Theoretical Performance of a Magnetohydrodynamic-Bypass Scramjet Engine with Nonequilibrium Ionization

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The theoretical performance of a scramjet propulsion system in which a magnetohydrodynamic (MHD) energy bypass scheme is operated in a nonequilibrium ionization environment is evaluated. In the MHD generator, the incoming airflow at a temperature too low to produce the required ionization is seeded with cesium and ionized by the use of an unspecified external power source. The resulting nonequilibrium environment is described using a two-temperature model. The accelerator is operated with equilibrium ionization. The expansion through the nozzle is calculated with account taken of finite rate reactions, and the boundary layer is assumed to be fully turbulent. The results show that the required external power is of the same order of magnitude as that of the power generated in the MHD generator. An MHD energy bypass propulsion scheme looks to be more promising with the equilibrium ionization method than with the nonequilibrium ionization method.

Nomenclature

\( A \) = cross-sectional area of flow path, m²

\( B \) = magnetic field strength, T

\( E_e \) = vibron (vibrational-electron) energy, J/kg, Eq. (1)

\( E_{eq} \) = \( E_e \) evaluated at \( T_e; \) J/kg

\( E_x \) = axial voltage gradient, V/m

\( E_y \) = transverse electrical field strength, V/m

\( H \) = static enthalpy excluding \( E_e \), J/kg

\( I_{sp} \) = specific impulse for \( h = 1 \), s

\( I_{sp_e} \) = specific impulse for \( h \neq 1 \), Eq. (12), s

\( j \) = electrical current density, A/m²

\( M \) = Mach number

\( M_c \) = combustor entrance Mach number

\( P \) = external ionizing power, W

\( p \) = pressure, Pa

\( q \) = density of external ionizing power, W/m³

\( R_1 \) = electron-heavy particle energy transfer rate, Eq. (8), W/kg

\( R_2 \) = vibration-heavy particle energy transfer rate, Eq. (8), W/kg

\( T \) = gas (heavy particle) temperature, K

\( T_e \) = vibron (vibrational-electron) temperature, K

\( T_{eq} \) = \( T \) required for magnetohydrodynamic generator, K

\( t \) = time, s

\( u \) = axial velocity, m/s

\( V \) = flight velocity, m/s

\( x \) = axial distance, m

\( x_{eq} \) = equilibration distance, m

\( y \) = transverse (normal to \( B \) and \( u \)) distance, m

\( \gamma_i \) = concentration of species \( i \), mol/kg

\( \eta_1 \) = external-to-combustor power ratio

\( \eta_2 \) = external-to-generator output power ratio

\( \eta_3 \) = energy bypass ratio

\( \eta_4 \) = fuel-to-external power conversion efficiency

\( \theta \) = ramp angle, deg

\( \rho \) = density, kg/m³

\( \sigma \) = electrical conductivity, mho/m

Introduction

In a recently proposed concept, the performance of a scramjet propulsion system is improved by the use of a magnetohydrodynamic (MHD)-energy bypass scheme. In this scheme, an MHD generator, in the course of generating electrical power, decelerates the flow entering the combustor, thereby providing a greater time to complete supersonic combustion. The electrical power generated in the process is used in an MHD accelerator positioned after the combustor to accelerate the flow before expanding through the nozzle.

To apply MHD to a flow, the flow must be electrically conductive. This can be achieved by seeding the flow with either potassium or cesium. Even with seeding, the flow must be sufficiently hot to ionize the seed species; typically, 3000 K is needed. In the accelerator, which accepts the flow emerging from the combustor, the flow is usually sufficiently hot for ionization. However, in the generator, which accepts the flow emerging from the inlet compression process, the flow is not necessarily sufficiently hot.

In the theoretical studies made in Refs. 2 and 3 in which the performance of MHD-energy bypass propulsion systems is analyzed theoretically, ionization is achieved in both the generator and the accelerator in a thermochemical equilibrium environment. Liquid hydrogen is the fuel for the system. Ionization in the generator is achieved by the shock compression of the oncoming airflow. In this equilibrium scheme, the static pressure reaches a few atmospheres in both generator and accelerator. The calculations²-³ show that the MHD bypass scheme offers a possibility of improvement of the scramjet performance in a certain range of flight speeds. The performance gain in this scheme is limited by the strong compression needed to produce ionization also producing a large drag.

There is also a concept, in which the MHD generator is operated in a thermochemical nonequilibrium regime. In this concept, the MHD generator will be operated in a low-temperature, low-pressure environment. Ionization is produced by the application of external power, for example, with an electric discharge. The advantage of...
such a scheme is that a much weaker shock compression is needed upstream of the generator, which would produce considerably less drag.

In Ref. 5, we estimated the overall performance of a scramjet system operating on such a nonequilibrium ionization principle. Flowpath calculations were performed for five different MHD-energy bypass system configurations, one producing two shock waves and four producing four shock waves. The nonequilibrium environment in the generator was analyzed using a two-temperature model. Comparisons are made of the specific impulse values with those of a conventional scramjet without MHD bypass.

However, two small errors of a numerical nature were found in Refs. 2, 3, 5: 1) The enthalpy values calculated by the equilibrium subroutine used was mistakenly considered to be referenced to 0 K, whereas, in fact, it was referenced to 298 K. 2) The sound speed was calculated imprecisely. These errors are corrected herein. Because the present results are at least qualitatively the same as those in Ref. 5, some results, those considered nonessential in the flow of logic, are not shown in the present work. Readers are referred to Ref. 5 for those missing details.

Method

Density Regime

Operating an MHD generator with nonequilibrium ionization implies that the flow density is low therein. The simplest scramjet system operating on the nonequilibrium MHD principle is one that produces two oblique shock waves of equal turning angles, one over a ramp and the other on the cowl that forms a second ramp. The MHD generator is placed after these two shocks. The density over the first ramp is lower than that over the second ramp, but it would not be practical to place the MHD generator over the first ramp because doing so would produce a nonuniform flow.

The density level in the MHD generator, in such a system is determined by the ramp angle, flight dynamic pressure, and flight velocity. The dynamic pressure will be assumed to be 1 atm in most of the present work. The flight velocity will be varied from 2500 to 4000 m/s. There is a practical lower limit to the ramp angle; below the limit, it is not possible to construct a viable airframe structure. In the present study, the lower limit will be considered to be 5 deg. This choice of dynamic pressure, flight velocity, and ramp angle leads to a minimum density, pressure, temperature, and total number density achievable in the MHD generator: \(7.0 \times 10^{-2} \text{ kg/m}^3\), \(1.4 \times 10^4 \text{ Pa} \) (0.14 atm), 715 K, and \(1.5 \times 10^{24} \text{ m}^{-3}\), respectively. The temperature is too low to produce the equilibrium ionization level needed for MHD action.

Ionization Schemes

In both equilibrium and nonequilibrium schemes, ionization can be facilitated when the flow is seeded with an alkali metal such as potassium or cesium. In Refs. 2 and 3, it is concluded that potassium and cesium are approximately equally effective in an MHD bypass scheme based on the equilibrium principle. In the present work, cesium in atomic form is assumed to be uniformly injected at the entrance of the MHD generator. In Refs. 2 and 3, the ionization fraction attained was in the order of \(10^{-4}\). The same level of ionization will be assumed in the present work. Because the lowest number density under consideration is \(1.5 \times 10^{24} \text{ m}^{-3}\), the lowest electron density to be considered is about \(10^{20} \text{ m}^{-3}\).

Cesium can be ionized, in a nonequilibrium condition, by several means.

Consequence Equations

As was done in Refs. 2 and 3, an ideal one-dimensional flow with Faraday-type MHD devices (i.e., with the electrical current flowing in the direction perpendicular to the direction of flow) will be assumed. A power density \(q\) is assumed to be expended to produce the required ionization.
To operate an MHD generator in a nonequilibrium regime, $E_r$ at the entrance of the generator $E_{r,0}$, which is low, must be raised to a required level $E_{r,r}$. This can be achieved, for example, by letting the airflow pass through a curtain of ultraviolet radiation. The power required to do so is

$$P_0 = \rho u A (E_{r,r} - E_{r,0})$$  \hspace{1cm} (1)$$

The power density $q$ required to deposit $P_0$ into the vibron mode is obtained by dividing $P_0$ by $\Delta A_x$, where $\Delta A_x$ is the distance over which this $q$ value is maintained. It is assumed that $\Delta A_x$ is infinitesimally small. That is, $P_0$ is applied impulsively.

After the required vibron energy $E_{r,r}$ is achieved, finite external power must still be applied to keep $E_r$ at that level. Otherwise, the collisions between the vibron group and the molecules at a lower temperature will lower the vibron energy. The external power required to keep $E_r$ at $E_{r,r}$ can be determined from the conservation relations, which include the following:

Species mass

$$\frac{d\gamma}{dx} = \frac{\rho \gamma}{\partial t}$$  \hspace{1cm} (2)$$

Global mass

$$\rho A = \text{const}$$  \hspace{1cm} (3)$$

Momentum

$$\rho \frac{du}{dx} = -\frac{dp}{dx} + jB$$  \hspace{1cm} (4)$$

Total energy

$$\frac{\rho u dE_r}{dx} \left( H + E_r + \frac{u^2}{2} \right) = \frac{i^2}{\sigma} + uBj + q$$  \hspace{1cm} (5)$$

And vibron energy

$$p \frac{dE_r}{dx} = \frac{i^2}{\sigma} - \frac{\partial E_r}{\partial t} + q$$  \hspace{1cm} (6)$$

The quantity $\gamma/\partial t$ in Eq. (2) is the rate of chemical reaction for species $\gamma$. The expression for this rate can be found, for example, in Ref. 10. We have used the relation $j = \sigma (E - uB)$ to obtain the form of Eq. (5) shown. The quantity $\partial E_r/\partial t$ in Eq. (6) is the rate of energy transfer from the vibron mode to the translational-rotational mode. It has two components: $\partial E_r/\partial t$ is the collisional energy transfer rate, which equals the electron-heavy particle energy transfer rate, plus the vibration-translation energy transfer rate, which equals $R_1 + R_2$. $R_1 + R_2$ is positive when $T_r > T$. The expressions for the two components can be derived with the information in Ref. 6.

To operate an MHD device, $E_r$ must be kept nearly constant. By the requirement that $E_r$ be constant, the left-hand side of Eq. (6) is set to zero, leading to

$$q = \rho \frac{\partial E_r}{\partial t} - \frac{i^2}{\sigma}$$  \hspace{1cm} (7)$$

The total energy equation (5) becomes

$$\frac{\rho u dH}{dx} \left( H + \frac{u^2}{2} \right) = \frac{\partial E_r}{\partial t} + uBj$$  \hspace{1cm} (8)$$

There exist two constraints to this system of equations, that is, on load factor $E_r/uB$ and the allowed axial voltage gradient $E_z$ (Refs. 2 and 3). These two constraints specify the magnetic field strength $B$ and the electrical current density $j$.

For a specified cross-sectional area distribution, one can determine all properties in the nonequilibrium region by solving Eqs. (2–4), (7), and (8) simultaneously, under the last two constraints. By adding $P_0$ [Eq. (1)] to the integration of $q$ over the volume, one obtains the external power required $P$:

$$P = P_0 + \int q A \, dx$$  \hspace{1cm} (9)$$

**Performance Calculation**

The performance of the overall system is calculated in the same manner as in Refs. 2 and 3, except for the nonequilibrium region in the MHD generator just discussed. In addition to the configurations mentioned earlier, which produces two oblique shock waves, four configurations producing four shock waves are considered. The five configurations studied are shown schematically in Figs. 1a–1e. The two-shock configuration in Fig. 1a is identified as configuration 0. The four shock configurations, 1–4, are considered because they generally produce lower Mach numbers and higher pressures at the entrance of the combustor than the two-shock scheme. In configurations 1 and 2, the flow exiting from the MHD generator undergoes two more shock compressions. In configurations 3 and 4, the two additional shock compressions occur before entering the generator. In configurations 1 and 3, the third and fourth shocks occur in the yaw plane. Such a two-plane four-shock compression system was considered in Ref. 3. In configurations 2 and 4, the third and fourth shocks occur in the pitch plane. Such a single-plane four-shock compression system was considered in Ref. 2.

There are four ratios that are associated with the performance of the nonequilibrium MHD scheme: (1) the ratio of the external power $P$ to the power of the fuel consumed in the combustion chamber $\eta_1$; (2) the ratio between the external power $P$ and the power output from the MHD generator (the integral of the product of electrical current and voltage over the flow volume in the generator) $\eta_2$; (3) the energy bypass ratio, which is the ratio between the electrical power output by the MHD generator to the flow power $uA(H + u^2/2)$ at the entrance of the generator $\eta_3$; and (4) the ratio of the external power to the power of the fuel expended to produce the external power, for example, by a hydrogen–oxygen fuel cell, $\eta_4$.

In the calculation of $\eta_1$, the fuel energy expended in the device, for example, the fuel cell, is calculated under the assumption that molecular hydrogen is converted completely to form water vapor. The temperature of the combustion product is brought to the room temperature, 298 K, which gives 119.9 MJ/kg. The fuel flow rate consumed to produce the external power is then $P/(1.199 \times 10^4 \eta_1)$. The specific impulse for $\eta_1 = 1$ will be designated $I_{sp}$. The calculation results will be presented in terms of this quantity. For cases $\eta_1 \neq 1$, the specific impulse, $I_{vp}$, is calculable by

$$I_{vp} = I_{sp} \frac{1 + \eta_1}{1 + \eta_1/\eta_4}$$  \hspace{1cm} (10)$$

**Assumptions and Parameters**

The assumptions stated, and other parameters not specified, are summarized here.

**Vehicle**

1) The flight dynamic pressure is taken to be 1 atm for most of the calculations. Two other values, 0.5 and 2 atm, will be considered toward the end of the work.

2) The width of the vehicle is infinite. However, all performance values, such as thrust, are given per 1 m of width. For configurations 1 and 3, each engine units is typically 2–3 m wide. The performance values for each unit are divided by the width of the unit to obtain the per meter value. The length of the vehicle is chosen arbitrarily to be 46 m. The lengths of the MHD generator, combustor, and MHD accelerator are equal to or less than 2.72, 0.48, and 2.84 m, respectively. For the four-shock system, the ramp angle for the third shock is the same as that for the first shock. For configurations 1 (Fig. 1b) and 3 (Fig. 1d), the cross section of the combustor is a square. All of these parameters are the same as those used in Refs. 2 and 3. The nozzle geometry is also that given in Refs. 2 and 3.

3) The length of the first ramp is 30 m for the two-shock configuration (Fig. 1a), 20 m for configurations 1–3, and 10 m for configuration 4. These lengths result in approximately the same length nozzle. However, there are small differences in the height of the vehicle and, therefore, in the airflow rate captured.

4) The fuel equivalence ratio is 1. Fuel is uniformly mixed with air, and equilibrium is reached within the combustor.
a) Two-shock design, configuration 0

b) Four-shock design, configuration 1

c) Four-shock design, configuration 2

d) Four-shock design, configuration 3

e) Four-shock design, configuration 4

Fig. 1 Perspective view of the designs considered.

5) The ratio between the exit cross section and the entrance cross section of the combustor, which is usually slightly larger than unity to prevent choking, is kept to the lowest nonchoking value.

6) The boundary layer is fully turbulent, starting from the nose tip. Turbulent skin friction in the flowpath is calculated by the use of the method of Ref. 11. The thrust of the system is calculated in the present work as the inviscid thrust minus the skin-friction drag in the flowpath.

7) The heat transfer phenomenon and multidimensional flow phenomena such as separation are neglected.

8) Calculations are made at ramp angle intervals of 0.1 deg to determine the maximum specific impulse.

MHD Devices

1) The load factor $E_y/uB$ is taken to be 0.95 for the generator and 1/0.95 for the accelerator for most of the calculations. The $E_y$ and $u$ are those in the inviscid region. The pairs (0.9, 1/0.9), (0.85, 1/0.85), and (0.8, 1/0.8) are considered toward the end of the work.

2) Both the generator and the accelerator are considered to be Faraday devices (electrical current in the direction normal to the flow direction).

3) The cross-sectional area $A$ in Eq. (3) is constant in the nonequilibrium region of the MHD generator.

4) The seed material is cesium in an atomic form. Its mass fraction with respect to air is $3 \times 10^{-3}$. This means that the mass of the seed material needed is 10.3% of that of fuel. Seeding is instantaneous, and no energy is expended in seeding.

5) The required vibron temperature for configurations 0–2 is $T_e = 2800$ K. For configurations 3 and 4, $T_e = 3200$ K is used because the gas density in the generator is higher for these configurations. These result in an electrical conductivity between 40 and 110 mho/m, depending on flight velocity and ramp angle.

6) The maximum allowed axial voltage gradient $E_x$ is about 5000 V/m. This value was assumed in Refs. 2 and 3.

Results

Nonequilibrium Behavior

In Fig. 2, the behavior of the gas temperature $T$ and the external power density $q$ are shown for the case where the flight velocity $V$ and ramp angle $\theta$ are 3500 m/s and 6 deg, respectively. The gas temperature in the generator $T$ approaches $T_e$ and eventually reaches a plateau. One can define the equilibrium distance $x_{eq}$ as the distance at which the difference between $T_e$ and $T$ is 10% of its value at the entrance. When $dT_e = dx$, the nonequilibrium calculation is terminated, and the equilibrium calculation is started.

There is a small discontinuity in $T$ and $T_e$ at the switchover point. However, this discontinuity is believed to be inconsequential.

The initial impulsive external power needed for ionization, Eq. (1), is 102 MW for this case, as indicated in Fig. 2. The total external power to the point where the nonequilibrium analysis is finished, given by Eq. (9), is 464 MW, also indicated in Fig. 2. The difference, 362 MW, is expended in preventing the vibron temperature $T_e$ from falling to the gas temperature $T$. The electron density generated by the external power, and held approximately constant thereafter, is $1.5 \times 10^{20}$ m$^{-3}$, which results in electrical conductivity of 105 mho/m. Of the vibron energy, 99.5% consists of the vibrational energy. The external energy expended $E_{ext}$ is 58% of the static enthalpy and about 7% of the total enthalpy of the flow.

Cooling of
the vibron energy is almost entirely (98.6%) by the vibron–heavy particle translation energy transfer phenomenon $R_2$. Immediately after the application of $P_0$, the first term comprises about 90% of the sum of the two terms in the expression for $q$ in Eq. (7). When $T$ equilibrates with $T_e$, the entire flow is heated to a temperature slightly below the prescribed $T_e$ of 2800 K. Thus, the effect of application of the external power is to heat the entire gas mass to approximately $T_e$. This makes the application of external power an expensive measure. If potassium is used instead of cesium, the required $T_e$ and, consequently, the required external power, will be higher. If the flow is not seeded, the required $T_e$ and external power will be higher still.

In Fig. 3, the equilibration distance is shown as a function of flight velocity $V$ and ramp angle $\theta$. As seen here, the relaxation distance is less than 1.6 m for all cases.

### External Power Required

The external power required, normalized by the flow power $\rho uA(H + u^2/2)$, for configurations 0–2 is shown in Fig. 4. From the definition of $\eta_2$ and $\eta_3$, the ratio shown in Fig. 4 becomes $\eta_2/\eta_3$. As Fig. 4 shows, the external power is a significant fraction of the flow power.

For configurations 1 and 2, the ratio of the external power $P$ to the generator output power and the combustor entrance Mach number $M_c$ are calculated as a function of $V$ and $\theta$ and are, respectively, plotted in Figs. 5a and 5b. The upper limit in $\theta$ in Figs. 5a and 5b for each $V$ value is set by either of the following: 1) The gas temperature after the first two shocks is greater than the temperature required for ionization ($T_e = 2800$ K). 2) the Mach number after the third shock is too small to support an oblique fourth shock.

As Fig. 5a shows, in most cases, the external power is larger than the output from the generator ($\eta_2 > 1$). When $\eta_2$ is less than unity, which occurs at high ramp angles and high flight velocities, the MHD process can be self-sustaining by diversion to and expenditure of a portion of the generator output as the external power. For example, for $V = 3750$ m/s and $\theta = 19$ deg, 56% of the generator output can be expended to provide the needed external power, and the remainder, 44%, can be expended for acceleration. The power required is higher at low speeds, as expected. The power approaches zero as $\theta$ increases. This is also expected: As $\theta$ increases, the gas temperature after two shocks becomes high, and only a small amount of energy is needed to raise the gas temperature to that required for MHD action.

According to Fig. 5b, the combustor entrance Mach number $M_c$ is low at low $V$ and high $\theta$, which is expected. At the highest $\theta$ and the lowest velocity values calculated, $M_c$ decreases toward unity.

### Specific Impulse

As mentioned in the “Introduction,” the main purpose of consideration of MHD schemes for a scramjet system is to lower the flow Mach number at the entrance of the combustor $M_c$ and to improve the performance of the scramjet system. The flowpath calculations...
are made with the five configurations shown in Figs. 1a–1e at different flight velocities while $M_c$ is constrained to be less than either 1.5, 2.0, or 2.5. The ramp angle $\theta$ giving the highest viscous specific impulse is the optimum angle for that particular design and flight velocity. According to this procedure, $M_c$ varies from a low supersonic value at low velocities to the allowed maximum value at high velocities. In Figs. 6a–6c, the specific impulse values for the optimum conditions are compared. The specific impulse of 450 s for the space shuttle main engine (SSME) is also shown for comparison.

According to Figs. 6a–6c, at low speeds and $M_c > 1.5$, configurations 3 and 4 give higher specific impulses. These configurations lead to equilibrium ionization at high speeds, and, therefore, the nonequilibrium scheme does not function there, as mentioned earlier. Of these two, configuration 4 is better. At higher velocities, only configurations 1 and 2 function as a nonequilibrium device. Of these, configuration 1 gives a higher specific impulse.

In Fig. 7a, specific impulse values are compared for configuration 4 between the MHD and non-MHD cases. As mentioned in “Assumptions and Parameters,” the length of the first ramp is held the same between the MHD and non-MHD designs. Because the ramp angle is higher for the non-MHD case, the air mass captured in the non-MHD vehicle is higher than that for the MHD vehicle. Under this constraint, Fig. 7a shows that the non-MHD cases result in higher specific impulse values, except for $2 < M_c < 2.5$, at high flight speeds.

In Fig. 7b, a similar comparison is made for configuration 1. Here again, the MHD values are generally lower than the non-MHD values. Only for flight speeds equal to or greater than 3750 m/s is the MHD value greater than the non-MHD value. For $M_c < 1.5$ and at the flight speed of 4000 m/s, the MHD case is better than the non-MHD case. However, the absolute value of specific impulse is no better than that of the SSME there.

**Typical Operating Condition**

The foregoing results lead to the conclusion that the nonequilibrium MHD scheme is generally no better than the non-MHD scheme. The overall best scheme is configuration 1. At $V = 3750$ m/s, $\theta = 18.9$ deg and $M_c < 1.5$, the MHD scheme has a substantial advantage. $I_{sp} = 646$ s as compared to $I_{sp} = 556$ s for the non-MHD scheme. The results for this case are listed in Tables 1–4, and the overall performance is presented in Table 5. In this case the lengths of the generator, the combuster, and the accelerator are 2.681, 0.464, and 2.772 m, respectively.
Table 1  Compression stages

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Freestream</th>
<th>First shock</th>
<th>Second shock</th>
<th>Third shock</th>
<th>Fourth shock</th>
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<tr>
<td>Pressure, Pa</td>
<td>1.039 × 10^6</td>
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<td>Temperature, K</td>
<td>250</td>
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<td>371</td>
<td>3265</td>
<td>3683</td>
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<td>Density, kg/m^3</td>
<td>1.44 × 10^-2</td>
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<td>Velocity, m/s</td>
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<td>3427</td>
<td>3062</td>
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<td>Mach number</td>
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<td>4.616</td>
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Table 2  Entrance of MHD devices and combustor

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<th>Parameter</th>
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<th>Compressor</th>
<th>Accelerator</th>
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<td>Channel height, M</td>
<td>0.6027</td>
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<td>Pressure, Pa</td>
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<td>Temperature, K</td>
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<td>Velocity, m/s</td>
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<td>Impulsive external power, W</td>
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<td>External power density, q, W/m^3</td>
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<td>Equilibration distance, x eq, m</td>
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Table 3  Exit of MHD devices and combustor

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<td>Channel height, M</td>
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<tr>
<td>Pressure, Pa</td>
<td>1.581 × 10^6</td>
<td>1.426 × 10^6</td>
<td>1.371 × 10^6</td>
</tr>
<tr>
<td>Temperature, K</td>
<td>2770</td>
<td>3742</td>
<td>3736</td>
</tr>
<tr>
<td>Velocity, m/s</td>
<td>2597</td>
<td>1446</td>
<td>2105</td>
</tr>
<tr>
<td>Mach number</td>
<td>2.66</td>
<td>1.12</td>
<td>1.63</td>
</tr>
<tr>
<td>Conductivity, mho/m</td>
<td>39.1</td>
<td>NA</td>
<td>72.6</td>
</tr>
<tr>
<td>Magnetic field, T</td>
<td>4.50</td>
<td>NA</td>
<td>7.53</td>
</tr>
<tr>
<td>Hall parameter</td>
<td>7.40</td>
<td>NA</td>
<td>1.27</td>
</tr>
<tr>
<td>Axial voltage gradient, V/m</td>
<td>4321</td>
<td>NA</td>
<td>1058</td>
</tr>
<tr>
<td>Transverse voltage gradient, V/m</td>
<td>11,090</td>
<td>NA</td>
<td>13,900</td>
</tr>
<tr>
<td>Current density, A/m^2</td>
<td>3.944 × 10^4</td>
<td>NA</td>
<td>6.007 × 10^4</td>
</tr>
<tr>
<td>Voltage, V</td>
<td>8763</td>
<td>NA</td>
<td>8353</td>
</tr>
<tr>
<td>External power, W</td>
<td>0</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

The pressure and temperature in the combustion chamber are 1 × 10^6 Pa and 3683 K, respectively, which are of similar magnitudes as in the equilibrium schemes. However, unlike in the equilibrium schemes, the required magnetic field strengths are only 4.50 and 7.53 T for the generator and accelerator, respectively, which can be achieved with the present-day technology.

The external power required, 273 MW per each meter of the vehicle width, is 19.3% of the fuel power generated in the combustor and 56.1% of the output from the generator. This power level is much higher than envisioned in the original nonequilibrium ionization concept. The technology necessary for providing an external power of this magnitude to the vibron mode with a high efficiency is presently nonexistent.

### Variations

The effect of varying the flight dynamic pressure on specific impulse is shown in Fig. 8 for the optimum condition. Fig. 8 shows that specific impulse improves with increasing dynamic pressure. However, the advantage of the MHD decreases with increasing dynamic pressure. The MHD scheme produces a significantly higher specific impulse than SSME at all flight dynamic pressures considered.

![Fig. 8 Effect of flight dynamic pressure on best specific impulse: configuration 1, V = 3750 m/s, and M_e constrained at 1.5.](image-url)
The effect of varying the load factor $E_\gamma/\mu B$ on specific impulse is shown in Fig. 9 for the same case. Figure 9 shows that the viscous specific impulse increases as the load factor approaches unity. This is expected: As the load factor approaches unity, there is less and less "slip" between the flow and the field driving the flow. Therefore, the energy loss, or entropy increase, becomes smaller. Note that the strengths of the applied field become larger as the load factor approaches unity. At load factors below 0.89, the MHD system produces specific impulses smaller than the non-MHD system.

The effect of varying the fuel-to-external power conversion efficiency $\eta_4$ on specific impulse is shown in Fig. 10 for the same case. These values are obtained with Eq. (10). When the $\eta_4$ value is lower than 0.1, specific impulse varies approximately linearly with $\eta_4$. The $\eta_4$ values greater than unity, shown in Fig. 10, represent the case where the external power is drawn partly or wholly from a nuclear power source. As shown, specific impulse improves significantly if external power can be obtained without using fuel. Conversely, if the energy conversion efficiency is poor, the specific impulse will be low.

In Fig. 11, comparison is made between the present nonequilibrium MHD scheme and the equilibrium MHD scheme presented in Ref. 3. The two best-performing configurations, configuration 4 at flight speeds below 3000 m/s and configuration 1 for higher speeds, are selected for this comparison. The equilibrium MHD values are calculated with the method presented in Ref. 3, but by the use of the same constraints on the combustor area ratio as for the nonequilibrium case. They are, in the definition of the present work, for configuration 3. The two minor errors mentioned in the Introduction were also corrected in the equilibrium MHD calculation. The nonequilibrium MHD calculation for configuration 4 was stopped at $V = 3000$ m/s because the postshock temperature became so high that the nonequilibrium scheme was not necessary. Figure 11 illustrates that the thrust performance of the nonequilibrium MHD scheme can not exceed that of the equilibrium scheme.

Finally, in Fig. 12, the highest attainable specific impulse values are compared for the optimum condition among 1) the non-MHD, 2) the present MHD scheme with three different $\eta_4$ values, 3) the present MHD scheme in which the external ionizing power is drawn from the generator output, and 4) the equilibrium MHD scheme. As seen here, the present nonequilibrium scheme produces a higher specific impulse than the non-MHD case if $\eta_4$ is nearly unity or higher. However, the specific impulse of the nonequilibrium scheme is lower than that for the equilibrium scheme if $\eta_4$ is 1. For the self-sustaining case where the external power is drawn from the output of the generator, the specific impulse is calculated to be 401 s.

**Discussion**

The foregoing results contain two minor inaccuracies and one point of caution. The first inaccuracy is on the enthalpy of the flow.
As mentioned in “Ionization Schemes,” the equilibrium flow calculation was made in the present work with the JANNAF coefficient, which is known to be inaccurate at low temperatures. This inaccuracy introduces a small inaccuracy in the equilibrium flow conditions over the first ramp. This inaccuracy remains throughout the flowpath because energy flow is conserved thereafter. This inaccuracy is tolerated here because the same inaccuracy exists for both the MHD and non-MHD cases, and, therefore, the relative merit of the MHD scheme is unaffected.

Secondly, there exist small inconsistencies in the cross-sectional areas because of normalization. Inside each component, a small inaccuracy accumulates along the flow because of numerical truncation. At the exit of each component, the cross-sectional area of the flowpath is reevaluated to conserve mass and energy, that is, normalized. This procedure produced a small but abrupt change in the cross-sectional area at the juncture points.

Additionally, one is cautioned that the sound speed is evaluated under the assumption of equilibrium in the equilibrium region, as well as the assumption of a frozen flow in the nonequilibrium region. This changing of the definition of sound speed appears as a sudden jump in Mach number at those juncture points.

As stated in Refs. 2, 3, and 12, research and technology developments are required toward making the MHD energy bypass propulsion concept a reality. These references have identified issues such as nonequilibrium ionization vs equilibrium ionization, acceleration of flow with Lorentz forces with minimum Joule heating, various losses in MHD devices, efficient and effective seeding for ionization, and a need for lightweight MHD devices for use on spacecraft. Herein, we have addressed the issue of nonequilibrium ionization.

The calculation results presented show first that the external power required for achieving the degree of nonequilibrium ionization necessary for meaningful MHD action is a substantial fraction of the total enthalpy of the airflow (Fig. 4). This is because of the strong coupling between the ionization phenomenon and vibrational excitation phenomenon: external energy is expended mostly to raise the vibrational temperature of air molecules and to maintain it against the collisional cooling by the cold airflow. The external power required is of the same order as the power output from the generator (Fig. 5a).

The nonequilibrium ionization scheme was proposed for efficient operation at low ramp angles. However, at low ramp angles, the propulsion system produces only a relatively small momentum increase. The turbulent skin friction, which is a relatively weak function of flow properties, is a relatively large portion of the total drag at low ramp angles. As a result, the specific impulse is low at low ramp angles.

The simplified analysis presented herein suggests that the load factor of the MHD devices must be higher than 0.83 for the nonequilibrium MHD scheme to be superior to the SSME, if the flight speed is 3750 m/s and if the combustor inlet Mach number is limited to 1.5. The assumed uniform ionization will require uniform distribution of cesium. Because of the large Hall parameters, it may be beneficial to use a diagonally connected electrode arrangement3 rather than Faraday electrode connections. The high Hall parameter does also raise concerns about the possibility of instabilities.

At the ramp angles where the MHD system begins to be comparable to or better than the non-MHD system, the flow reaches equilibrium within a short distance from the entrance. Most of the MHD action occurs in the equilibrium region. A relatively large external power must be delivered to the flow in the small nonequilibrium region. Based on the quasi-one-dimensional analysis, the equilibrium scheme is superior in performance to the nonequilibrium scheme. The benefit of the nonequilibrium MHD scheme exists only when the external power can be produced by converting fuel energy very efficiently, or by not expending fuel at all.

Conclusions

In an MHD-energy bypass scramjet system that uses external power to produce electrically conductive flow, the external power is expended mostly to heat the vibrational mode of air molecules. The nonequilibrium region is small, and most of the MHD action occurs in the equilibrium region. The external power needed, partly for initiating the nonequilibrium ionized flow and partly for maintaining it, is of the same order as the power generated in the MHD generator and is a significant fraction of the power derived from the fuel in the combustor. The specific impulse depends on the efficiency of conversion of the fuel energy to the external power and the load factor. The highest attainable specific impulse is slightly higher than that of the conventional non-MHD scramjet, if the external ionizing power can be obtained by an efficient means, but it is lower than that of an MHD system relying entirely on equilibrium ionization. An MHD energy bypass propulsion scheme looks to be more promising with the equilibrium ionization scheme than with the nonequilibrium scheme.

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References


