berlin, 1991).
2. J.B. Barbour and H. Pfister (eds.), *Mach's Principle: From Newton's Bucket to Quantum Gravity*. Einstein Studies, vol. 6 (Birkhäuser, Boston–Basel–Berlin, 1995).
3. P.G. Bergmann, *The Riddle of Gravitation* (Charles Scribner's Sons, NewYork, 1968).
4. P.G. Bergmann, "Cosmology as a Science," *Foundation of Physics* 1 (1979) 19.
5. H. Bondi, *Cosmology* (Cambridge UP, Cambridge, 1952).
6. H.-H. v. Borzeszkowski and H.-J. Treder, "Mach-Einstein Doctrine and General Relativity," *Foundations of Physics* 26 (1996).
7. H.-H. v. Borzeszkowski and H.-J. Treder, "Mach's Principle and Hidden Matter," *Foundations of Physics* 27 (1997).
8. H.-H. v. Borzeszkowski and H.-J. Treder, "Mach's Principle Could Save the Gravitons," *Apeiron* 5 (1998) 135.
9. H.-H. v. Borzeszkowski and H.-J. Treder, "Bohr's and Mach's Conception of Non-Locality in Gravity," in *Advances in the Interplay between Quantum and Gravity Physics (Proceedings of the 17th Course of Cosmology and Gravitation)*, ed. by V. de Sabbata (Kluwer, Dordrecht-Boston-London, 2002).
10. H.-H. v. Borzeszkowski and R. Wahsner, "Mach's Criticism to Newton and Einstein's Reading of Mach: The Stimulating Role of Two Misunderstandings," in *Mach's Principle: From Newton's Bucket to Quantum Gravity*, ed. by J.B. Barbour and H. Pfister, Einstein Studies, vol. 6 (Birkhäuser, Boston–Basel–Berlin, 1995).
11. H.-H. v. Borzeszkowski and R. Wahsner, *Action and Reaction. Studies on Motion and Contradiction in Physics* (Logos Verlag Berlin, Berlin, 2001).
12. M. Bunge, "Mach's Criticism of Newtonian Mechanics," *American Journal of Physics* 34 (1966) 585 (also in: *Ernst Mach—A Deeper Look*, ed. by J. Blackmore (Kluwer, Dordrecht-Boston-London, 1992) pp. 243-261).
13. A.S. Eddington, *The Expanding Universe* (Cambridge UP, Cambridge, 1933).
14. A. Einstein, "Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktion analog ist?" *Vierteljahresschrift für gerichtliche Medizin* 44 (1912) 37.
15. A. Einstein, "Zum gegenwärtigen Stand des Gravitationsproblems," *Physikalische Zeitschrift* XIV (1913) 1249 (also in *The Collected Papers of Albert Einstein*, vol. 4, "The Swiss Years: Writings 1912-1914," ed. by M.J. Klein, A.J. Kox, J. Renn, and R. Schulmann (Princeton UP, Princeton, 1995), *No*. 17.
16. A. Einstein, "Prinzipielles zur verallgemeinerten Relativitätstheorie und Gravitationstheorie," *Physikalische Zeitschrift* XV (1913) 176 (also in *Collected Papers*, vol. 4, No. 25).

17. A. Einstein, "Letter of June 25, 1913 to E. Mach," in *Collected Papers*, vol. 5, No. 448.
18. A. Einstein, "Bemerkungen," Appendix to the reprint of the 1913 paper of Einstein and Grossmann in: *Zeitschrift für Mathematik und Physik* 62 (1914) 260 (also in *Collected Papers,* vol. 4, *No*. 26).
19. A. Einstein. "Zum Relativitätsproblem," *Scientia.* 15 (1914) 328 (also in *Collected Papers* vol. 4, No. 31).
20. A. Einstein, "Formale Grundlage der allgemeinen Relativitätstheorie," *Berliner Berichte* (1914), pp. 1030-1085.
21. A. Einstein, "Letters to Mach, Lorentz, Ehrenfest, and Besso," in: *The Collected Papers of Albert Einstein*, vol. 5, "Correspondence 1902-1914," ed. by M.J. Klein, A.J. Kox, and R. Schulmann (Princeton UP, Princeton, 1993), No. 470, 481, 484, 496, and 519).
22. A. Einstein, "Prinzipielles zur allgemeinen Relativitätstheorie," *Annalen der Physik* 55 (1918) 240.
23. A. Einstein, *Relativity: The Special and General Theory*, translated by R.W. Lawson (Henry Holt and Co., New York, 1920).
24. A. Einstein, "Riemann-Geometrie mit Aufrechterhaltung des Begriffs des Fernparallelismus," *Berliner Ber.* (1928), pp. 3-7.
25. A. Einstein, *The Meaning of Relativity*, 5th ed. (Princeton UP, Princeton, 1955).
26. A. Einstein, "Ernst Mach," in: *Ernst Mach—A Deeper Look*, ed. by J. Blackmore (Kluwer, Dordrecht-Boston-London 1992) pp. 154-159.
27. A. Einstein and M. Grossmann, *Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation* (Teubner, Leipzig, 1913) (also in *The Collected Papers*, vol. 4, No. 13).
28. H. Goenner, "Machsches Prinzip und Theorien der Gravitation," in: *Grundlagenprobleme der modernen Physik,* ed. by J. Nitsch, J. Pfarr and E.-W. Stachow (B.I.- Wissenschaftsverlag, Mannheim–Wien–Zürich, 1981) pp. 85-101.
29. P. Jaeglé, *Essai sur l'espace et le temps* (Edition sociales, Paris, 1976).
30. I. Kant, *Critique of Pure Reason*, transl. and ed. by P. Guyer and A.W. Wood (Cambridge UP, Cambridge 1998).
31. I. Kant, *Kritik der Urteilskraft*, in: *Immanuel Kant, Werkausgabe*, ed. by W. Weischedel, vol. X (Suhrkamp, Frankfurt a.M., 1974).
32. L. Lange, *Die geschichtliche Entwicklung des Bewegungsbegriffs und ihr voraussichtliches Endergebnis. Ein Beitrag zur historischen Kritik der mechanischen Prinzipien* (Engelmann, Leipzig, 1886).
33. P. Lorenzen, "Theorie des technischen Wissens," in: *Protophysik. Für und wider eine konstruktive Wissenschaftstheori*e (Suhrkamp, Frankfurt a.M, 1976) pp. 40-82.
34. E. Mach, *The History and the Root of the Principle of the Conservation of Energy*, transl. and annotated by Ph.E.B. Jourdain (Chicago—The Open Court Publishing Company, London, 1911).
35. E. Mach, *The Science of Mechanics: A Critical and Historical Account of Its Development*, transl. by Th.J. McCormack (The Open Court Publishing Company, La Salle, 1960).
36. E. Mach, *Principles of the Theory of Heat. Historically and Critically Elucidated*, ed. and transl. by B. McGuiness (Reidel, Dordrecht–Boston–Lancaster–Tokyo, 1986).
37. C. Møller, "Conservation Laws and Absolute Parallelism in General Relativity," *Math. Fys. Skr. Dan. Vid. Selskab* 1 (1961), *No*. 10.
38. C. Mœller, "Survey of Investigations on the Energy-Momentum Complex in General Relativity," in *Entstehung, Entwicklung und Perspektiven der Einsteinschen Gravitationstheorie*, ed. by H.-J. Treder (Akademie-Verlag, Berlin,1966).
39. C. Neumann, *Über die Principien der Galilei-Newtonschen Theorie* (Teubner, Leipzig, 1870).
40. I. Newton, *Philosophiae Naturalis Principia Mathematica* (Andae et Joannis M. Duncan, Glasgue, 1822).
41. A. Pais, *'Subtle is the Lord'. The Science and the Life of Albert Einstein* (Oxford UP, Oxford–NewYork, 1982).
42. E. Schrödinger, *Nature and the Greeks*, in *Nature and the Greeks/ Science and Humanism*, with a foreword by R. Penrose (Cambridge UP, Cambridge, 1996).
43. E. Schrödinger, *Mind and Matter* (Cambridge UP, Cambridge, 1967).
44. J. Stachel, "Einstein and Quantum Mechanics," in *Conceptual Problems of Quantum Gravity*, ed. by A. Ashtekar and J. Stachel (Birkhäuser, Boston, *etc.*, 1991).

45. H. Thirring, and J. Lense, "Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie," *Physikalische Zeitschrift* 19 (1918) 156.
46. R. Wahsner and H.-H. v. Borzeszkowski, "Epilog to the work: E. Mach, *Die Mechanik in ihrer Entwicklung. Historisch-kritisch dargestellt [The Science of Mechanics: A Critical and Historical Account of Its Development]*," ed. and annotated by Renate Wahsner and Horst-Heino v. Borzeszkowski (Akademie-Verlag, Berlin, 1988).
47. R. Wahsner and H.-H. v. Borzeszkowski, *Die Wirklichkeit der Physik. Studien zu Idealität und Realität in einer messenden Wissenschaft [The Reality of Physics. Studies on Ideality and Reality in a Measuring Science]* (Peter Lang, Frankfurt a.M.– Berlin–Bern–NewYork–Paris–Wien, 1992) especially pp. 287-339.
48. R. Wahsner, "Dualism between Space-Time and Matter Analyzed from Epistemological Point of View," in *Advances in the Interplay between Quantum and Gravity Physics (Proceedings of the 17th Course of Cosmology and Gravitation)*, ed. by V. de Sabbata (Kluwer, Dordrecht-Boston-London, 2002).

Experiments Motivated by Mach's Principle: A Review with Comments

C. S. Unnikrishnan*

Mach's principle underlined the possible relational link between local physical phenomena and the rest of the material Universe. General Relativity (GR) is strongly Machian in many respects, and it is also considered to be anti-Machian is some other aspects. Here, I review experimental tests that have a bearing on a minimal Mach's principle. I argue that some of the successful tests of general relativity are also tests of the Machian thought on relativity of inertia. Also, Machian reasoning helps to understand and appreciate the fundamental pillar of GR—the equivalence principle. In a way, the fact that no violation of the equivalence principle is seen gives good support to Mach's principle. I discuss a fundamental problem of quantum cosmology where application of the uncertainty principle and relativistic gravity to the origin of the Universe naturally selects the cosmological parameters appropriate for Mach's principle to be valid—a Universe evolving at the critical density. Finally, I explore Mach's principle in the context of quantum mechanics and show that inertia in the context of quantum mechanics holds interesting new aspects.

Keywords: Mach's principle, experimental tests, General Theory of Relativity, Frame dragging, Quantum Mechanics, Quantum Cosmology.

Introduction

Mach's principle has been discussed extensively in the context of what is believed to be the complete classical theory of gravitation—the general theory relativity (GR). The motivations that lead to the GR were firmly rooted in Mach's principle and even in some of Mach's thoughts on the equivalence principle. Yet, at present, many physicists who have studied both subjects state often that general relativity is strongly anti-Machian. I will argue that the general relativity is in fact minimally Machian, and some of the tests of general relativity can also be interpreted as tests of a minimal Mach's principle, in the way Mach originally stated it.

I will start with a working set of statements constituting a minimal Mach's principle. Then I will briefly mention some experiments that have been suggested as tests of Mach's principle, but are not. I will examine the relation between general relativity and Mach's principle and then discuss the experimental tests of the Machian nature of GR. I will make comments on some anti-Machian views and show that some of the claims about the anti-Machian nature of GR were premature and depended on a misinterpretation. Then we will discuss the important topic of Mach's principle in the context of Quantum Mechanics. The Universe, just once given, has interesting aspects that fit well with Mach's principle, almost as if the principle is a fundamental truth. I will dis-

* Gravitation Group, Tata Institute of Fundamental Research, Mumbai – 400 005, India & NAPP Group, Indian Institute of Astrophysics, Bangalore – 560 034, India. E-mail: unni@tifr.res.in.

cuss this, and point out the connection in the context of what could be called the quantum birth of the Universe. Finally I will discuss the role of Mach's principle in a Universe dominated by the dark energy of the vacuum.

We need to state a definition of Mach's principle; from the point of view of experiments or observations, what is a working set of statements that would be interpreted as statements of Mach's principle? Once we have done this, we can examine whether there is experimental support for Mach's principle.

It is sensible to reiterate Mach's statement [1] and then derive testable statements based on the original formulation. According to Mach "Newton's experiment with a rotating bucket of water simply tells us that the relative rotation of water with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces are produced by its relative motion with respect to the mass of the Earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the bucket were increased in thickness and mass until they were several leagues* thick." Simply put, Mach's principle implies that acceleration with respect to massive objects would generate inertial forces. This is more than saying that there is no absolute space, and that space is defined only in relation to material particles. For example it is possible to describe motion with respect to light material particles filling the whole Universe, or, say, with respect to the cosmic microwave background (CMB). But the centrifugal force that matches quantitatively what is seen the laboratory, or the equivalence (reciprocity) of the rotating bucket of water with respect to the distance cosmos, and the rotation of the cosmos itself with respect to a 'stationary' bucket can be stated only when the quantity $8\pi G\rho T^2/3 \approx 1$. ρ is the average density of the Universe with a finite age, T. On the other hand GR can describe motion with respect to a material reference frame in space irrespective of the 'mass' of the reference frame. For Einstein, CMB is a valid preferred frame, though for Mach it is not enough, since its density falls far short of the criticality condition mentioned above.

Thus, we take the following as the primary statement of Mach's principle:

Inertial forces are generated by motion with respect to other massive objects in the Universe, and quantitatively the inertial forces experienced by a body will be proportional to the sum of (mass/distance) of all the particles of the rest of the Universe. Gravitational interaction as a basis for inertia is implied.

Other possible statements of Mach's principle are [2,3]:

1. *Inertial frames are determined by the matter (its average properties) in the Universe.*
2. *Inertia originates due to acceleration with respect to the rest of the matter-energy in the Universe (An isolated body in an otherwise empty Universe has no inertia).*
3. *The total energy, angular momentum and linear momentum of the Universe are zero.*
4. *The quantity $8\pi G\rho T^2/3 \approx 1$ is nearly constant.*
5. *The ratio of the gravitational mass to inertial mass m_g/m_i is universal and is equal to unity.*

* A league can vary from 3.9 to 7.4 kilometres.

Experiments Motivated by Mach's Principle

In general, a Machian effect is defined as a physical effect that could be linked to the influence of distant matter in such a way that meaningful description of the effect is inseparably linked to the existence and quantity of distant matter. This is a vastly more general concept than the original Mach's principle, as Mach stated it. Therefore, one can easily arrive at opposite conclusions as to whether a particular experiment supports or contradicts 'Mach's Principle'. Instead of getting into such a difficulty, I choose to discuss a subclass of experimental tests that are closely linked to the minimal Mach's principle on the influence of distant matter on inertial effects. There are excellent reviews on a more general class of experiments, and also particular experiments related to frame dragging, *etc*. by Will [4], Nordtvedt [5], and Ciufolini [6]. For example, Will's review links PPN formalism and PPN parameters to Machian effects.

Experiments That are Not Tests of Mach's Principle

Based on a suggestion by Cocconi and Salpeter [7], two groups [8,9] performed experiments to test for anisotropy of inertia and interpreted these experiments as tests of Mach's principle. The basic idea was that the degeneracy of the various atomic or nuclear energy levels arising from rotational symmetry will be lifted by breaking of this symmetry due to the interaction between anisotropically distributed matter within the galaxy. Application of Mach's principle was thought to imply anisotropy of inertia itself. The anisotropic inertia was assumed to originate in an interaction that might depend on the direction of the velocity or acceleration of the test system with respect to the rest of the matter in the Universe. No anisotropic effect was observed in the experiments by Hughes *et al.* [8] and Drever [9], even at the level of $\Delta m/m \leq 10^{-21}$. Modern experiments using laser-cooled and trapped atoms [10] have pushed these limits to about $\Delta m/m \leq 10^{-26}$, and it is generally concluded that Mach's principle is not supported by observations. However, Dicke [11] had argued that if the Mach program is implemented by nature through a single tensor field g_{ij} and possibly a scalar field φ, then the effect of the scalar field is to make the mass a function of space, $m \to m(\varphi) = mf(\varphi)$. The new metric $g_{ij}(new) = g_{ij}(old)f^2$ can be made locally Minkowskian. Then, the anisotropy will not be observable since the interaction is universal. This will not be possible if there is a second tensor field, since in general it will not be possible to make both the tensor fields locally Minskowskian. Thus, the Hughes-Drever type experiments are tests of whether there is more than one tensor field that couples to matter in the Universe. They are not tests of the minimal Mach's principle.

Mach's Principle, General Relativity and Experiments

One might say that General theory of Relativity is 'as good as it gets' in the context of Mach's principle. I think that GR embodies Mach's principle as it was intended, even though there are particular solutions of the Einstein's equations in various contexts that could be interpreted as anti-Machian. There are two kinds of physical situations where the Machian nature of GR manifests itself in a clear way; one is effects near rotating massive objects [12], and the other, related, context is when there are accelerations with respect to the rest of the matter in Universe [13,14]. GR naturally allows a vector potential in the low field approximations (and equivalent effects in the high field regime as well) and this leads to the widely discussed gravito-magnetic effects [12]. For example, a gyroscope in the presence of a rotating massive object will precess just as a magnetic moment will precess in a magnetic field. Such frame dragging effects, and in particular the Lense-Thirring precession, have been a topic of experimental tests for several decades now. There is direct and indirect evidence that this feature of GR is correct [6,12]. This being the most direct correspondence between GR and Mach's principle, any experiment that studies gravito-magnetic phenomena also studies the Machian connection between local inertial forces and the interaction with the rest of the Universe.

The off-diagonal components of the metric tensor g_{0i} acts as the vector potential, and the curl of this vector potential $\nabla \times g_{0i}$ is the gravito-magnetic field, $\vec{H}_g$. A gyroscope in such a field, orbiting around the Earth, for example, will feel the torque $\tau = \vec{S} \times \vec{H}_g / 2$. The resulting spin precession is the goal of experiments like the gravity-probe B (GP-B) [15]. GP-B is the most sophisticated and sensitive experiment that will study the various gravito-magnetic effects in the near future. The frame-dragging effect of the gravito-magnetic field has been measured to about 25% accuracy by the LAGEOS mission a few years ago [16]. There are other proposed experiments, but it is unlikely that any of them will actually be performed before the GP-B.

Experiment	**Date**	**Results**	**Reference**
Lunar Laser Ranging + VLBI	1987	Measurement of de Sitter precession of the Earth-Moon 'gyroscope' orbiting the sun (10% accuracy)	[17]
Lunar Laser Ranging	1988 – 1993	Measurement of de Sitter precession of Moon's perigee (1%-2% accuracy)	[18]
LAGEOS I	1998	Measurement of the Lense-Thirring precession of the LAGEOS orbital plane in Earth's gravito-magnetic field (25%)	[16]
Gravity-Probe B	soon	Proposed measurements of de Sitter precession and Lense-Thirring effect on gyroscope orbiting the rotating Earth (1%)	[15]

The table above summarizes experiments and results that are relevant to Mach's principle, all performed or designed as tests of GR. One could say that these are also tests of Mach's principle, the imprecision in the statement arising solely from the imprecision in our statement of Mach's principle. But there is no doubt that these effects are Machian in the original sense of Mach's principle.

An interesting related issue is Thomas precession. In atomic physics as well as in the general context of special relativity, Thomas precession is seen as a geometric effect, resulting from noncommutativity of Lorentz boosts and rotations. So, Lorentz boosts of a reference frame in two different directions will lead to a net rotation of the reference frame. From the point of view of gravito-magnetism, a test particle that is accelerated with respect to the frame that is at rest with respect to the distant stars will see the gravito-magnetic field generated by all the matter in the Universe. A particle with a spin, like the electron, when taken around in an orbit, will experience a nonzero gravito-magnetic field in its frame arising from acceleration with respect to the respect of the Universe. We could either see it as an accelero-magnetic field [19], $H_a \propto \vec{v} \times \vec{a} / c$, exactly similar to the induced de Sitter gravito-magnetic field $H_g = \vec{v} \times \vec{g} / c$, or as the gravito-magnetic field generated by the relative acceleration with respect to the whole Universe. With our general policy of describing every inertial effect, including the inertial mass, as Machian in terms of gravitational charge and interactions, we identify here the Thomas precession as due to precession in the Universal gravito-magnetic field. It is startling to think that half the total magnitude of fine-structure splitting in atoms is due to a direct gravitational interaction between the atomic electron and the rest of the Universe, and due to the torque on the electron spin in the Universal gravito-magnetic field! The important point we want to stress is that if GR is considered true in all its predicted effects, then Thomas precession is due to the motion of the electron with respect to the massive Universe. In the average rest frame of the orbiting electron, the Universe is rotating, and there is a vector potential whose curl is the gravimagnetic field responsible for the precession. (Now, there cannot be an alternate interpretation invoking either inertial fields or geometric effects, since that would be double counting). Clearly, this is a deep issue and requires further consideration.

Another issue of relevance is the statement that de Sitter precession in the motion-induced gravito-magnetic field, derivable from Lorentz invariance alone, is fundamentally different from the Lense-Thirring precession in the gravito-magnetic field of a body with angular momentum. The difference has to do with the fundamental status of angular momentum in general relativity, as measured asymptotically far away [6]. I wish to state, without proof, that this perceived difference may not be of relevance in the classical context, especially for experiments performed in the low field regime. The point is that, for a massive rotating object, the dragging effect of the angular momentum can be obtained by adding up individual contributions from the vector potential of motion for the elementary mass elements of the rotating object. Only in a quantum situation, where 'spin' is fundamentally different from the $\vec{r} \times \vec{p}$ angular momentum, there could be a fundamental difference between frame dragging due to spin and frame dragging from relative motion and the Lorentz transformation. Therefore, I will consider the present experimental results at 1% accuracy for the existence of frame dragging effects as proof of Lense-Thirring effect as well.

There is a deep connection between GR and Mach's principle that is not stressed often enough, though well known. Ever since Sciama's demonstration [13] of the 'origin of inertia' from the change in the vector potential of the whole Universe arising in a frame that accelerates with respect to the rest of the Universe, it is known that the inertial mass of a test body is perhaps just a weighted sum of the gravitational influences of the whole Universe on the body. If this is the case, then the concept of inertial mass is secondary, and there is only the gravitational mass as an independent attribute to matter. Thus, the equivalence principle is a consequence of Mach's principle. In this sense, the null results of the tests of the equivalence principle indicate support for Mach's principle.

Comments on Some Anti-Machian Observations

There is just one instance I want to point out where a 'shift in the reference frame' has lead to the incorrect conclusion that frame dragging is anti-Machian. It turns out that such statements have been repeated, and the cause of confusion is easy to spot, and worth pointing out. Consider a charged sphere of total charge Q, inside a massive hollow spherical shell of mass M, and mean radius R, rotating about an axis at angular velocity Ω. GR as well as Mach's principle predict that there will be inertial forces generated inside the hollow shell. The expectation in this case is that the charged sphere will be endowed with a magnetic dipole moment, as if it is being rotated. Now the crucial question: in which reference frame will such a magnetic dipole field appear? I quote from Rindler [20], who investigated this example with Ehlers [21]. "We expected, by appeal to Mach's principle, to find a magnetic dipole in the inertial frame inside the shell." "To our surprise, we found indeed a magnetic dipole field of the expected magnitude but in the wrong frame, namely the frame at rest relative to infinity." This 'surprise', according to me signifies the fundamental problem in interpreting the result from GR. The prediction is that a reference frame inside the shell will be dragged with the rotating shell, with an effective angular velocity given by $\vec{\omega} = 4\vec{\Omega} \cdot GM / 3c^2 R$. Therefore it might seem reasonable to expect the magnetic dipole field with respect to such a frame. But that is not really what GR meant! Its prediction is that any legitimate frame inside the rotating shell will be dragged around, and the charged sphere itself is such a frame. There is no abstract frame in relativity. The reference frames have to be made out of material particles, though often this is forgotten and an abstract frame is imagined. So, in this example, what is being dragged around is the charged sphere, as well as any other material or abstract frames inside the shell. Clearly, the magnetic dipole field will show up only with respect to a frame that is *not* being dragged around with the shell, and that is the frame that is nonrotating with respect to the distant stars. Therefore, the result obtained by Ehlers and Rindler is a perfectly Machian result within GR, and their interpretation that it was an anti-Machian result was incorrect [22]. We stress that a charged sphere inside a massive rotating shell will develop a magnetic dipole moment in any frame for which $\vec{\omega} \neq 4\vec{\Omega} \cdot GM / 3c^2 R$.

The same conclusion applies to a more recent discussion by Rindler on the same issue[20]. (See also ref. [3] for a discussion). It was claimed that an anti-

Machian result was obtained in a situation where a gyroscope is orbiting the a stationary massive object. This was obtained by comparing the results in a frame in which the gyroscope is orbiting the stationary and nonrotating central object, with another frame that is rotating such that the gyroscope's center of mass is stationary with respect to the center of mass of the central object. In the second frame, the central object is rotating. The claimed result was that the precession of the gyroscope in the two situations could be of opposite signs. This claim can be shown to be incorrect. In fact, the precession rate is given by [3,20]

$$\vec{\alpha} = \left(\frac{2GM}{c^2 R} - \frac{R^2\Omega^2}{2c^2} \right) \vec{\Omega} = \frac{2GM}{c^2 R}\left(1 - \frac{1}{4}\right)\vec{\Omega} = \frac{3GM\vec{\Omega}}{2c^2 R}$$

The second step is obtained by substituting for the Keplerian orbital velocity. Hence we see that the precession α is in the same sense as the angular velocity Ω as required by the Mach principle, unless the orbital velocity exceeds a critical value. But to do that, additional forces are required and then there are additional contributions to precession that Rindler did not consider. It seems to me that there is still confusion between a 'frame' in Mach's program and a 'frame' in the Newtonian sense.

Mach's Principle and Cosmology

Mach's principle underlines the essential umbilical link between local physics and the Universe as a whole. One significant implication of the truth of Mach's principle is that the Universe contains enormous amounts of matter, such that the physical effects of motion of a test body with respect to this Universal frame is equivalent to that of motion of the entire Universe with respect to the test body in the reverse sense. This reciprocity can be true quantitatively only if the evolving and expanding Universe contains a precise quantity of matter. In fact, only if it contains enough matter to satisfy the 'criticality' condition, $8\pi G\rho R^2 / 3c^2 \simeq GM_U / c^2 R_U \approx 1$. ρ is the average density of the Universe, and R is size scale of the presently observable Universe. M_U is the total mass-energy contained within a causal size of R_U. This is an amazing result, since the most recent observations confirms [23] with great certainty that indeed the density of the Universe satisfies the criticality criterion! There could be many possible reason why things fit in so well. Without going into possibilities like inflation, it is possible to understand this result by merely noting that if the Universe was created from 'nothingness' then we should expect that the total energy of any of its constituent is always zero. This is precisely the condition required to have a Universe with critical density, or equivalently, spatial curvature $k = 0$.

Unnikrishnan, Gillies and Ritter have recently shown that an origin of the Universe determined by the fundamental principles of quantum physics and relativistic gravity will endow the Universe with precisely the critical density [24], consistent with Mach's principle and the zero-energy condition. This is a very satisfying result since otherwise the observed fact that the Universe is evolving at the critical density would have to be explained by attributing certain purpose to Mach's principle. But getting the result of critical evolution from fundamental theories of nature more or less tells us why Mach's principle is likely to be a true fundamental principle of nature; Mach's principle is a consequence of quantum physics and gravity operating together in the early Universe!

If the Universe is dominated in its density by strange dark energy with equation of state $p=-\rho$, then the expansion is accelerating and the scale factor increases exponentially. Since the density of dark energy remains a constant during such expansion, the Machian condition, $8\pi G\rho R^2/3c^2 \approx 1$, cannot be met. This, I feel should lead to interesting observable effects that are at present overlooked. One possibility is that there is anomalous *blueshift* of spectral lines amounting to $\Delta\nu/\nu \simeq \Delta R/R \simeq \Delta t/T$. This is expected since the (magnitude of) average gravitational potential at any point in the Universe is increasing if the vacuum energy density is a constant, and there is an additional gravitational blueshift between the time of emission and the time of detection. For a light travel time of 10^8 light years, this will amount to about 10^{-2}, which is measurable above scatter from proper motions. Of course, this is of the same order of magnitude as the Hubble redshift, but of opposite sign due to the fact that the magnitude of the gravitational potential is smaller at the time of emission compared to that at the time of reception at a later time. Since this is a very important issue, a detailed study within the general relativistic formalism is required, and I hope to say something more conclusive soon. Other Machian effects, possibly undetectable, are continuous change in the inertial mass, broken reciprocity between local inertial effects and Machian inertial effects, *etc*. In fact, from the point of view of Mach's principle, a Universe with its dynamics dominated by a vacuum energy density does not seem appealing.

Mach's Principle and Quantum Mechanics: New Considerations

One of the major implications of Mach's principle is that inertial forces are a consequence of the gravitational interaction of a body with the rest of mass-energy in the Universe. This has been extended to the statement that the property of inertia itself—the concept of an inertial mass—is a consequence of gravitational interaction with the whole Universe when a body is accelerated with respect to the Universe. Since inertial forces are apparent only when a body is accelerated with respect to a reference frame, it might seem acceptable to state that a body possesses inertia only when it is accelerated and this is accounted for by Mach's principle. Sciama's famous derivation [13] of this fact is what I will discuss now. In Sciama's picture, a body of gravitational mass m_g experiences a force due to a changing vector potential generated by the whole Universe, when the body is accelerated with respect to the whole Universe. In the rest frame of the accelerated body, the force is

$$F = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} = -\frac{m_g}{c^2}\left(\frac{4\pi G\rho R^2}{3}\right)\frac{dv}{dt} = -m_i\vec{a}\,.$$

This immediately suggests that the inertial mass is a derived concept and it is essentially the gravitational mass multiplied by the 'universal' quantity $4\pi G\rho R^2/3c^2 \simeq GM_U/c^2R_U$. It is a remarkable fact that this universal gravitational influence is almost exactly unity according to most recent cosmological observations and therefore, the ratio of the inertial mass to gravitational mass could be exactly unity.

This means that there is only one kind of mass: the gravitational charge or the gravitational mass. It follows that in every physical context in which the inertial mass appears, the parameter is actually the gravitational mass, even if the physical context has nothing to do with gravity directly. We conclude that the mass parameter appearing in the Schrödinger equation and all of quantum mechanics is really the gravitational charge. This suggests indirectly that quantum mechanics has some deep relation to gravitational phenomena. I am not able to even speculate on this possible connection, but it seems to me that there is a deep connection and this aspect needs to be contemplated on.

This forces us to change the standard way of thinking about inertia in the context of Mach's principle. The mass parameter appears in the quantum contexts that do not involve any acceleration. The inertial mass is defined in the quantum equations of motion not through forces, but through the evolution equation for the wave function (The Heisenberg equation has a direct correspondence with Newton's force equation, but the Schrödinger description does not refer to forces or accelerations). The concept of energy is more fundamental than that of force. Description of energy also requires the concept of inertia and therefore the origin of inertia cannot be in the acceleration with respect to the rest of the Universe. The inertial forces themselves seem to a consequence of acceleration with respect to the average Universe, but inertia or inertial mass itself is a more primal concept. Therefore, it should be possible to establish the relation $m = m_i = m_g(4\pi G\rho R^2/3c^2)$ without taking the Sciama route. The consequence of such a derivation would be to establish that quantum mechanics and Schrödinger equation has a direct connection with gravity and rest of the Universe, in the Machian sense.

Summary

I have tried to present Mach's principle as a natural principle of nature, arising from requirements imposed on the Universe at its birth by quantum physics and relativistic gravity. In this sense, Mach's principle and relativistic theory of gravity, its development inspired by Mach's principle itself, are in remarkable mutual consonance. This is evident in the experimental tests that attempts to see the velocity dependent new gravitational forces. These tests are simultaneously tests of general theory of relativity and Mach's principle. I have argued that some claims about the anti-Machian nature of GR are in fact in error. I pointed out that Machian (or relativistic) influence in a Universe dominated by vacuum energy has local observational implication, like an anomalous blueshift of spectral lines. A discussion of Mach's principle in the context of nonrelativistic quantum mechanics and the Schrödinger equation is seen to lead to new avenues that need further exploration.

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References

[1] E. Mach, *The Science of Mechanics: A critical and historical account of its development*, (Open Court Publishing Co. La Salle, 1942).

[2] See, for a detailed account, J. Barbour and H. Pfister (Eds), *Mach's principle: From Newton's bucket to Quantum gravity* (Birkhauser, Boston, 1995).

[3] H. Bondi and J. Samuel, The Lense-Thirring effect and Mach's principle, *Phys. Lett*. A, **228**, 121-126, (1997).

[4] C. M. Will, "Testing Machian effects in laboratory and space experiments," in: *Mach's principle: From Newton's bucket to Quantum gravity* (J. Barbour and H. Pfister, Eds.), 365-385, (Birkhauser, Boston, 1995).

[5] K. Nordtvedt, "Machian effects in physical law and the field paradigm of modern physics," *ibid*, 422-435.

[6] I. Ciufolini, Dragging of inertial frames, gravito-magnetism and Mach's principle, *ibid*, 386-402.

[7] G. Cocconi and E. Salpeter, *Nuovo cimento*, **10**, 646, (1958).

[8] V. W. Hughes, H. G. Robinson, and M. V. Beltran-Lopez, Upper limit for the anisotropy of inertial mass from nuclear resonance experiments, *Phys. Rev. Lett*., **4**, 342, (1960).

[9] R. P. W. Drever, A search for anisotropy of inertial mass using a free precession technique, *Phil. Mag*. **6**, 683, (1961).

[10] T. E. Chupp *et al*., Results of a new test of local Lorentz invariance: A search for mass anisotropy in ^{21}Ne, *Phys. Rev. Lett*. **63**, 1541 (1989); S. K. Lamoreaux, J. P. Jacobs, B. R. Heckel, F. J. Raab, and E. N. Fortson, New limits on spatial anisotropy from optically-pumped ^{201}Hg and ^{199}Hg, *Physics Review Letters* **57**, 3125 (1986).

[11] R. H. Dicke, Experimental tests of Mach's principle, *Phys. Rev. Lett.*, **7**, 359, (1961).

[12] I. Ciufolini and J. A. Wheeler, *Gravitation and Inertia*, (Princeton series in Physics, 1995).

[13] D. Sciama, Origin of Inertia, *MNRAS*, **113**, 34, (1953).

[14] D. J. Raine, Mach's principle and space-time structure, *Rep. Prog. Phys*., 44, 1151-1195, (1981).

[15] Gravity Probe B is being prepared for space launch at the Stanford University. The details are available at the website http://einstein.stanford.edu

[16] I. Ciufolini, *et al*., Test of General Relativity and Measurement of the Lense-Thirring Effect with Two Earth Satellites, *Science*, **279**, 2100-2103 (1998).

[17] B. Bertotti, I. Ciufolini, and P. L. Bender, New test of general relativity: Measurement of the de Sitter geodetic precession rate for lunar perigee, *Phys. Rev. Lett.*, **58**, 1062-1065, (1987).

[18] I. I. Shapiro, R. D. Reasenberg, J. F. Chandler, and R. W. Babcock, Measurement of the de Sitter precession of the moon: A relativistic three-body effect, *Phys. Rev. Lett*., **61**, 2643-2646 (1998).

[19] C. S. Unnikrishnan, The accelero-magnetic field, Thomas precession, and an equivalence principle for spin, *Mod. Phys. Lett*. A**16**, 429, (2001).

[20] W. Rindler, "The Lense-Thirring effect exposed as anti-Machian," *Phys. Lett.* A **187**, 236-238 (1994).

[21] J. Ehlers and W. Rindler, An electromagnetic Thirring problem, *Phys. Rev.* D **4**, 3543 (1971).

[22] C. S. Unnikrishnan, Is the electromagnetic Thirring moment anti-Machian ?, in preparation (2002).

[23] P. de Bernadis *et al*., BOOMERANG collaboration, A flat universe from high-resolution maps of the cosmic microwave background radiation, *Nature*, **404**, 955, (2000).

[24] C. S. Unnikrishnan, G. T. Gillies, and R. C. Ritter, A quantum origin of the Universe determined by quantum physics and relativistic gravity, *Pramana-Jl. Phys.* **59**, 369-374 (2002).

A Perspective on Mach's Principle and the Consequent Discovery of Major New Phenomenology in Spiral Discs

D.F. Roscoe*

This paper begins with speculation on the realization of Mach's Principle; we came to the details of the present analysis *via* the formulation of two questions: *(a) Can a globally inertial space & time be associated with a non-trivial global matter distribution? (b) If so, what are the general properties of such a global distribution?*

The analysis (see Roscoe, *GRG* 2002 (Astro-ph/0107397) for the details) led us to conclude that a globally inertial space & time *can* be associated with a non-trivial global matter distribution, and that this distribution is necessarily fractal with $D = 2$.

Gravitation processes are then understood in terms of perturbations of this equilibrium space-matter-time structure. We give a very brief overview of these gravitational processes, specifically applied to spiral galaxies. These considerations led directly to the discovery of a completely new phenomenology in spiral discs, which is now in the main-stream astrophysical literature (Roscoe *A & A* 2002, astro-ph/0107300). We review this phenomenology, and give a brief account of how this work is currently progressing.

1. Introduction

The ideas underlying what is now known as 'Mach's Principle' can be traced to Berkeley (1710, 1721) for which a good contemporary discussion can be found in Popper (1953). Berkeley's essential insight, formulated as a rejection of Newton's ideas of absolute space, was that the motion of any object had no meaning except insofar as that motion was referred to some other object, or set of objects. Mach (1960, reprint of 1883 German edition) went much further than Berkeley when he said:

> I have remained to the present day the only one who insists upon referring the law of inertia to the earth and, in the case of motions of great spatial and temporal extent, to the fixed stars.

In this way, Mach formulated the idea that, ultimately, inertial frames should be defined with respect to the average rest frame of the visible universe.

It is a matter of history that Einstein was greatly influenced by Mach's ideas as expressed in the latter's *The Science of Mechanics* (see for example Pais 1982) and believed that they were incorporated in his field equations so long as space was closed (Einstein 1950). The modern general relativistic analysis gives detailed quantitative support to this latter view, showing how Mach's Principle can be considered to arise as a consequence of the field equa-

* Applied Maths Dept, Sheffield University, Sheffield, UK. Email: D.Roscoe@shef.ac.uk

tions when appropriate conditions are specified on an initial hypersurface in a closed evolving universe. In fact, in answer to Mach's question asking what would happen to inertia if mass was progressively removed from the universe, Lynden-Bell, Katz & Bicak (1995) point out that, in a *closed* Friedmann universe the maximum radius of this closed universe and the duration of its existence both shrink to zero as mass is progressively removed. Thus, it is a matter of record that a satisfactory incorporation of Mach's Principle within general relativity can be attained when the constraint of closure is imposed.

However, there is a hardline point of view: in practice, when we talk of physical space (and the space composed of the set of all inertial frames in particular), we mean a space in which *distances* and *displacements* can be determined; but these concepts only have any meaning insofar as they refer to relationships within material systems. Likewise, when we refer to elapsed physical time, we mean a measurable degree of ordered change (process) occurring within a given physical system. Thus, all our concepts of measurable 'space & time' are irreducibly connected to the existence of material systems and to process within such systems—which is why the closed Friedmann solutions are so attractive. However, from this, we can also choose to conclude that any theory (for example, general relativity, notwithstanding its closed Friedmann solutions) that allows an internally consistent discussion of an empty inertial spacetime must be non-fundamental at even the classical level.

To progress, we take the point of view that, since all our concepts of measurable 'space & time' are irreducibly connected to the existence of material systems and to process within such systems, then these concepts are, in essence, *metaphors* for the relationships that exist between the individual particles (whatever these might be) within these material systems. Since the most simple conception of physical space & time is that provided by inertial space & time, we are then led to two simple questions:

> Is it possible to associate a globally inertial space & time with a non-trivial global matter distribution and, if so, what are the fundamental properties of this distribution?

In the context of the simple model analysed, the present paper finds definitive answers to these questions so that:

- A globally inertial space & time *can* be associated with a non-trivial global distribution of matter;
- This global distribution is necessarily fractal with $D = 2$

2. General Overview

We start from the position that conceptions of an *empty* inertial spatio-temporal continuum are essentially non-physical, and are incapable of providing sound foundations for fundamental theory.

According to this view, the fact that general relativity admits an empty inertial spatio-temporal continuum as a special case (and was actually originally derived as a generalization of such a construct) implies that it is not sufficiently primitive to act as a basis from which fundamental theories of cosmology can be constructed.

By starting with a model universe consisting of objects which have no other properties except identity (and hence enumerability) existing in a formless continuum, we show how it is possible to project spatio-temporal metric properties from the objects onto the continuum. By considering idealized dynamical equilibrium conditions (which arise as a limiting case of a particular free parameter going to zero), we are then able to show how a globally inertial spatio-temporal continuum is necessarily identified with a material distribution which has a fractal dimension $D = 2$ in this projected space. This is a striking result since it bears a very close resemblance to the cosmic reality for the low-to-medium redshift regime.

However, this idealized limiting case material distribution is distinguished from an ordinary material distribution in the sense that the individual particles of which it is comprised are each in a state of arbitrarily directed motion, but with equal-magnitude velocities for all particles—and in this sense is more like a quasi-photon gas distribution. For this reason, we interpret the distribution as a rudimentary representation of an inertial material vacuum, and present it as the appropriate physical background within which gravitational processes (as conventionally understood) can be described as point-source perturbations of an inertial spatio-temporal-material background.

2.1 Overview of the Non-Relativistic Formalism

In order to clarify the central arguments and to minimize conceptual problems in this initial development, we assume that the model universe is stationary in the sense that the overall statistical properties of the material distribution do not evolve in any way. Whilst this was intended merely as a simplifying assumption, it has the fundamental effect of making the development inherently non-relativistic (in sense that the system evolves within a curved metric three-space, rather than being a geodesic structure within a spacetime continuum).

The latter consequence arises in the following way: since the model universe is assumed to be stationary, then there is no requirement to import a predetermined concept of 'time' into the discussion at the beginning—although the qualitative notion of a generalized 'temporal ordering' is assumed. The arguments used then lead to a formal model which allows the natural introduction of a generalized temporal ordering parameter, and this formal model is invariant with respect to any transformation of this latter parameter which leaves the absolute ordering of events unchanged. This arbitrariness implies that the formal model is incomplete, and can only be completed by the imposition of an additional condition which constrains the temporal ordering parameter to be identifiable with some model of physical time. It is then found that such a model of physical time, defined in terms of 'system process', arises automatically from the assumed isotropies within the system.

In summary, the assumption of stationarity leads to the emergent concept of a physical 'spatio-temporal continuum' which partitions into a metric three-space together with a distinct model of physical time defined in terms of ordered material process in the metric three-space. The fractal $D = 2$ inertial universe then arises as an idealized limiting case.

3. The Starting Point

If it is *impossible* to conceive of a physical spatio-temporal continuum in the absence of material, then it follows that we need a theory of the world according to which (roughly speaking) notions of metrical space & time are somehow projected out of primary relationships between objects. Our starting point is to consider the calibration of a radial measure which conforms to these ideas.

Consider the following perfectly conventional procedure which assumes that we 'know' what is meant by a given radial displacement, R say. On a large enough scale ($>10^8 \approx$ light years, say), we can reasonably assume it is possible to write down a relationship describing the amount of mass contained within a given spherical volume: say

$$M = U(R) \tag{1}$$

where U is, in principle, determinable. Because M obviously increases as R increases, then U is said to be monotonic, with the consequence that the above relationship can be inverted to give

$$R = G(M) \tag{2}$$

which, because (1) is unremarkable, is also unremarkable.

In the conventional view, (1) is logically prior to (2); however, it is perfectly possible to reverse the logical priority of (1) and (2) so that, in effect, we can choose to *define* the radial measure in terms of (2) rather than assume that it is known by some independent means. If this is done then, immediately, we have made it impossible to conceive of radial measure in the absence of material. With this as a starting point, we are able to construct a completely Machian Cosmology in a way outlined in the following section.

4. The End Point

Invariant ideas of spatial and temporal measure arise naturally within the course of the analysis which led, finally, to the conclusion that it is possible to have a non-trivial matter distribution irreducibly associated with an equilibrium (inertial) universe—but only if mass is distributed with fractal dimension two. (Roscoe, *GRG* 2002 (Astro-ph/0107397)).

5. A Quasi-Fractal Mass Distribution Law, $M \approx R^2$: the Evidence

A basic assumption of the *Standard Model* of modern cosmology is that, on some scale, the universe is homogeneous; however, in early responses to suspicions that the accruing data was more consistent with Charlier's conceptions of an hierarchical universe (Charlier, 1908, 1922, 1924) than with the requirements of the *Standard Model*, de Vaucouleurs (1970) showed that, within wide limits, the available data satisfied a mass distribution law $M \approx r^{1.3}$, whilst Peebles (1980) found $M \approx r^{1.23}$. The situation, from the point of view of the *Standard Model*, has continued to deteriorate with the growth of the data-base to the point that, (Baryshev *et al.* (1995))

> ...the scale of the largest inhomogeneities (discovered to date) is comparable with the extent of the surveys, so that the largest known structures are limited by the boundaries of the survey in which they are detected.

For example, several recent redshift surveys, such as those performed by Huchra *et al.* (1983), Giovanelli and Haynes (1986), De Lapparent *et al.* (1988), Broadhurst *et al.* (1990), Da Costa *et al.* (1994) and Vettolani *et al.* (1994), *etc.*, have discovered massive structures such as sheets, filaments, superclusters and voids, and show that large structures are common features of the observable universe; the most significant conclusion to be drawn from all of these surveys is that the scale of the largest inhomogeneities observed is comparable with the spatial extent of the surveys themselves.

In recent years, several quantitative analyses of both pencil-beam and wide-angle surveys of galaxy distributions have been performed: three recent examples are give by Joyce, Montuori & Labini (1999) who analysed the CfA2-South catalogue to find fractal behaviour with $D = 1.9 \pm 0.1$; Labini & Montuori (1998) analysed the APM-Stromlo survey to find fractal behaviour with $D = 2.1 \pm 0.1$ whilst Labini, Montuori & Pietronero (1998) analysed the Perseus-Pisces survey to find fractal behaviour with $D = 2.0 \pm 0.1$. There are many other papers of this nature in the literature, all supporting the view that, out to medium depth at least, galaxy distributions appear to be fractal with $D \approx 2$.

This latter view is now widely accepted (for example, see Wu, Lahav & Rees (1999)), and the open question has become whether or not there is a transition to homogeneity on some sufficiently large scale. For example, Scaramella *et al.* (1998) analyse the ESO Slice Project redshift survey, whilst Martinez *et al.* (1998) analyse the Perseus-Pisces, the APM-Stromlo and the 1.2-Jy IRAS redshift surveys, with both groups finding evidence for a cross-over to homogeneity at large scales. In response, the Scaramella *et al.* analysis has been criticized on various grounds by Joyce *et al.* (1999).

So, to date, evidence that galaxy distributions are fractal with $D \approx 2$ on small to medium scales is widely accepted, but there is a lively open debate over the existence, or otherwise, of a cross-over to homogeneity on large scales.

To summarize, there is considerable debate around the question of whether or not the material in the universe is distributed fractally or not, with supporters of the big-bang picture arguing that, basically, it is not, whilst the supporters of the fractal picture argue that it is with the weight of evidence supporting $D \approx 2$. This latter position corresponds exactly with the picture predicted by the present approach.

6. Gravitating Systems

Theories of gravitation are derived by assuming some conception of an inertial background (for example, Newton's absolute space, or the flat spacetime of Einstein's special relativity) and perturbing it in someway to generate the required theory. Clearly, the assumed nature of the inertial background has a fundamental influence on the structure of the final theory. So, we can expect that perturbations of an inertial background which is irreducibly associated with a fractal $D = 2$ matter distribution will produce a gravitation theory that is quite distinct from existing theories.

We proceeded as follows:

- Perform a point-source perturbation of the inertial background to obtain a point-source theory, and confirm that it reproduces Newtonian gravitation in the point-source case;
- Extend to a two point-source perturbation, and confirm that the Newtonian two-body theory is reproduced;
- Use the insights gained to write down the *N*-body theory—which is generally intractable;
- Note that in conditions of very high symmetry, and in the limit from an *N*-body theory to a continuum theory, there exists the possibility of a tractable form emerging;
- As an example, apply conditions of cylindrical symmetry and use the resulting equations as a model of an idealized spiral galaxy (*i.e.*, a perfect disc with no lumps and bumps.)

7. Spiral Galaxies

When applied to a mass distribution with perfect disc symmetry, the theory leads to a set of highly non-linear equations of motion. The application of additional constraints concerning the nature of the *idealized disc* leads to a power-law class of solutions for the circular velocities, given by

$$V = AR^{\alpha}$$

where V is the circular velocity, R is the radial displacement and (A,α) are parameters which vary between distributions.

We use the power law as a model of the circular velocities in idealized spiral galaxies, and demonstrate how well it works by reference to the data from a sample of 305 spiral galaxies published by Courteau 1997. Figure 1 gives the basic scatter plot of $(\ln A,\alpha)$ computed for all galaxies in the sample.

Detailed investigation shows that the variation in this plot can be virtually all accounted for by variations in the luminosity properties of the galaxies used in the sample. In particular, we find that the model

FIG 1: Courteau data:

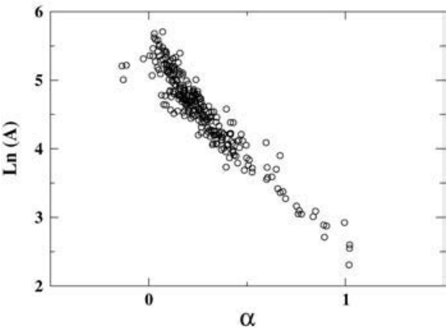
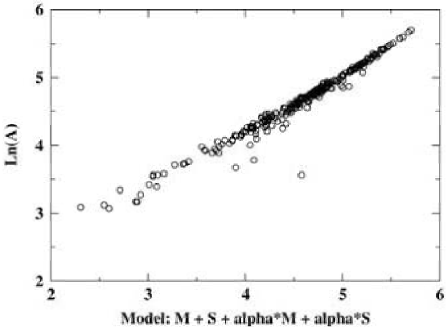


FIG 2: Courteau data



$$\ln A = -2.870 - 0.380M + \alpha(7.648 + 0.503M + 0.012S)$$

accounts for about 98% of the variation in Fig 1. Here M represents the *absolute magnitude* and S represents the *surface brightness* of each object in the sample. Fig 2 plots $\ln A$ against the model, and we see an almost perfect fit.

Similar results can be obtained from any of the four samples that we have analysed. Thus, we can consider the power-law description of circular velocities in idealized spiral discs to be virtually perfect when applied to large ensembles.

8. The Discrete States Phenomenology: Hypothesis

The work which considered the goodness of the power-law model (above) was preceded by a pilot study of a very small sample of 12 galaxies—used as a means of getting a 'feel' for how the analysis of large samples might progress. Apart from noting a strong $(\ln A, \alpha)$ correlation, we also noted that $\ln A$ computed for this small sample clustered, in a remarkably tight way, around the values (3.5, 4.0, 4.5, 5.0). We initially considered this to be almost certainly coincidence, but worth investigating at a later date. Note: these $\ln A$ values are dependent on the distance scaling used. In this case, the distance scaling assumed $H = 50$ km/sec/Mpc.

Subsequently, we obtained our first very large sample, of data on 900 spirals measured by Mathewson, Ford & Buchhorn 1992 and folded (a necessary data reduction process) by Persic & Salucci 1995. Most of the basic power-law fitting work was originally done on this sample (Roscoe 1999). The pilot study of the original 12 objects had raised the idea that $\ln A \approx (3.5, 4.0, 4.5, 5.0)$ for $H = 50$ km/sec/Mpc.

The current large sample was scaled using $H = 85$ km/sec/Mpc for which the corresponding $\ln A$ values are given by $\ln A \approx (3.85,\ 4.24,\ 4.72,\ 5.06)$. Thus, we had the strong *prior* hypothesis that, for new samples calibrated with $H = 85$ km/sec/Mpc, we should find $\ln A \approx (3.85, 4.24, 4.72, 5.06)$.

9. The Discrete States Phenomenology: Results

See Figures 3-7.

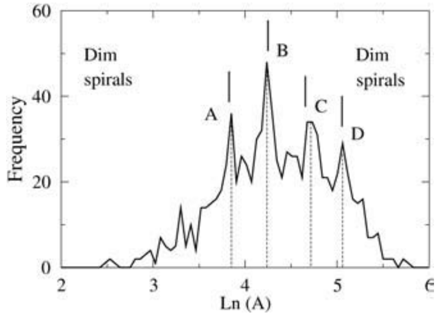


Figure 3: This shows the ln *A* frequency diagram arising from the Mathewson *et al.* 1992 data (900 objects) where the folding process (essential data reduction) was performed by Persic & Salucci 1995. The short vertical bars are the positions of the *predicted* ln *A* values, (3.85, 4.24, 4.72, 5.06) of the last section. There is clearly a remarkable correspondence.

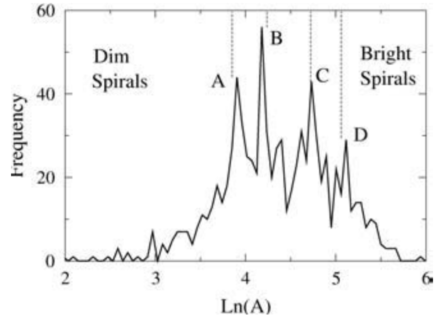


Figure 4: Again, Mathewson *et al.* 1992 data, but now the folding has been done by my own software—necessarily developed for new samples. The vertical dotted lines are the centres of the peaks in Figure 3. The discrete signal is much sharper and enhanced in strength.

10. Comments

- The four samples are completely independent of each other;
- With the exception of the Mathewson *et al.* 1992 sample (Figures 3,4) the *A*-peak is generally weak. This is because this peak corresponds to the low luminosity end of the samples. Such objects are very much under-represented in the samples, except for Mathewson *et al.* 1992 data: hence weak *A*-peaks.
- The discrete-state signal, in the form of the *A, B, C* and *D* peaks, is confirmed at vanishingly small odds of it being a chance effect—formally, at the level of 10^{-30} over the four samples considered as independent *given* the prior hypothesis raised on the sample of twelve objects.

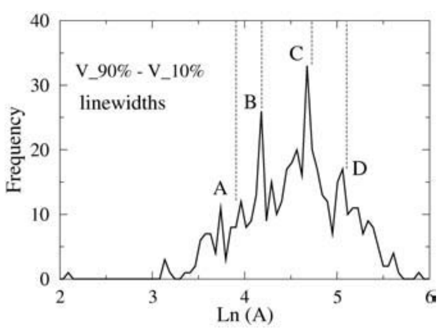


Figure 5: This shows the ln *A* frequency diagram arising from Dale *et al.* 1997 et seq data (483 objects). The vertical dotted lines are the centres of the Figure 4 peaks.

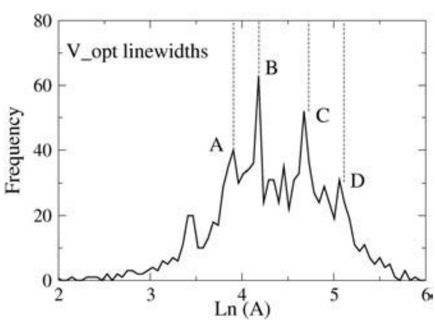


Figure 6: This shows the ln *A* frequency diagram arising from Mathewson & Ford 1996 data (1083 objects and completely independent of the Mathewson *et al.* 1992 sample used in Figs 3,4). The vertical dotted lines are the centres of the Figure 4 peaks.

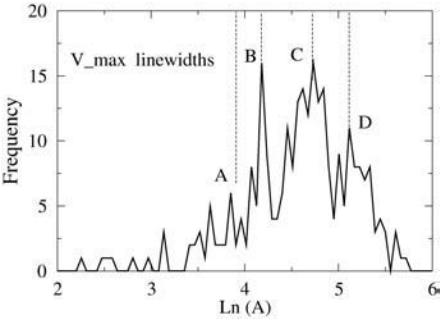


Figure 7: This shows the ln A frequency diagram arising from Courteau 1995 (305 objects). The vertical dotted lines are the centres of the Figure 4 peaks.

11. What Does it Mean?

We know, from Sec 7, that $\ln A = F(M,S,\alpha)$. Thus, if $\ln A$ is constrained to have discrete values, (k_1,k_2,k_3,k_4), say, then we find $F(M,S,\alpha) = k_i$, $i = 1,2,3,4$. Consequently, at a phenomenological level, we can say that spiral galaxies are constrained to evolve over one of four distinct surfaces in (M,S,α) space.

But what deeper meaning can we infer? A straightforward way of possibly understanding the meaning of this phenomenology goes as follows: in the wider world of non-linear dynamical systems, it is quite common to come across systems of non-linear differential equations that can only be consistently solved when a certain *algebraic* condition between system parameters is satisfied. If this algebraic condition happened to be a quartic equation in three parameters, then we would have a situation very similar to the *discrete states* phenomenology outlined above.

This was the state of play in December 2001. In January 2002, I returned to the analysis of the equations of motion in their full non-linear form, and found that I could, in fact, solve them exactly. The power-law is still the solution for the circular velocities—so no change there! *But*, it turns out that the equations of motion can only be consistently solved when a certain *quartic* (involving α) is satisfied between parameters in the system—exactly as inferred above. Of course, difficulties remain: for example, the parameters in the equations are dynamical parameters, whereas the parameters in the phenomenology are luminosity parameters—so much work remains. This work is still being written up.

13. Conclusions

By taking a particular interpretation of Mach's Principle seriously, we have arrived at a conception of inertial space & time which:

1. is irreducibly associated with a fractal $D = 2$ distribution of material which corresponds very closely with what is observed on medium scales at least;
2. leads to a theory of gravitation which, when applied to model spiral galaxies gives a description of circular velocities in idealized discs which, taken statistically over a large number of spirals, performs with a very high precision and:
   - led to the discovery of major new phenomenology in spiral discs;
   - provides a detailed explanation of this phenomenology.

References

Baryshev, Yu V., Sylos Labini, F., Montuori, M., Pietronero, L. 1995. *Vistas in Astronomy* **38**, 419.

Berkeley, G. 1710 *Principles of Human Knowledge*.

Berkeley, G. 1721 *De Motu*.

Broadhurst, T.J., Ellis, R.S., Koo, D.C., Szalay, A.S. 1990 *Nature* **343**, 726.

Charlier, C.V.L. 1908 *Astronomi och Fysik* **4**, 1 (Stockholm).

Charlier, C.V.L. 1922 *Ark. Mat. Astron. Physik* **16**, 1 (Stockholm).

Charlier, C.V.L. 1924 *PASP* **37**, 177.

Courteau S., 1997, *AJ* **114**, 6, 2402-2427.

Da Costa, L.N., Geller, M.J., Pellegrini, P.S., Latham, D.W., Fairall, A.P., Marzke, R.O., Willmer, C.N.A., Huchra, J.P., Calderon, J.H., Ramella, M., Kurtz, M.J. 1994 *ApJ* **424**, L1.

Dale DA, Giovanelli R, Haynes M, 1997 *AJ* **114** (2): 455-473.

Dale DA, Giovanelli R, Haynes MP, Scodeggio M, Hardy E, Campusano LE, 1998 *AJ* **115** (2), 418-435.

Dale D.A., Giovanelli R, Haynes M.P., 1999, *AJ* **118** (4), 1468-1488.

Dale D.A., Uson JM, 2000, *AJ* **120** (2), 552-561.

Dale D.A., Giovanelli R, Haynes M.P., Hardy E, Campusano LE, 2001, *AJ* **121**, 1886-1892.

De Lapparent, V., Geller, M.J., Huchra, J.P. 1988 *ApJ* **332**, 44.

De Vaucouleurs, G. 1970 *Science* **167**, 1203.

Einstein, A. 1950 *The Meaning of Relativity*, 3rd ed., Princeton Univ. Press.

Giovanelli, R., Haynes, M.P., Chincarini, G.L. 1986 *ApJ* **300**, 77.

Huchra, J., Davis, M., Latham, D., Tonry, J. 1983 *ApJS* **52**, 89.

Joyce, M., Montuori, M., Labini, F.S. 1999 *ApJ* **514**, L5.

Joyce, M., Montuori, M., Labini, F.S., Pietronero, L. 1999 *A&A* **344**, 387.

Labini, F.S., Montuori, M. 1998 *A&A* 331, 809.

Labini, F.S., Montuori, M., Pietronero, L. 1998 *Phys. Lett*. **293**, 62.

Lynden-Bell, D., Katz, J., Bicak, J. 1995 *MNRAS* **272**, 150.

Mach, E., *The Science of Mechanics - A Critical and Historical Account of its Development* 1960 Open Court, La Salle.

Mathewson D.S., Ford V.L., Buchhorn M. 1992*,* *ApJS* **81**, 413.

Mathewson D.S., Ford V.L. 1996, *ApJS* **107**, 97.

Martinez, V.J., PonsBorderia, M.J., Moyeed, R.A., Graham, M.J. 1998 *MNRAS*' **298**, 1212.

Pais, A. 1982 *Subtle is the Lord - The Science and Life of Albert Einstein*, OUP.

Peebles, P.J.E. 1980 *The Large Scale Structure of the Universe*, Princeton University Press, Princeton, NJ.

Persic M., Salucci P., 1995, *ApJS* **99**, 501.

Popper, K.R. 1953 *A Note on Berkeley as precursor of Mach, J. Phil. Sci.*, **4**, 26.

Roscoe D.F., 1999, *A&A*, **343**, 788.

Roscoe D.F., 2002*, A&A*, **385**, 431.

Roscoe D.F., 2002, *GRG*, **34**,5, 577.

Scaramella, R., Guzzo, L., Zamorani, G., Zucca, E., Balkowski, C., Blanchard, A., Cappi, A., Cayatte, V., Chincarini, G., Collins, C., Fiorani, A., Maccagni, D., MacGillivray, H., Maurogordato, S., Merighi, R., Mignoli, M., Proust, D., Ramella, M., Stirpe, G.M., Vettolani, G. 1998 *A&A* 334, 404.

Vettolani, G., *et al.*, *Proc. of Schloss Rindberg Workshop: Studying the Universe With Clusters of Galaxies*.

Wu, K.K.S., Lahav, O., Rees, M.J. 1999 *Nature* **397**, 225.

Mach's Principle and Quantum Mechanics Without Spacetime

A Possible Case for Noncommutative Differential Geometry?

T.P. Singh*

In the spirit of Mach's ideas, spacetime should itself be regarded as a derived construct, rather than an absolute one, the fundamental role being assigned to matter. In order to achieve such a development, we suggest that one needs to re-examine the basic theory we use to describe matter—quantum mechanics. The rules of quantum mechanics require a time coordinate for their formulation. However, a notion of time is in general possible only when a classical spacetime geometry exists. Such a geometry is itself produced by classical matter sources. Thus it could be said that the currently known formulation of quantum mechanics pre-assumes the presence of classical matter fields. A more fundamental formulation of quantum mechanics should exist, which avoids having to use a notion of time. In this paper we discuss as to how such a fundamental formulation could be constructed for single particle, non-relativistic quantum mechanics. We argue that there is an underlying non-linear theory of quantum gravity, to which both standard quantum mechanics and classical general relativity are approximations. The timeless formulation of quantum mechanics follows from the underlying theory when the mass of the particle is much smaller than Planck mass. On the other hand, when the particle's mass is much larger than Planck mass, spacetime emerges and the underlying theory should reduce to classical mechanics and general relativity. We also suggest that noncommutative differential geometry is a possible candidate for describing this underlying theory.

> *A reformulation is suggested in which quantities normally requiring continuous coordinates for their description are eliminated from primary consideration. In particular, since space and time have therefore to be eliminated, what might be called a form of Mach's principle be invoked: a relation of an object to some background space should not be considered—only relationships of objects to each other can have significance.*
>
> \- Roger Penrose (1971)

The spirit of Mach's principle goes beyond the proposal that inertial frames and the nature of mass be determined by the distribution of matter in the Universe. It is in fact desirable that at a more basic level, spacetime should itself be regarded as a derived construct, and one should be able to describe matter without reference to spacetime. This in fact is one of the objectives of string theory. In this article we discuss how the Machian objective of constructing spacetime from a spacetime-less physics might be achieved by re-examining the basic theory one uses to describe matter – quantum mechanics.

The rules of quantum mechanics require the concept of time, for their formulation. The time coordinate determines the choice of canonical position

* Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India. e-mail: tpsingh@tifr.res.in

and momenta, the normalization of the wave function, and of course, the evolution of the quantum system. From the point of view of the special theory of relativity, time is a component of spacetime; furthermore the general theory of relativity endows the spacetime with a pseudo-Riemannian geometry. This geometry however, is determined by the distribution of *classical* matter—such matter is of course a limiting case of matter obeying the rules of quantum mechanics. Hence, via its dependence on time, quantum mechanics pre-assumes the existence of classical matter, whose very properties it should explain in the first place. A more fundamental formulation of quantum mechanics should exist, which does not refer to a background time. (for a recent discussion on the difficulties associated with quantum clocks see, *e.g.*, [1]). The purpose of the present paper is to outline a proposal for such a fundamental formulation, which we call Fundamental Quantum Mechanics (FQM).

The need for FQM also follows from noting that the Universe could, in principle, be completely devoid of classical matter. For instance, the Universe could consist entirely of non-relativistic, non-classical microscopic particles, which would be described by standard non-relativistic quantum mechanics, if a background spacetime were to be available. Since here classical matter and consequently classical spacetime geometry are completely absent, FQM is necessary for describing this Universe. Under very special circumstances (suitably chosen quantum states, *etc.*) such a Universe could be described by a semi-classical theory of gravity (classical gravity produced by quantum matter), but the semiclassical description is in general not valid (for a useful discussion see [2]). We observe that FQM can become necessary even if the typical energy scale of the particles in the system is much less than the Planck scale, and the particles are 'moving' non-relativistically.

In order to attempt tackling the simpler problem first, in this paper we address only the issue of a FQM for non-relativistic quantum mechanics. Even though its true that here time and space are absolute Newtonian concepts, they are nonetheless attributes of a classical world, and a nonrelativistic approximation to the geometric description provided by the special and general theories of relativity. It is therefore meaningful to search for an FQM corresponding to non-relativistic quantum mechanics. (for another discussion on quantum mechanics without time, see [3]).

We can learn about the properties of FQM by constructing a simple model of the Universe. Imagine, for the sake of visualization, the Universe to be a manifold which is a 2-sphere, with one angular coordinate representing space, and the other time (unphysical properties like closed timelike curves are not relevant here since the spherical topology is only chosen as a model which makes it easy to form pictures). Next, let us imagine that there is only one object in this Universe—a macroscopic, localized object of mass m_1. If m_1 can be treated classically, then it appears reasonable to propose that the spacetime on this manifold, as well its accompanying pseudo-Riemannian geometry, are due to this object. The object's own state is of course a trajectory in this spacetime, or equivalently, in the phase space. A key feature of this classical state is the absence of superpositions.

Next, we consider the case that the mass m_1 is so small that the object is no longer classical, but must be treated according to quantum mechanics. This would be straightforward if there were a background spacetime on the 2-sphere. Now, however, we have a completely delocalized particle which can be thought of as residing on the 2-sphere, but there is no longer any Riemannian spacetime geometry on the manifold, nor any concepts of space and time. The particle, manifold and its pseudo-Riemannian geometry have all merged into one, in some sense. The dynamics of the system can be described, possibly, as a geometry of the 2-sphere—we call this dynamics Nonrelativistic Quantum Gravity (NQG). The state of the system can no longer be said to have a causal evolution, but exists as a whole, once and for all.

Another way to arrive at this conclusion is to consider the situation where the 2-sphere already possesses a classical spacetime geometry because of existent classical matter, and the value of the mass m_1 (now assumed to be a test particle) is gradually reduced. When m_1 is classical, its state is a spacetime trajectory, but when it becomes quantum mechanical, the state becomes an element of Hilbert space, labelled by a time coordinate. It is rather unnatural to imagine that as the value of m_1 is reduced, the state at some stage jumps from being in physical spacetime, to being in a Hilbert space. It is more natural to assume that the state of m_1 always belongs to the fundamental NQG 2-sphere—which looks like physical spacetime for large values of m_1 and like the direct product of a Hilbert space with time, for small values of m_1.

The only natural scale which separates a large classical value for the mass m_1 from a small 'quantum mechanical' value should be determined by Planck mass $m_{Pl} = (hc/G)^{1/2} \approx 10^{-5}$ gm. Now one expects, from observation, the mass scale separating the 'classical' from the 'quantum' to be smaller than m_{Pl} by a few orders of magnitude. However, for want of a better understanding, we will for the time being continue to refer to this separation scale as m_{Pl}, with the understanding that a more refined analysis will yield a somewhat smaller separation scale. For $m_1 >> m_{Pl}$ (equivalently $h \to 0$ or $G \to \infty$) NQG should reduce to a classical spacetime trajectory for the particle, and to the nonrelativistic Einstein equations (*i.e.* Newtonian gravity) for the emergent spacetime geometry. Since classical objects are never observed in superposed states, it has to be the case that NQG is a non-linear theory—two solutions of the theory cannot be superposed.

It is important to note that NQG is *not* the timeless description of nonrelativistic quantum mechanics (*i.e.*, FQM) that we are seeking. Unlike standard quantum mechanics and unlike FQM, NQG is non-linear, and involves the gravitational constant G, since it must also describe the gravitational effects of m_1. It is clear that if we take the limit $G \to 0$ (equivalently, $m_{Pl} \to \infty$ or $m_1 << m_{Pl}$) then NQG no longer describes the gravity of m_1 and should reduce to FQM: a linear theory which is the timeless formulation of standard nonrelativistic quantum mechanics. In FQM too, the physical state of m_1 does not have a causal evolution, but exists as a timeless whole on the 2-sphere.

We thus have the picture that the dynamics of the object m_1 on the 2-sphere is in general described by NQG—here spacetime geometry and matter cannot be separated from each other. NQG reduces on the one hand (when $m_1 >> m_{Pl}$) to non-relativistic Einstein equations and classical mechanics (the coordinates of the sphere now become space and time), and on the other hand (when $m_1 << m_{Pl}$) to a timeless description of standard quantum mechanics.

The time-dependent equivalent version of the FQM for m_1 is obtained when the 2-sphere possesses a classical spacetime geometry due to other matter sources. This could happen, for instance, if there is present on the 2-sphere, another mass $m_2 >> m_{\text{Pl}}$ which hence endows it with a classical spacetime. m_1 is now a test particle on this 2-sphere in the sense that $m_1 << m_{Pl} << m_2$. The FQM for m_1 now has the standard interpretation of non-relativistic quantum mechanics. It could be concluded from this discussion that there is an apparent time evolution in quantum mechanics only because the Universe today is dominated by classical matter, which produces classical spacetime. At a deeper level, quantum mechanics describes the physical state of the system m_1 not as an evolution in time, but as a dynamics in which spacetime and matter cannot be separated.

Important clues as to the nature of NQG can be obtained by examining the possible gravitational interaction a test particle m_1 has with the background gravitational field provided by the particle m_2. The various possibilities are shown in the Table below, depending on how m_1 and m_2 compare with m_{Pl}.

$m_1 \downarrow \| m_2 \rightarrow$	$m_2 << m_{Pl}$	$m_2 \sim m_{Pl}$	$m_2 >> m_{Pl}$
$m_1 << m_{Pl}$	NO	Test 1	NRQM
$m_1 \sim m_{Pl}$	X	NQG	Test 2
$m_1 >> m_{Pl}$	X	X	GR

The 11 entry (NO) indicates that gravitational interaction is absent when both the masses are much smaller than m_{Pl}. The 21 and 31 entries (X) indicate that in these cases m_1 is not a test particle. The 12 entry (Test1) is the most important, as this is the domain where laboratory experiments could in principle search for possible signatures of NQG, by studying the gravitational interaction of a quantum mechanical particle m_1 with the NQG 'gravitational field' of m_2. The 22 entry (NQG) represents the genuine gravitational interaction of two particles in NQG: this would describe the dynamics that replaces Newtonian gravity in a quantum theory of gravity. The 13 (NRQM) entry is the standard non-relativistic quantum mechanics on a background spacetime, whereas the 23 entry (Test2) probes the response of a particle in NQG to a classical spacetime geometry. The final 33 entry is Newtonian gravity (nonrelativistic GR). Looking at the last column, since both the first (NRQM) and the third (GR) entry satisfy the equivalence principle, one could expect that NQG on a background spacetime (middle entry Test2) satisfies the equivalence principle too—a possible indicator that NQG is a generally covariant theory.

We have been able to infer some properties of nonrelativistic quantum gravity by starting from the premise that there should be a description of quantum mechanics which does not refer to a time coordinate. The theory to which one is led (NQG) has as its limits both quantum mechanics ($G \to 0$) as well as nonrelativistic gravity ($h \to 0$). It is hence possible that the dynamics of NQG (in our model the dynamics of the 2-sphere) is described by a noncommutative differential geometry (NDG) [4], because NDG has within itself, as special cases, a commutative spacetime geometry which describes spacetime and gravity, and also, a noncommutative algebra structure which can describe quantum mechanics on an ordinary spacetime.

An outline of the dynamics of NQG on the 2-sphere is suggested here. We shall assume that the 2-sphere is a noncommutative space on which the algebra of functions is in general noncommuting. Let A and B denote the 'coordinates' on the 2-sphere, and let there be, associated with these 'coordinates', 'momenta' p_A and p_{B}. These four quantities are assumed to describe a particle m on the 2-sphere. We propose, on the 2-sphere, commutation relations of the following kind:

$$[A,B] = L_{Pl}^2 F_1\left({m}/{m_{Pl}} \right)$$

$$[A,P_A] = \left[B, B_{p_B}\right] = ihF_2\left({m}/{m_{Pl}} \right)$$

$$[p_A, p_B] = ih^2 L_{Pl}^{-2} F_3\left({m}/{m_{Pl}} \right)$$

The functions F_1, F_2 and F_3 are assumed to go to zero in the limit $m >> m_{Pl}$. Thus all the four quantities describing the particle become commuting in the large mass limit. Now one may identify A and B with ordinary spacetime coordinates on the 2-sphere, and p_{A}, p_{B} with ordinary energy and momentum. In the limit $m << m_{Pl}$ these commutation relations describe FQM. Since a background spacetime is absent, the function F_1 m/m_{Pl}) has to be non-vanishing in this limit. FQM is hence described in terms of quantities which are not standard spacetime coordinates and momenta. This description becomes equivalent to standard quantum mechanics in the presence of an external spacetime, though it is unclear at present as to how that happens. The presence of the mass m in the commutation relation [A,B] should be seen as analogous to the fact that in Riemannian geometry the non-commutativity of covariant derivatives is determined by the Riemann tensor, which itself is related to the distribution of matter. Hence we are suggesting that at a fundamental level not only the covariant derivatives, but the coordinates as well, do not commute, and the non-commutativity of the latter may also be related to matter.

The physical state of the system in NQG is the noncommutative analog of a vector field, or equivalently a noncommutative analog of a derivation, on the 2-sphere. Furthermore, associated with the differential structure of the A,B space there is a concept of curvature of the noncommutative space. The exact implementation of the notion of curvature in NDG is at present an issue that has not been fully resolved [5]. We propose that this curvature is induced by the presence of the mass m, in the spirit of general relativity. In the commutative limit the dynamical equation relating curvature to m reduces to Einstein's equations. The detailed nature of the dynamics of NQG in the language of noncommutative geometry is under investigation.

It appears to us that, like spacetime, the metric is also an emergent concept, valid only when the Universe is dominated by classical matter. Our nonrelativistic considerations here cannot explain the Lorentzian signature for the emergent metric. We hope to address relativistic generalizations of these ideas in the near future.

It is also of interest to investigate what possible connection this work might have with string theory and M-theory, wherein the position coordinates of a D-brane become non-commuting.

We conclude by suggesting that Einstein's criticism that quantum mechanics is incomplete may now be understood as the theory having to refer to a background time. A version of the theory which does not refer to such a background time possibly removes this incompleteness. We also suggest that fulfilling the Machian objective of constructing a space-timeless description of the Universe provides a natural path toward the correct quantum theory of gravity.

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Note added in revised version: Some more recent developments are discussed in [6].

References

[0] R. Penrose (1971), The theory of quantized directions, in *Quantum Theory and Beyond*, Ed. T. Bastin, Cambridge University Press.

[1] S. Carlip, Quantum Gravity: a Progress Report, gr-qc/0108040, *Rept. Prog. Phys.* 64, 885 (2001).

[2] Chung-I Kuo and L. H. Ford, Semiclassical Gravity Theory and Quantum Fluctuations, *Phys. Rev.* D47, 4510 (1993).

[3] C. Rovelli, Quantum mechanics without time: a model, *Phys. Rev.* D42, 2638 (1990).

[4] Alain Connes, *Noncommutative Geometry*, Academic Press (1994).

[5] J. Madore, *An Introduction to Noncommutative Differential Geometry and its Physical Applications*, 2nd Edition, Cambridge University Press (1999), Chapter 3.

[6] T.P. Singh, *Quantum mechanics without spacetime II*, gr-qc/0205056 (2002).

Mass and Gravitation in a Machian Universe

H. Broberg
henrik@atelsat.com

The essence of the "Mach Principle," a term coined by Albert Einstein in 1918, is that "the inertial force which acts on an accelerating object is due to its interaction with all the matter present in the rest of the Universe." In the following we outline a theory of interactions between matter and vacuum energy-fields that incorporates this principle.

Introduction

The classical question of the origin of the inertial force and the relation between gravitational and inertial mass is dealt with in Section 1, using the concept of "inertial induction" proposed by Sciama and further developed by A. Ghosh. Here we have added a relational form for the gravitational constant and a universal mass density distribution for the fields of particles, which lead to identity between the gravitational and inertial masses. The "gravitational paradox" is also resolved.

In Section 2, the concepts of mass-flows in time and space are discussed in order to provide a foundation for the later discussion of the gravitational field. An important aspect is how particle mass interfaces with these flows, and the consequences in terms of transfer of forces in a vacuum. Linear acceleration is discussed in Section 3. A displacement velocity acting through the vacuum frame is introduced as a precursor to a more detailed analysis of the gravitational field in the following section. Gravitation is described in Section 4, using a space-time geometry that reflects a displacement velocity in the vacuum mass-density field as the carrier of the gravitational effect. The quantization of the field is an important result, leading to the appearance of gravitational force. In Section 5, the essence of Mach's principle is defined in a discussion of the roles and topologies of particles as components of the Universal mass-system. In Section 6, some deductions from the results in the other sections are demonstrated. The mass of the electron is calculated from its charge and the particle mass-time interface, while links are made also to other particle masses.

1 Inertial and Gravitational Mass

The Inertial Force

In his book *Origin of Inertia*, [2] A. Ghosh, following an idea first proposed by Sciama, [11] suggests that the gravitational interaction between two bodies has an acceleration dependent force-term if the bodies posses a relative acceleration (a). The result is that, in addition to the well-known distance-dependent force between two bodies (m_1 and m_2), namely, $F = G \cdot m_1 \cdot m_2/r^2$, there will be an additional acceleration-dependent term $\Delta F = (G \cdot m_1 \cdot m_2/c^2 \cdot r) \cdot a$, which Sciama referred to as "inertial induction." This extra term can be regarded as the force associated with the acceleration of the mass-equivalent of the gravitational field energy of one body in the field of the other. For a test particle, the ΔF terms arising from all the other masses in the Universe in the inertial induction approach should sum up to the inertial force on the particle when it is accelerated.

A. Ghosh proposes that, as the Universe can be considered to be isotropic in the large scale, the position-dependent gravitational force terms will cancel out on the Universal scale, and only the acceleration dependent terms remain. The resultant force on a test particle under acceleration will then be the totality of inertial induction on the particle from all the masses in the Universe, as follows.

The mass of a spherical shell can be given as:

$$dm = 4\pi r^2 \cdot \rho \cdot dr$$

Integrating over spherical shells around the test particle from distance zero up to a radius R_U gives the force on the test particle from the inertial induction due to all the masses in the Universe:

$$F = \int_0^{R_U} \frac{G \cdot m \cdot \rho \cdot 4\pi r^2 dr}{c^2 r} \cdot a \equiv \frac{2\pi G \rho R_U^2}{c^2} \cdot ma$$

If the force in question is actually the inertial force, in line with Mach's principle, the parameter modifying ma must be equal to 1. Using Hubble's constant [12] to establish R_U, and assuming an average density of the Universe from astronomical observations, A. Ghosh evaluates this parameter to be approximately 1.

Using the same approach, an absolute identity can be found by using the expression for G proposed on the basis of our earlier work [5] (further discussed in Section 2):

$$G = \frac{Ac^2}{4\pi R_U}, \tag{1}$$

where A ($\approx 1.4\ \mathrm{m^2/kg}$) is the universal parameter for the interface between mass and vacuum fields in a quantum system.

We have also suggested [5] (further developed in Section 2) that in respect of each particle, the average mass-density in vacuum space at a distance r from a particle can be expressed by the following function, valid up to the event horizon of the Universe:

$$\rho_{vac} = \frac{1}{Ar}. \tag{2}$$

With these relations, the above integral function assumes the form:

$$F = \int_0^{R_U} \frac{Gm\left(\rho 4\pi r^2 dr\right)}{c^2 r} \cdot a \equiv \frac{4\pi Gm}{c^2} \cdot a \int_0^{R_U} \rho r dr \equiv \frac{m}{R_U} \cdot a \int_0^{R_U} dr \equiv a \cdot m . \qquad (3)$$

This result, which is independent of the size of the Universe, supports Mach's principle and agrees with the acceleration-dependent term proposed by A. Ghosh for the identity between the inertial and gravitational mass.

The Gravitational Paradox

A. Ghosh [2] and A.K.T. Assis [4] have discussed a difficulty that emerges in Newton's inverse square law of gravitation, the so-called "gravitational paradox." If the Universe is assumed to be homogenous, infinite and Euclidean, then the energy potential of a particle *m* relative to the surrounding Universe integrated over spherical shells up to a radius R_U becomes:

$$U = -\int_0^{R_U} \frac{Gm}{r} \cdot 4\pi r^2 \rho dr .$$

When the parameters involved are static and constant, this integral implies $U = -2\pi Gm\rho R_U^2$, which becomes infinitely large when $R_U \to \infty$. A. Ghosh removes this paradox by invoking cosmic drag, whereby the parameter *G* would decay exponentially with distance.[2]

The paradox can also be eliminated if the same parametric relations as above [1] are assumed for Newton's "constant" (1) and the universe density (2). In this case the integral becomes

$$U = -4\pi Gm_0 \int_0^{R_U} \rho r dr .$$

Inserting the expressions for the gravitational constant and the density gives:

$$U = -4\pi \left(\frac{Ac^2}{4\pi R_u} \right) m_0 \int_0^{R_U} \left(\frac{1}{Ar} \right) r dr \equiv -\frac{m_0 c^2}{R_u} \int_0^{R_U} dr \equiv -m_0 c^2 . \qquad (4)$$

This result is also independent of the "size" of the Universe, and removes the "gravitational paradox." [9] It points instead to a solution whereby the total energy of the Universe is always zero.

2. The Mass-Time Interface and Vacuum Density

Particles will be treated here as extended quantum systems in space *and* time, *i.e.*, as time-lines, which have acquired a thickness *via* an interface between mass and time, thus extending the time-dimension to a tube-like space, say time-tubes or time-fibres, postulated by the author. [5,6,8] (Unlike the situation in relativity, the "world-line" of a particle here has a thickness.) The interface is introduced as a surface proportional to the mass of the particle, with the proportionality constant *A* (see Section 1).

We first treat the simplest case of mass transported from one instant of time to another in the particle system. The interface between the time and mass is defined as a surface perpendicular to the time-line, $\Phi(M) = AM$. The time-like distance is $\Delta t = t_2 - t_1$, given here geometrically as $R = c\Delta t$. The mass M is thus transported as a flow of density ρ_t in the time-dimension through the mass-time interface from t_1 to t_2, and this process restores M at t_2: Thus we have $M = \Phi(M) \cdot \rho_t \cdot R \equiv AM \rho_t R$, from which it follows that

$$\rho_t = 1/AR \tag{5}$$

In the four-dimensional space-time, this density would result if the mass is smeared out with equal density over the time-tube with extension Δt. It can be seen that this function has a singularity at $R = 0$, which represents the present time of the mass-object. At this time the density is mathematically equal to ∞.

Another case is the flow of energy through the vacuum toward a local sink, represented by a singular point where the mass-density is ∞. Associated with this vacuum flow is a symmetric energy-density field, $\rho(r)$ (here expressed in mass units). The mass of the flow within a radius R_p becomes

$$M_p = \int_0^{R_p} 4\pi r^2 \rho(r) dr$$

We postulate that the mass-time interface to M_p has the form:

$$\Phi(M_p) = 2AM_p = 4\pi R_p^2 \tag{6}$$

The relation between the space-time interface and the vacuum density becomes:

$$M_p = \frac{2\pi R_p^2}{A} \equiv \int_0^{R_p} 4\pi r^2 \rho(r)\, dr . \tag{7}$$

Differentiation gives:

$$dM_p = \frac{4\pi R_p}{A} dR_p \equiv 4\pi R_p^2 \rho(R_p)\, dR_p . \tag{8}$$

From this it follows that the vacuum flow density is the same as the density in the time dimension given earlier:

$$\rho_{vac} = \frac{1}{AR} \tag{9}$$

The vacuum density function also becomes infinite when $R \to 0$, which signifies a singular point for the particle in space as well as in time.

We will now examine the following relation between radii and masses from equation (7):

$$M_p = \frac{2\pi R_p^2}{A} .$$

This relation can be compared with observations on elementary particle level, as well as for the large scale Universe. For both cases the relation between radius and mass gives sensible results with A as a fixed scale parameter. Using the radius of $\approx 10^{-14}$ m for a nucleon and the Hubble radius for the Universe gives $A \approx 1$ m²/kg. The value has been more precisely fixed at $A \approx 1.4$ m²/kg based on calculation of the electron mass from its charge (see Section 6 of this paper). On this basis, with further support from the relation between the inertial and gravitational mass, as well as the solution to the gravitational paradox from Section 1, the relations found here are assumed to be applicable to particles and their associated vacuum fields.

If the particle radius is combined with the concept of a "gravitational radius," we obtain the following equation:

$$\begin{cases} AM_p = 2\pi R_p^2 \\ R_p = \dfrac{2GM_p}{c^2} \end{cases}, \tag{10}$$

resulting, for the case of the Universe treated as an autonomous particle, or a single particle treated as an autonomous system (a Universe of its own) in the following relations (the subscripts "U,p" mean that relations are valid either with U for the mass of the Universe or p for a particle mass)

$$\begin{cases} G_{U,p} = \dfrac{Ac^2}{4\pi R_{U,p}} \\ G_{U,p} = \frac{1}{2}\sqrt{\dfrac{Ac^4}{2\pi M_{U,p}}} \end{cases}, \tag{11}$$

and, for the particle as a member of the Universe, the Schwarzschild gravitational radius becomes:

$$R_g(M_p) = 2\cdot\left(G_U\right)\cdot\frac{M_p}{c^2} \equiv 2\cdot\left(\frac{Ac^2}{4\pi R_U}\right)\frac{M_p}{c^2} \equiv \frac{AM_p}{2\pi R_U}, \tag{12}$$

which can be further developed to:

$$AM_p = 2\pi R_U R_g \equiv 2\pi R_p^2 \tag{13}$$

When the gravitational parameter is set equal to Newton's constant, the resulting mass and radius of the system assume cosmic dimensions, allowing the radius to be identified with the Hubble radius and the Universe to be treated as a black hole, confirming equation (1). Therefore, the particle has a dual nature:

(I) it has a radius R_p for its mass-time interface, defining it as an autonomous gravitational system;

(II) it has a gravitational radius R_g for the confinement of its mass as a part of the gravitational mass-system of the Universe.

In both cases the Schwarzschild radius applies, although scaled as in equation (11) above, where M is the gravitational mass of the system. For the geometric relation between the two particle aspects I and II, please refer to Figure 4 and the corresponding text in Section 5, Vacuum Flow Dynamics.

If the Universe is scaled down to the mass of a nucleon, while A is kept constant, also the gravitational constant and the radius are scaled accordingly.

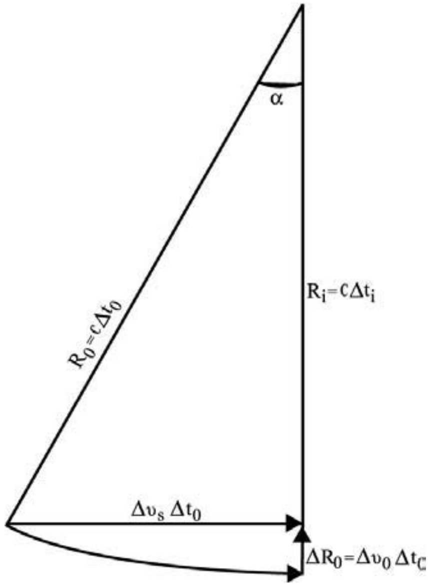

Figure 1. Space-time relations at linear acceleration.

The gravitational constant becomes about 10^{40} times larger, while the radius R_p shrinks to $\approx 10^{-14}$ m, which fits well for a nucleon both in terms of nuclear force and dimensions.

3 Linear Acceleration

Acceleration can be defined as a transfer of a test-particle from one time-system into another. For the case of linear acceleration, the space-time geometry of the transfer is illustrated in Fig. 1. The time increment during which the acceleration takes place is Δt_0 in the original system, reduced to Δt_i during the process, while the velocity of the test particle changes by the increment $\Delta \upsilon_s$.

The vacuum displacement velocity ($\Delta \upsilon_0$) reduces the time-like distance R_0 to R_i while the space-like velocity changes from 0 to $\Delta \upsilon_s$ in the system of the test-particle relative to its earlier state of motion. In other words: $c\Delta t_i = c\Delta t_0 - \Delta \upsilon_0 \Delta t_0$. The relativistic factor then emerges as

$$\gamma = \frac{R_0}{R_i} \equiv \frac{1}{\sqrt{1-\left(\frac{\Delta \upsilon_s}{c}\right)^2}} \equiv \frac{1}{1-\frac{\Delta \upsilon_0}{c}} \tag{14}$$

and

$$\gamma - 1 = \frac{\frac{\Delta \upsilon_0}{c}}{1-\frac{\Delta \upsilon_0}{c}}$$

which can be developed into $\Delta \upsilon_s \, d\Delta \upsilon_s = (c - \Delta \upsilon_0)\, d\Delta \upsilon_0$, indicating the difference between the normal concept of velocity applicable to $\Delta \upsilon_s$ and the displacement velocity $\Delta\upsilon_0$. (In respect of the angle α in Figure 1, we have $d\Delta\upsilon_0/d\Delta\upsilon_g = \tan\alpha$). The kinetic energy required for the acceleration can be described in terms of a vacuum flow in the time dimension:

$$\begin{aligned} E_k &= \Phi(M_0)\cdot\rho(R_i)c^2\cdot\Delta R_0 \equiv AM_0\cdot\frac{c^2}{AR_i}\cdot\Delta\upsilon_0\Delta t_0 \equiv \\ &\equiv M_0c^2\cdot\frac{\Delta\upsilon_0}{c}\cdot\frac{\Delta t_0}{\Delta t_i} \equiv M_0c^2\cdot\frac{\Delta\upsilon_0}{c}\cdot\frac{1}{1-\Delta\upsilon_0/c} \equiv M_0c^2\cdot(\gamma-1) \end{aligned} \quad . \tag{15}$$

In this way, the kinetic energy can be described as resulting from a flow of energy from the surrounding space into the mass of the test particle (compare our daily experience of using fuel to accelerate an automobile).

It is important to observe that $\Delta\upsilon_0$ can take on any value, although it has to be limited to $\Delta\upsilon_0 \le c$ as long as it is required that imaginary or negative time-distances are to be avoided. This solution for the linear acceleration may seem trivial. However, its key importance is that it indicates a way to describe the gravitational field and its quantization, which can be further generalized to quantum systems in general.

4 The Gravitational Field

4.1 Space-Time Concepts

Using the concepts introduced above, a central force field in the vacuum can be described assuming that the timelike distance R_0 from Fig. 1 is the distance from a test particle to the field-centre before the particle is acted on by the central mass M, while R_i is the distance to the centre when the contraction of the space in the vicinity of M is taken into account. This case is illustrated in Fig. 2.

The acceleration field in the vacuum space surrounding M is described using the velocity $\Delta\upsilon_0$ applicable to the displacement of the vacuum itself. The figure illustrates that, whereas a test particle m would "fall" from R_0 to R_i during the time Δt_0, the same particle positioned at R_i would have "fallen" to R_X during the time Δt_i (assuming the same $\Delta\upsilon_0$ to apply during the whole cycle). The conventional gravitational field follows if the distance $R_g(M)$ from R_0 to R_X during each such cycle is given by:

$$R_g(M) = \Delta\upsilon_0(\Delta t_0 + \Delta t_i), \tag{16}$$

leading to

$$\Delta\upsilon_0 = \frac{R_g(M)}{\Delta t_0 + \Delta t_i} \equiv c\cdot\frac{R_g(M)}{R_0 + R_i}. \tag{17}$$

From Figure 2 and Figure 1 we have the relation between R_i and R_0 given by equation (14), which together with equation (17) results in the following second degree equation:

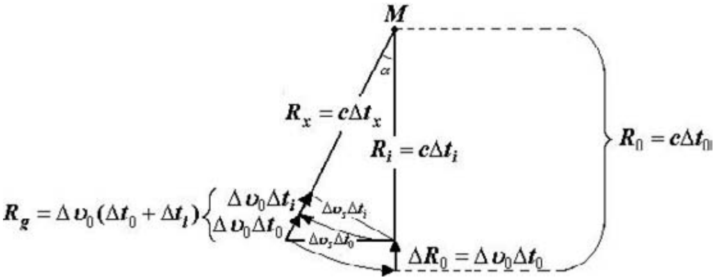

Figure 2 The geometry in the presence of a central mass *M*.

$$\left(\frac{\Delta\upsilon_0}{c}\right)^2 - 2\frac{\Delta\upsilon_0}{c} + \frac{R_g}{R_0} = 0 .$$

The solutions are:

$$\Delta\upsilon_0 = c\left(1 \pm \sqrt{1 - \frac{R_g}{R_0}}\right).$$

To ensure $\Delta\upsilon_0 \le c$ we use the solution from above with the minus sign $\Delta\upsilon_0 = c[1 - \sqrt{(1 - R_g/R_0)}] \approx cR_g/2R$, where the approximation is valid for $R_0 >> R_g$. In this limit, considering the definition $R_0 = c\Delta t_0$ from Figures 1 and 2, the gravitational acceleration becomes $a_g = -cR_g/2R_0^2 \cdot dR_0/dt_0 \cong -R_gc^2/2R_0^2$, which is recognized as Newton's law.

Derivation of $\Delta\upsilon_0 = f(t_0)$ as if it were a continuing function gives the somewhat more accurate expression $a_g = -R_gc^2/2R_0R_i$. However, also to include the region close to R_g, the latter function should be replaced by another solution respecting the quantized nature of equation (17). For this purpose, reference is made to equation (15), which can be rewritten as $E_k = m_p\gamma\cdot(\Delta\upsilon_0/\Delta t_0)\cdot R_0 \equiv m_p\gamma\cdot(\Delta\upsilon_0/\Delta t_i)\cdot R_i$. Considering E_k equivalent to the negative energy potential of the particle with rest mass m_p in the gravitational field of the mass M from Figure 2, the acceleration in the quantized field can be recognized as

$$a_g(M) = -\frac{\Delta\upsilon_0}{\Delta t_0} \equiv -\frac{R_g(M)}{\Delta t_0(\Delta t_0 + \Delta t_i)} \equiv -\frac{R_g(M)c^2}{R_0(R_0 + R_i)} \tag{18}$$

This is equivalent to Newton's law at a sufficient distance from the central mass, while it gives the conditions for trapping light as it approaches the gravitational radius of a black hole.

It should be noted that the gravitational radius of the central mass is equal to the sum of all the gravitational radii of its constituent particles, since there is a fine structure in $R_g(M) = \sum_v R_{gv}(m_v)$ and, accordingly, in the possible subdivisions of R_0.

A relation between R_0 and R_i is given earlier in equation (14). Combining that relation with the solution given above for $\Delta\upsilon_0$ gives, in accordance with the time dilation from the Schwarzschild solution, the following:

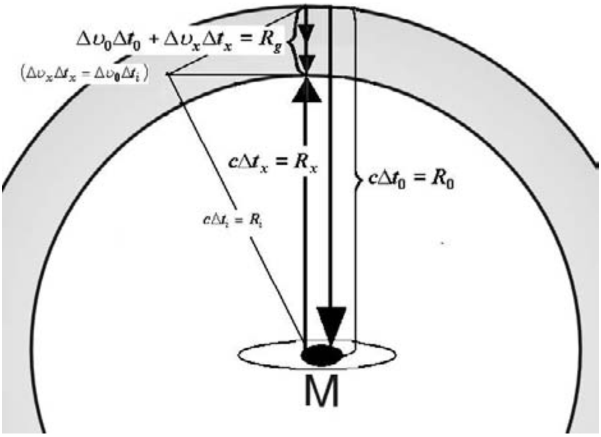

Figure 3 Vacuum flow directions in respect of a particle singularity.

$$\frac{R_i}{R_0} \equiv \frac{\Delta t_i}{\Delta t_0} = \sqrt{1 - \frac{R_g(M)}{R_0}} \tag{19}$$

With reference to Figure 2, we have therefore confirmed the relation

$$R_x = R_0 - R_g , \tag{20}$$

which can also be expressed as $R_X = R_0 - \Delta \upsilon_0 (\Delta t_0 + \Delta t_i) \equiv R_0 - (\Delta \upsilon_0 \Delta t_0 + \Delta \upsilon_x \Delta t_x)$. It follows that $\Delta \upsilon_0 \Delta t_i = \Delta \upsilon_x \Delta t_x$, and this leads to

$$\Delta \upsilon_x = \Delta \upsilon_0 \cdot \frac{\Delta t_i}{\Delta t_x} \equiv \Delta \upsilon_0 \cdot \frac{R_0}{R_i}$$

giving

$$R_i^2 = R_0 R_x . \tag{21}$$

The angle (α) between R_0 and R_i in Fig. 2 is given by:

$$\sin \alpha = \sqrt{\frac{R_g(M)}{R_0}} \tag{22}$$

When the vacuum energy “falls” toward the mass centre, each R_X in the preceding cycle becomes the R_0 in the next cycle. Therefore, if $R_0 = R_\nu$, we now have $R_X = R_{\nu-1}$, which becomes the new R_0 in the following cycle, *etc*. The R_i becomes $R_{\nu i}$ in each cycle.

The geometry can be described by two characteristic expressions:

$$\begin{cases} R_\nu = R_{\nu-1} + R_g(M) \\ R_{\nu i}^2 = R_\nu \cdot R_{\nu-1} \end{cases} ,$$

from which it follows that

$$R_\nu^2 - R_{\nu i}^2 = R_\nu R_g \tag{23}$$

This constitutes the “handshake” between two contiguous quantum regions in the vacuum space, following which the distance $\Delta \upsilon_s \Delta t_i$ in Fig. 2 will also rotate from its horizontal position and take the place of $\Delta \upsilon_s \Delta t_0$ during the following cycle. This process will gradually enlarge the angle until it reaches 90° when R_0 approaches the gravitational radius at the same time as R_i and R_X disappear.

4.2 Vacuum Flow Dynamics

With reference to equation (8), the flow through the spherical surface set up over the gravitational radius of the particle is:

$$\frac{dM}{dt_0} = 4\pi R_g^2 \rho\left(R_g\right) \Delta \upsilon\left(R_g\right). \tag{24}$$

With reference to equation (9) the flow density becomes $\rho(R_g) = 1/AR_g$. With reference to equation (17), $\Delta\upsilon(R_g) = c$ because $R_0 = R_g$ and $R_i = 0$. Therefore the absorption flow rate becomes:

$$\frac{dM}{dt_0} = \frac{4\pi R_g}{A} \cdot c.$$

The relation described above can be linked to Hubble's constant using $R_g = AM/2\pi R_U$ from equation (12):

$$\frac{dM_p}{dt_0} = \frac{M_p c}{R_U} \equiv 2H_0 M_p. \tag{25}$$

Therefore, if the particle singularity is an absolute sink for vacuum energy, the above described vacuum (energy-mass) absorption at the gravitational radius signifies that vacuum space is "falling" toward the mass-centre in contiguous steps, each separated from its neighbour by the distance R_g, while the displacement velocity $\Delta \upsilon_0$ from equation (17) is defined for each step.

The vacuum flow can also be described by one relativistic flow (positive energy) from each R_0 to the mass centre, and another relativistic flow (negative energy) from the mass centre to $R_x = R_0 - R_g$. The displacement flow fills up the difference between the two relativistic flows, as illustrated in Figure 3.

The relativistic flows will give rise to a resultant flow of:

$$\frac{\Delta M}{\Delta t_0} = 4\pi R_0^2 \rho_0 c - 4\pi R_x^2 \rho_x c \equiv \frac{4\pi R_g c}{A} \tag{26}$$

Because each R_0 has the role of the R_x in the preceding cycle, counted from the mass-centre outwards, it is important to note that equation (26) (in consideration of equation (21)) is equivalent to the following expression:

$$\frac{\Delta M}{\Delta t_0} = 4\pi R_0^2 \rho_0 c - 4\pi R_i^2 \rho_0 c \equiv \frac{4\pi R_g c}{A} \tag{26'}$$

These relations will enable a description of the gravitational interaction in line with the concept of virtual photon exchange in particle physics, and possibly the energetic vacuum concept in O(3) electrodynamics. [15]

From equation (26), we can obtain the corresponding displacement flow by replacing $cR_g(M)$ with $\Delta\upsilon(R_0 + R_i)$ from equation (17):

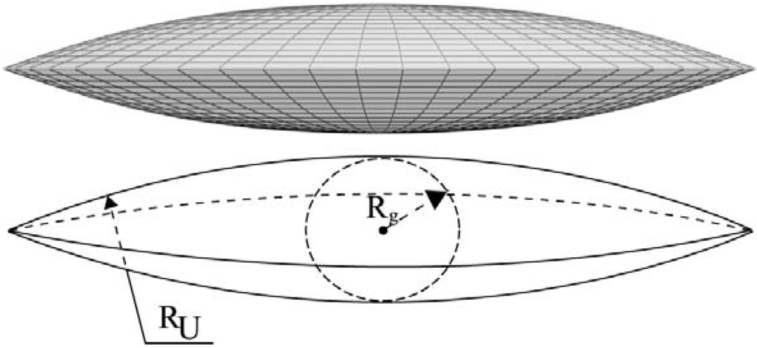

Figure 4 Illustration of a particle's vacuum interface at R_g and its time interface at R_U.

$$\frac{\Delta M}{\Delta t_0} = 4\pi R_0^2 \rho_0 \Delta \upsilon_0 + 4\pi R_i^2 \rho_i \Delta \upsilon_0 \equiv \frac{4\pi R_g c}{A} \tag{27}$$

Alternatively, using equation (21), this expression (27) can be transformed into:

$$\frac{\Delta M}{\Delta t_0} = 4\pi R_0^2 \rho_0 \Delta \upsilon_0 + 4\pi R_x^2 \rho_x \Delta \upsilon_x \equiv \frac{4\pi R_g c}{A}. \tag{27'}$$

The above description of the vacuum flow toward the mass-centre is in line with the two-sphere Schwarzschild solution. It has its counterpart in the flow in the time-dimension through the particle mass-time interface as defined by equation (6) and illustrated in Figure 4, further developed in the following section.

5. The Universe and the Particles

Following the discussion in Section 2 and the evidence from the other sections, a picture emerges of a Universe where each fundamental mass particle is associated with a singular point, at which the present time in the particle system resides. These points can be regarded as nodes in the space-time web of the Universe, connected *via* the gravitational fields. Further, the gravitational radii associated with the particles as members of the Universal system sum up to the radius (R_U) of the Universal time-sphere, while their mass-time interfaces sum up to the surface of the same sphere. This can be regarded as the essence of Mach's principle.

This quantitative surface relation is independent of how the gravitational particle radii are ordered along the radius of the Universal time-sphere. Therefore, in respect of an individual particle, its local character is preserved when its mass-time interface is composed by two opposite polar caps from the Universal sphere arranged into the shape of a disk, as illustrated in Figure 4.

With reference to equation (6) and Figure 4, the total surface of the two sides of the disk is:

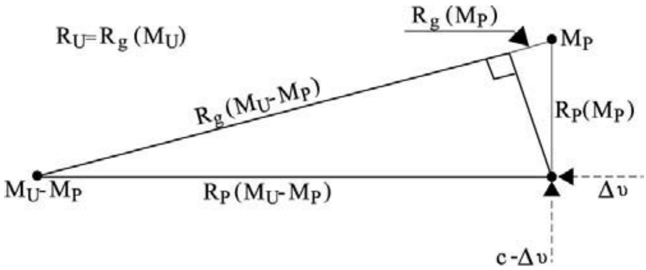

Figure 5 Illustration of Mach's Principle. The relation between the Universe and one of its constituent particles. (Compare with Figure 2: $R_0 \rightarrow R_U$.)

$$\Phi(disk) = 2AM \equiv 2 \cdot 2\pi R_u R_g \equiv 4\pi R_p^2,$$

which is identical with the particle mass-time interface from equation (6).

The core sphere with radius R_g in the centre of the disk has the density function $\rho(r) = 1/Ar$ and embraces the singularity at $r = 0$. Its mass is:

$$M_{core} = \int_0^{R_g} 4\pi r^2 \rho(r)dr = \frac{2\pi R_g^2}{A}$$

The remainder of the disk has the volume:

$$V_{disk} - V_{core} \equiv \left(2\pi R_u R_g^2 - \frac{2}{3}\pi R_g^3\right) - \left(\frac{4}{3}\pi R_g^3\right) \equiv 2\pi R_g\left(R_p^2 - R_g^2\right).$$

Applying the density parameter $\rho_g = 1/AR_g$ to the above volume gives the mass:

$$M_{disk} - M_{core} = \frac{2\pi\left(R_p^2 - R_g^2\right)}{A}$$

Adding M_{core} gives the total mass of the disk

$$M_{disk} = \frac{2\pi R_p^2}{A},$$

which is identical with equation (7).

The main flow-dynamic element of the disk is therefore the core with its singularity, while the remainder (the main part) can be regarded as a mass storage between the mass-time interface and the core. The interface of the particle is the particle's share of the interface of the Universe to the time-dimension, which is assumed to cover the event horizon of the Universe, Therefore, the density of the flow rate through the particle mass-time interface is $\rho_U = 1/AR_U$, where R_U is assumed to be the Hubble radius. With reference to equations (17) and (26') for $R_0 \rightarrow R_U = R_g$ and $R_i \rightarrow 0$, the flow velocity is c, and applying these conditions to equation (27) gives the flow rate:

$$\phi(M_p) = \Phi(disk) \cdot \rho_U \cdot c \equiv 4\pi R_p^2 \rho_U c$$
$$\equiv 4\pi R_U R_g \frac{1}{AR_U} c \equiv \frac{4\pi R_g c}{A} \equiv 2H_0 M_p \tag{28}$$

The right hand side of the relation is equivalent to that of equation (24), which proves the equivalence of the two particle-aspects (I) and (II) above, although their geometries differ.

This disk would have been a sphere if the particle was alone in the Universe or functioning as an isolated autonomous system. In such cases, the radius of the mass-time interface would also serve as the gravitational radius of the particle in respect of equation (12). It is therefore the gravitational interaction with the Universe at large which dilates R_p to become the gravitational radius of the particle, as shown in Figure 5.

The consequences of energy absorption into particle masses are discussed elsewhere [5,6]. However, here it may be mentioned that if particles are treated as black holes, according to Hawking [7,8] they should radiate energy as with a modified "black body spectrum." In the case where the spectrum takes the form of a traditional black body spectrum over a surface set up outside the event horizon by the particle's Compton or de Broglie wavelength, it should be possible to find the equilibrium radiation temperatures for electrically charged particles. For example, if particles radiate over spherical surfaces with radii corresponding to their Compton wavelengths, the temperatures would range from ~3 K for an electron to higher temperatures for heavier particles. Other alternatives are particle decay or a scenario where the particles are continuously scaled with the Universe.

Reference may also be made to the Hubble redshift, which can be expressed as:

$$\frac{d\lambda}{dt} = H_0 \lambda . \tag{29}$$

Therefore, if Planck's constant is also constant on the cosmic level, the photon would lose a quantum of energy during each cycle, independent of its wavelength, equal to:

$$\Delta E(Photon) = -hH_0 . \tag{30}$$

This fundamental observation motivated the concept of an elementary quantum with energy *hH*, representing the smallest possible discrete unit of energy in the universe, first proposed by Nernst [13] and later independently suggested by us [1]. The elementary quantum could, for example, be a carrier of the gravitational force within the vacuum mass flows discussed above.

In this regard, it is noteworthy that the mass-time interface radius for this elementary quantum would be equal to the Planck length, while its wavelength or "uncertainty in position" would correspond to the Hubble radius.

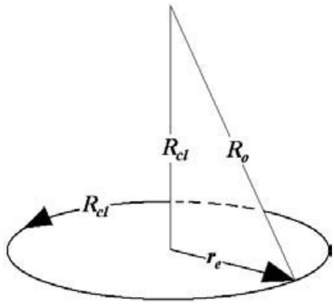

Figure 6 The geometry used for the mass of the electron.

6. Elementary Particle Masses

In the preceding sections we have introduced some new concepts, such as particle mass-time interfaces and the quantization of the gravitational field. Here we study, somewhat speculatively, the possibility that these concepts can have a more general validity for quantum physics in terms of particle masses. The mass of the electron is used to fine tune the universal parameter A. The consistency of the numerical value given to A here (1.4 m^2/kg) is then tested by application to other particles, the muon and the pions.

The Mass of the Electron

The mass of the electron can be related to a simple geometry in the vacuum.

The classical radius of the electron is:

$$R_{cl} = \mu e^2 / 4\pi m_e \tag{31}$$

The classical radius will represent a measure of time in the electron system. In the space-time geometry it will also be curled along the equivalent to the event horizon of a black hole. This geometry arises when the angle between R_0 and R_i in Fig. 2 becomes 90° at the gravitational radius, equation (22). For the electron as an isolated quantum system—like a self-contained Universe—the gravitational radius will be adapted to the size of the system according to equation (12). The radius vector to the curled R_{cl} is $r_e = \Delta \upsilon \cdot \Delta t_0$. This gives the following equation system (see Figure 6):

$$\begin{cases} r_e^2 + R_{cl}^2 = \left(c\Delta t_0\right)^2 \\ R_{cl} = 2\pi r_e \end{cases}$$

The electron mass corresponds to the interface surface from equation (6) with the radius r_e:

$$Am_e = 2\pi \left(r_{cl} / 2\pi \right)^2 . \tag{32}$$

This gives (for $A \approx 1.4$ m^2/kg)

$$m_e = \frac{1}{4\pi} \sqrt[3]{\frac{2(\mu e^2)^2}{A}} \approx 9.1 \cdot 10^{-31} \text{ Kg} \tag{33}$$

The Planck Particle

This particle was briefly introduced as the elementary quantum at the end of Section 5 above. It is a candidate for the role of the graviton, as well as the material content of mass. Here we characterize it further in its double role, as follows.

Applying the formalism of equation (6) gives:

$$\begin{cases} \Phi_{eq} = 2AM_{eq} = 4\pi R_{eq}^2 \\ R_{eq} = R_{Pl} \equiv \sqrt{\dfrac{2Gh}{c^3}} \end{cases} \tag{34}$$

From this, and with G from equation (1) follows that the mass can be expressed as:

$$\begin{cases} M_{eq} = \dfrac{2\pi R_{eq}^2}{A} \equiv \dfrac{4\pi Gh}{Ac^3} \equiv \dfrac{hH}{c^2} \approx 1\cdot 10^{-68}\,\mathrm{Kg} \\ G = \dfrac{Ac^2}{4\pi R_U} \equiv \dfrac{AcH}{4\pi} \end{cases}, \tag{35}$$

where h is Planck's constant and H is Hubble's constant. This particle has the topology illustrated in Figure 4, with its singularity at the central point of the figure.

If we introduce the "quantum expression" for the Plank length, with h replaced by $\hbar$, the surface-to-mass expression from equation (35) becomes:

$$\tfrac{1}{2}\Phi = AM_{eq} \equiv (2\pi)^2 \cdot \left(\frac{2G\hbar}{c^3}\right) \equiv \left(2\pi R_{Plq}\right)^2 \tag{36}$$

The right hand side of the above expression shows a possible shift to a geometry where the particle takes the shape of a torus. The radius of its cross-section (R_t in Figure 7) has the modified Planck radius

$$R_t = R_{Plq} = \sqrt{\frac{2G\hbar}{c^3}}, \tag{37}$$

Whereas the elementary particle topology in Figure 4 has a point-like singularity, the singularity of the torus topology in Figure 7 is distributed over a circular line, $L_t = 2\pi R_t$, as if the singularity of Figure 4 was rotated along the circle L_t. Within the torus, along any radial distance r perpendicular to L_t, the density function (from equation (9)) is $\rho = 1/Ar$. Hence, $\rho = \infty$ along L_t. The mass of the toroidal system becomes

$$M_{eq} = L\int_0^{R_{tor}} 2\pi r\rho dr \equiv \frac{2\pi LR_{tor}}{A} \equiv \frac{\left(2\pi R_{Plq}\right)^2}{A} \equiv \frac{hH}{c^2}, \tag{38}$$

as in (35) above.

Hence, while the elementary quantum was described as a dish-like object from the gravitational point of view in Section 5 here it has been developed

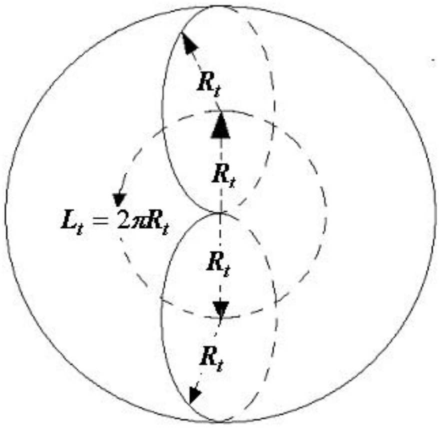

Figure 7 A toroidal topology for closed particle systems.

into a local toroidal particle when the viewpoint is transferred from the Schwarzschild to the Kerr metric system. Variations of the models developed here can also be applied to the better known elementary particles, as discussed for a few cases in the following section.

Particle Spin

The difficulties involved in making a geometrical model of the intrinsic particle spin parameter are well known. However, in this subsection, we speculate as to the possibility of linking this parameter to the confinement of matter in a toroidal topology (Figure 7).

By analogy with the introduction of the mass-time interface in Section 2, the toroidal model can be treated as a closed tube-like space in the time-dimension, within which the mass takes the form of a relativistic fluid along curved time-lines and with the density defined in equation (5). In the context of this model, particle spin can be given the following two definitions:

1. J_i being the angular momentum resulting when the torus volume of Figure 7 rotates in a plane perpendicular to the curved line L_t, with its spin axes tangential to L_t.
2. J_t being the angular momentum resulting from a rotation of the torus volume along the ring L_t, with the spin axis perpendicular to the paper.

The angular momentum J_i can be calculated as follows, with dM given by analogy with the preceding equation (38):

$$dJ_i = r \cdot r\omega \cdot dM \equiv r \cdot r\omega \cdot \rho L_t 2\pi r dr \ .$$

With $\rho = 1/Ar$ and $L_t = 2\pi R_t$, while assuming that $r\omega = c$ across the integration plane, and with M from equation (36), the angular momentum becomes:

$$J_i = \frac{2\pi c L_t}{A}\int_0^{R_t} r dr \equiv \frac{2\pi c L_t R_t^2}{2A} \equiv \frac{N(2\pi R_t^2)}{A} \cdot \frac{cR_t}{2} \equiv M \cdot \frac{cR_t}{2} \ .$$

With $R_t = \hbar/cM$, the angular momentum is further developed to $J_i = \frac{1}{2}\hbar$. Because of the assumption $r\omega = c$, all the rotating circumferences in torus space can be assumed to become Lorenz-contracted to points projected on the ring L_t. From the above it follows that $\omega = Ac\rho$, while $\omega \to \infty$ and $\rho \to \infty$ at L_t.

The angular momentum J_t, was defined by a rotation along the ring L_t. Assuming $\omega R_t = c$ as above, J_t becomes

$$J_t = M \cdot R_t \cdot c \equiv \hbar$$

This latter rotation would subsequently Lorenz-contract the ring L_t to a point.

Without going further into the delicate matter of particle spin, the above summary may be of help as a mental picture of particle spin and its role in the dual aspects of a particle's wave-like and point like representations, the former being the due to the non-contracted radii of the rotating frames, while the latter would arise from the successive Lorenz-contraction of the whole system to a point.

Finally, the spin quanta may provide the truncating conditions for equation (7), which set the conditions for establishing particles as isolated, quantified elements of the Universe.

Links to other Particle Masses - M_π^0, $M_\pi^\pm$ and M_μ

Determinations of the other elementary particle masses generally arise from the same geometry in space-time discussed above in relation to the elementary quantum. For example, the μ-meson mass conforms with a toroidal system, or a Kerr metric, similar to equation (38), although with twice the size of L_t in Figure 7, or $L_t = 4\pi R_t$:

$$AM = 2(2\pi)^2 \left(\frac{\hbar}{cM} \right)^2, \tag{39}$$

resulting in the mass

$$M_\mu = \sqrt[3]{\frac{2}{A}\left(\frac{h}{c} \right)^2} = 1.90 \cdot 10^{-28}\,\text{Kg}. \tag{40}$$

An estimate of the neutral pion mass is achieved by the same approach, although with four times the length L_t of the torus from Figure 7, or $L_t = 8\pi R_t$:

$$AM = 4(2\pi)^2 \left(\frac{\hbar}{cM} \right)^2,$$

resulting in the mass:

$$M_\pi^0 = \sqrt[3]{\frac{4}{A}\left(\frac{h}{c} \right)^2} = 2.406 \cdot 10^{-28}\,\text{Kg} \tag{41}$$

A mass estimate close to the charged π-meson is found as an adjunct to the electron solution, with a surface-to-mass interface $AM = 4\pi r_e^2 \cdot \alpha^{-1}$, where α is the fine structure constant ($\alpha^{-1} \approx 137$) and $r_e = R_{cl}/2\pi$, giving the mass

$$M_\pi^\pm = \frac{4\pi r_e^2}{A} \cdot \alpha^{-1} = 2.49 \cdot 10^{-28}\,\text{Kg}. \tag{42}$$

In earlier work we showed relations to other particle masses, *i.e.*, nucleons [14].

The above examples indicate that the concepts introduced here, in particular the mass interface to time and vacuum space, are relevant not only to the gravitational field, but also to particles and their mass-fields in general.

7. Conclusion and Results

The concept of quantum systems extended in the time dimension, their associated vacuum flows and the singularities introduced here appear to be fundamental concepts for the understanding of how the Universe functions as a coherent (Machian) system.

Examples of the application of this concept have been given in terms of the equivalence between the inertial and the gravitational mass, the demonstrated common nature of acceleration and gravitation, the quantizing of the gravitational field, the topology of particle mass and its role in the system of the Universe, and the ratio of electron mass to charge. The masses for the π-mesons and the μ-meson have been derived.

The scaling of radii, masses and force constants that results when the parameter A is kept constant may also be of relevance to the Principle of Physical Proportions proposed by A.K.T. Assis [see article in this volume].

Based on the results achieved, it is proposed that a new and promising path has been found leading to a deeper understanding of the fundamental nature of particles and fields in the Universe.

References

[1] Henrik Broberg, *ESA STM* 223, Dec. 1981, Chapter 7 (Gravity).

[2] Amitabha Ghosh, *Origin of Inertia* (Montreal, Apeiron, 2000), Chapter 3.2.

[3] Tim Maudlin, *Quantum Non-Locality and Relativity* (Oxford, Blackwell 1994).

[4] Andre K.T. Assis, *Relational Mechanics* (Montreal, Apeiron, 1999), Chapter 4, Chapter 6.8

[5] Henrik Broberg, "Geometry of Acceleration in Space-Time," in: *Gravitation, Electromagnetism and Cosmology: toward a new synthesis* (Montreal, Apeiron, 2001).

[6] Henrik Broberg, "The Interface between Matter and Time; a key to Gravitation," in: *Open Questions in Relativistic Physics* (Montreal, Apeiron, 1998).

[7] Stephen W. Hawking, *A Brief History of Time* (New York, Bantam, 1988).

[8] Charles W. Misner *et al.*, *Gravitation* (San Francisco, Freeman, 1973), Section 21.12 (Mach's Principle and the Origin of Inertia).

[9] A.A. Michelson, *Studies in Optics* (Chicago, University of Chicago Press, 1962).

[10] Ernst Mach, *The Science of Mechanics* (Lasalle IL, Open Court, 1960), ch.2.

[11] Dennis W. Sciama (1953). On the Origin of Inertia, *Monthly Notices of the Royal Astronomical Society* **113**, p.34.

[12] Edwin Hubble, *The Realm of the Nebulae* (New Haven, Yale University Press, 1936).

[13] Walther Nernst, *Zeitschrift für Physik* **106**, pp. 633-661, 1937.

[14] Henrik Broberg, On the Kinetic Origin of Mass, *Apeiron* 15, 1993.

[15] D. Clements *et al.*, Gravitation and Electromagnetic Energy from Curved Space-Time, *Found. Phys. Lett.*, in press.

Mach's Principle and the True Continuum

George Galeczki[*]

I. Atomism and Matter

There is a long-standing, continuous preoccupation with the continuum. By *continuum* I mean here *gapless, true continuum*, as epitomized by the *real axis* of algebra. The paradoxical nature of this continuum is revealed in Cantor's set theory, where the countable set of natural numbers (*aleph zero*) is the first so-called *cardinal number,* while the real axis (*aleph one*) is the non-countable set of real numbers, isomorphic with the interval [0;1].

Cantor's set theory seems to be incompatible with the *structure* of matter, as forcefully pointed out by Schrödinger in his charming little book *Science and Humanism* [1]. There Schrödinger quotes Anaximenes, in order to show that the *atomism* of the ancient Greeks was based on careful considerations of everyday observations, rather than a lucky guess:

> If you try to assimilate Anaximenes' idea, you naturally come to think that the change of properties of matter, say on rarefaction, must be caused by its parts receding at greater distances from each other. But it is extremely difficult to accomplish this in your imagination, if you think of matter as forming a gapless continuum. What should recede from what? The mathematicians of the same epoch considered a geometrical line as consisting of points. That is perhaps all right if you leave it alone. But if it is a *material* line and you begin to stretch it, would not its points recede from each other and leave gaps between them? For the stretching cannot *produce* new points and the same set of points cannot go to cover a greater interval. From these difficulties, *which reside in the mysterious character of the continuum*, the easiest escape is the one taken by the atomists, namely to regard matter as consisting from the outset of isolated 'points' or rather small particles, which recede from each other on rarefaction and approach to closer distances on condensation, while remaining themselves unchanged.

Atomism is fundamental to physics, since without it *structure, statistical mechanics, variable density* and even *motion* are impossible. The rejection of atomism by Mach and Oswald in the second half of the 19th century drove Ludwig Boltzmann to suicide. Later, the young Einstein—after completing his work on Brownian motion and the reality of molecules—travelled to his spiritual mentor Ernst Mach in Prague and tried hard to convince him of the discreteness of matter. Characteristically, Einstein published in the same 1905 volume of *Annalen der Physik* the "special" theory of relativity, which relies upon Maxwell's *continuous field* theory, as well as his articles on the *discrete* nature of matter (as revealed by the Brownian motion) and the discrete nature of radiation as manifested in the photoelectric effect. He also pointed out that

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[*] Society for the Advancement of Physics, R.S., Flittarder Hauptstrasse 22, D-51061 Cologne, Germany. Email: galeczki@mgx.de.

"If physics could not be formulated in terms of pure field theory, nothing will remain from my castles in air."

II. Waves as Multi-Particle Phenomena

It has to be clear that the ubiquitous description by means of continuous functions and by equations with *partial derivatives* in physics is no more than a useful approximation, the so called *hydrodynamic approximation*. Indeed, although all fluids seem to be continuous and all material waves are well described by the wave-equation—an equation with partial derivatives—everybody agrees that waves in gaseous, liquid and solid media are macroscopic manifestations of collective, coherent motions of discrete entities of matter, be they atoms, or molecules. Even electromagnetic waves are macroscopic manifestations of *coherent*, *self-organized ensembles* of photons [2], provided a *critical number* is exceeded [3]. Once the discrete microstructure of waves is realized, the (in)famous "particle-wave dualism" vanishes, since a wave always consists of many discrete particles, but a single particle can never be a wave! This obvious conclusion implies that all claimed "single particle interference," "single atom lasers" and similar fictitious phenomena, are no more than erroneous interpretations of *ensemble, multi-particle effects.* Even the usual Schrödinger equation describes an ensemble of identical particles, every particle being under the influence of all others. This holds even for gases, as proven by the interference and diffraction effects realized with the help of classical, so called *thermal sources*. No interference phenomena would be possible, if the thermal source were *partially coherent*. The coherence of a laser beam is no more than the amplification of a pre-existing coherence, as the very acronym *laser* states.

III. The Mass Equivalence of Potential Energy

After pleading the cause of the fundamentally discrete nature of both matter and radiation, we turn now our attention to the *mass-energy equivalence* $m = E/c^2$, expressing the fact that any form of energy E possesses an inertial mass m. A blatantly curious fact is the total absence—with very few exceptions, like Leon Brillouin's last book *Relativity Re-examined* [4], from all textbooks and monographies of *potential energy.* All books on "special" relativity show a pathological amnesia by talking about *rest* and *kinetic energy*, but carefully avoiding even the mention of *potential energy*. Consciously, or not, this seems (to me) to be related to the necessarily *negative sign* of E_{pot}, implying a *negative mass*. Most interesting, but no less important, is the necessary association of potential energy with a *truly continuous mass-density distribution,* seemingly contradicting the fundamentally discrete structure of matter. The truly continuous mass distribution is, however, compulsory, if we are to take the relation $m = E/c^2$ seriously, which—in view of the energy liberated in fission processes—we are compelled to do. The Wesley theory of gravitation beyond

Newton [5] accepts mass-energy equivalence as a general principle, thus adding to the source mass density a gravitational field energy density. The total mass density : ρ(total) = $\rho + \rho'$ with:

$$\rho' = \frac{W}{c^2} = -\frac{(\nabla\Phi)^2}{8\pi G c^2} \tag{1}$$

where Φ and *G* are the Newtonian gravitational potential field and the universal gravitational constant, respectively, inserted in Poisson's equation:

$$\nabla^2\Phi = -4\pi G(\rho + \rho') = -4\pi G\rho + \frac{(\nabla\Phi)^2}{c^2} \tag{2}$$

provides a non-linear equation for the gravitational potential beyond Newton, which can be linearized by the transformation:

$$\Phi = -2c^2 \ln\zeta \tag{3}$$

The linear equation thus obtained:

$$\nabla^2\Phi = \frac{2\pi G\rho\zeta}{c^2} \tag{4}$$

is much simpler than the ten non-linear field equations of the general theory of gravitation with Riemannian metric, and explains equally well the motion of a test mass in a gravitational field, the precession of Mercury's perihelion, and gravitational redshift, among other effects.

As first pointed out by Heisenberg, the energy liberated by the explosion of an atomic bomb originates in the strong *binding energy* of the nucleus—evidently a specific *potential energy*. On purely logical grounds then, we have to accept that matter consists of a *superposition of discrete* (elementary) *particles* and a *truly continuous, gapless, density distribution* associated with the *ubiquitous potential gravitational energy.* We stress once more that this is a logical implication, quite independent of the penetrability of the *continuous background* to discrete, ordinary matter.

IV. The Ideally Rigid Continuous Medium

The unexpected conclusion of the present analysis is the necessity of a *physical coexistence* of *material discretum* with *material, true continuum*. *This continuum is of gravitational origin*, since gravitation is the only long-range, universal interaction, as early realized by the genius of Newton. I was told by Nathan Rosen ("the EPR one") that the need for a continuous, background metric, assuring energy-momentum conservation (valid only in a 'flat', non-Riemannian metric) and possibly accounting for Dayton Miller's aether drift experiments led him to start working on his *bi-metric theory of gravity* in 1940 [6]. Charged systems, from atoms to galaxies, display a general tendency toward neutrality; therefore a truly continuous, electrical background is non-existent.

The fundamental difference between gravitation and electricity is manifested in the everyday presence of all kinds of electromagnetic waves and in *the total absence of gravitational waves.* Gravitational interaction is apparently *instantaneous*, as reluctantly assumed by Newton and latter suggested by many researchers from Laplace to Eddington and Van Flandern. Indeed, from the astronomical evidence within the solar system, Laplace concluded that the speed of propagation of gravity has to be at least 10^8 *c*. This limit has been pushed to 10^{10} *c* in the last years by Van Flandern, thus strongly suggesting instantaneous propagation of gravity. To quote Van Flandern:

> Anyone with a computer and an orbit computation or numerical integration software can verify the consequences of introducing a delay into gravitational interaction. The effect on computed orbits is usually disastrous because conservation of angular momentum is destroyed. Expressed less technically by Sir Arthur (Eddington), this means: 'If the Sun attracts Jupiter toward its present position S, and Jupiter attracts the Sun toward its present position J, the two forces are in the same line and balance. But if the Sun attracts Jupiter toward its previous position S', and Jupiter attracts the Sun toward its previous position J', when the force of attraction started out to cross the gulf, then the two forces give a couple. This couple will tend to increase the angular momentum of the system. And, acting cumulatively, will soon cause an appreciable change of period, disagreeing with observations if the speed is at all comparable with that of light.'

Since, according to both Anaximenes and Schrödinger, the true continuum cannot be either compressed, or dilated, the *continuous gravitational background* could well be seen as the long sought-after *ideally rigid medium allowing instantaneous action at a distance.*

Here I could mention again Rosen's "bi-metric" gravitational theory started in 1940 [6], which also assumed a *truly continuous, static gravitational background*, with an associated *background metric* distinct from the *Riemannian metric* associated with ordinary matter, but nevertheless, predicting (just like Einstein's "general relativity") *gravitational waves* supposedly propagating with the same velocity *c* as the electromagnetic waves.

V. The Folly of the Search for Gravitational Waves

87 years ago Einstein predicted in his *linearized general relativity theory* (GRT) gravitational waves propagating with the velocity of light in vacuum, *c*. The detection of these elusive waves has been pursued since 1969 by Joseph ("Jo") Weber, who claimed several times to have detected gravitational waves. His supposedly gravitational effect was, however ten thousand (10^4) times larger than predicted by GRT and has been questioned by several researchers not committed to GRT. After 1975, although Weber has further developed his heavy aluminum cylindrical detectors weighing several tons, no one takes the supposed detection of gravitational waves seriously. Nevertheless, since the three classical test suggested by Einstein (the precession of Mercury's perihelion, the bending of starlight near the Sun and the redshift caused by the strong gravitational field of the Sun) provided only a shaky experimental basis for GRT, the quixotic hunt for gravitational waves has continued. In the mid-

eighties the idea to use huge Michelson type interferometers to detect tiny changes in length—caused supposedly by gravitational waves—gained popularity in general relativistic circles. Three such interferometers have been built, all three requiring about 200 million US$: in Italy bellow Gran Sasso, in the USA the LIGO project, and in Germany, south of Hannover, the GEO 600, the number 600 standing for the length of the (evacuated) interferometer arms. The expected changes in length are of the order of 10^{-16} meter, about one one-thousandth of the diameter of an atomic nucleus! Recently (January 30, 2002) the prestigious German daily *Frankfurter Allgemeine Zeitung* reported the successful completion of two weeks of measurements, in which all three Michelson interferometers indicated—supposedly—the same effect. The results of the data analysis are not expected before June 2002 and there is great hope that gravitational waves will finally be confirmed. The *march of folly* continues.

VI. Continuous *vs.* Discrete Time

Approaching the end of this article, I shall leave the continuous gravitational background and discuss briefly the nature of the time parameter used in physics. Since the times of Newton and (latter) Hamilton, the identification of the time parameter with the continuum of real numbers has never been seriously questioned. Also, reinforced by Boltzmann's H-theorem, unlike the spatial parameters x, y, z, the time parameter has been accepted as one-directional, a fact known as *the arrow of time*. In the early days of the movie (*cinema*), the philosopher Henri Bergson—one of the most prominent thinkers about time—made a very important analysis of the *illusion of motion* introduced by the movie. To make a film, so Bergson, one takes a *discrete set* of pictures or *frames* and then unwinds—or rolls off—the recorded frames with a high enough speed, creating the illusion of motion. However, one has to clearly distinguish between *real, dynamical motion* and *illusory, movie motion, in which no real causes (forces) are involved.* The most baffling difference is that *unlike real motion, movie-like motion is reversible*, generating both paradoxical and comical processes. In modern language one can say that the making of a movie amounts to *discretizing* some *continuous, real process*, or making an *analog to digital conversion. The digitalized, discrete frame sequence is reversible, while real, natural processes are generally irreversible. Just as the physical continuum associated with the gravitational background is incompressible, physical time is irreversible.*

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References

[1] Schrödinger E., *Science and Humanism* (Cambridge, Cambridge Univ. Press, 1951).
[2] Wesley J. P., *Causal Quantum Theory* (Blumberg, Benjamin Wesley, 1996).

[3] Galeczki G. and Marquardt P., Order and Coherence: A Question of Size for Matter and Radiation, *Physics Essays* 10, 506, 1997.

[4] Brillouin L., *Relativity Re-examined* (New York and London, Academic Press, 1970) pp.20-25.

[5] Wesley J. P., *Advanced Fundamental Physics* (Blumberg, Benjamin Wesley, 1991) Chs. 2, 3, 4.

[6] Rosen N., Bi-metric Theory of Gravitation, *Phys. Rev*. **57**, 147, 1940.