Nonequilibrium Ionization Due to Electron Heating: I. Theory

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A two-temperature conduction law is proposed which accounts for the variation of the electrical conductivity of a plasma due to joule heating of the electrons. The conduction law represents the behavior of the plasma over a wide range of conditions likely to be encountered in MHD generators and accelerators. It differs from the usual Ohm's law in that the current density is proportional to about the fifth power of the electrical field for a slightly ionized gas with electron temperature more than 10% greater than the gas temperature. The process of relaxation from an initial condition of thermal equilibrium to the final nonequilibrium condition is treated for a channel flow that represents conditions in an MHD generator. For a mixture of argon and potassium at 1 atm pressure and 2000 K, the relaxation length is about 1 m at a Mach number of unity if the electric field is held constant. The stability of the two-temperature plasma to wave-like disturbances of electron energy and concentration is treated by the classical perturbation technique. It is concluded that such disturbances will amplify at large Hall parameters. The exponentiating time may be less than 10^4 sec, so that the disturbances could grow very large in a time of the order of the fluid residence time in an MHD channel.

Introduction

In most analyses of magnetogasdynamic devices, it has been assumed that the conductivity of the plasma is related to the thermodynamic state of the gas. Under some conditions, this approximation is valid. On the other hand, examples of nonequilibrium of energy are common in gaseous electronics. The glow discharge exists by virtue of an excess of the electronic thermal energy over the gas thermal energy. In this case, the electronic energy is determined by a balance between gain in the electric field and loss by collisions and interaction with the walls of the containing vessel. The electronic concentration is determined by an even more complicated balance between ionization due to electron impact and recombination by diffusion to the wall.

It was pointed out in Ref 1 that, in many plasmas of engineering interest, an intermediate situation may exist, wherein the electronic thermal energy is essentially determined by gain in the electric field and loss due to collisional processes, and the electronic concentration is determined by approximating ionization equilibrium at the electron temperature. Thus, we imagine a situation wherein the electrons comprise a gas that is weakly coupled thermally to the molecular gas, but very strongly coupled to the valence electrons of the gas, and thoroughly Maxwellized at its own mean thermal energy. In this case, the conductivity is determined by local conditions in the plasma, as in the equilibrium case; however, one more parameter is required, in addition to the thermodynamic variables of the gas, in order to determine the state of the plasma. The electron temperature is, of course, a fundamental variable that must be added; however, it was shown in Ref 1 that the electron temperature and concentration can be related simply to the electronic current density. Thus, it is possible to characterize the local state of the plasma in terms of the thermodynamic variables of the gas and the current density.

The advantage of this characterization is that it permits the generalization of the usual Ohm's law to include the nonequilibrium effect. The resulting "two-temperature conduction law," although nonlinear, is sufficiently simple to be useful in engineering analysis.

A preliminary experimental verification of the proposed conduction law was presented in Ref 1. More convincing experimental evidence is now at hand and will be described in Part II of this article.

The purposes of the present article are 1) to describe broadly the conditions under which the proposed conduction law should be valid, and 2) to consider some of its implications. In particular, we shall attempt to determine whether it is reasonable to apply the two-temperature conduction law locally in analyses of practical magnetogasdynamic devices, and whether the nonequilibrium state that it implies is stable.

The question of stability is not a trivial one. Karlowitz et al.1 considered this question in the course of their early work on MHD power generation and concluded that the nonequilibrium state would be unstable. Later analyses and the experimental data now at hand do not support their conclusion. However, most of the systematic experimental work has been done without magnetic fields. As we shall see, the present analysis indicates that instabilities may occur for large values of the Hall parameter.

Formulation

There are several ways in which the electron energy may be forced to be different from the atom energy. Any process that feeds energy preferentially to the electrons, or withdraws it, will raise or lower the electron energy. For example, recombination in a nozzle by a three-body process, in which the ionization energy is carried away by an electron, will cause an increase of the electron energy above the gas energy.

However, we shall be concerned with situations in which the increase of electron energy is due to energy gain in an electric field. There will then be a steady state in which this gain is offset by various loss mechanisms, including elastic collisions and radiation. Ionization and recombination will influence the energy balance only during relaxation to or from this steady state or in perturbations about it.
To obtain a complete description of this energy balance, we would of course need the complete electronic distribution function, from which the energy loss by the various collisional processes could be computed in detail. Rather than attempt such a complete description, we shall assume that the energy loss per unit volume from the electrons may be represented in terms of their mean thermal energy $\epsilon$ by

$$n \Delta e = \delta(m/m_0) n \nu(\epsilon - \epsilon_i) + n Q$$  \hspace{1cm} (1)

where $\epsilon_i = 3kT/e/2$ is the mean translational energy of the gas molecules, $m$ and $m_0$ are the masses of electron and molecule, $n$ is the electron concentration, $\nu$ is the electronic collision frequency, and $\delta$ is a constant. The second term represents energy loss by radiation, whereas the first represents the loss by all other collisional mechanisms. In writing the energy balance this way, we have assumed a single molecular species. For a mixture, we should replace $\delta \nu/m_0$ by $\delta \nu/m_0$ where the sum is taken over all molecular (and ionized) species. It should also be noted that, by assuming this form for $n \Delta e$, we have tacitly assumed that ionization is dominated by electron-atom collisions. Otherwise, we should have to add to $n \Delta e$ a term representing the production of electronic excitation by atom-atom collisions. At the high electron concentrations of interest here, this process is negligible compared to the electron-atom ionization process.

With this expression for $n \Delta e$, the electronic energy balance is obtained as a moment of the Boltzmann equation:

$$\{D[n(\epsilon + \epsilon_i)]/Dt\} + n_0(\epsilon + \epsilon_i) \text{div}_\nu + \text{div} \mathbf{j} = -J(e) - \mathbf{B}$$  \hspace{1cm} (2)

Here $\epsilon_i$ is the ionization energy of the species which contributes the electrons, $\nu_0$ is the mass-averaged velocity of the flow, and $\mathbf{J} = \mathbf{E} + n_0 \times \mathbf{B}$ is the electric field measured in the moving gas.

The current density $\mathbf{J} = \mathbf{E} + n_0 \mathbf{B}$ is the magnetic field. The second term represents the diffusion velocity resulting from an electron concentration gradient.

This expression may be put in the alternative form

$$\mathbf{j} = \sigma(n, \nu) \mathbf{E} + (kT/e) \text{grad} n \text{ln} n - (1/en) \mathbf{j} \times \mathbf{B}$$  \hspace{1cm} (3)

which is more convenient for analysis. In Cartesian form, the tensor $\mathbf{M}$ is

$$M = \frac{1}{1 + \beta^2} \begin{pmatrix} (1 + \beta^2) & (\beta \beta_2 + \beta) & (\beta \beta_2 - \beta) \\ (\beta \beta_2 + \beta) & (1 + \beta^2) & (\beta_2 \beta + \beta) \\ (\beta \beta_2 - \beta) & (\beta_2 \beta - \beta) & (1 + \beta^2) \end{pmatrix}$$  \hspace{1cm} (5)

where $\beta_2 = eB_2/m_0 \nu$ is the Hall parameter based on the $i$ component of $\mathbf{B}$ and $\beta = \beta_2 + \beta_2^2 + \beta_2^4$. The electronic heat-flux vector $\mathbf{q}$ will be taken as

$$\mathbf{q} = -\lambda \mathbf{M} \text{grad} T - (1/e) [\beta_2 \epsilon + \epsilon_i] \mathbf{j}$$  \hspace{1cm} (6)

For the sake of simplicity, the electronic heat-conduction coefficient $\lambda$ will be assumed constant. The second term represents the heat flux carried by the diffusing electrons.

For the situations of interest here, $n$ is sufficiently large that we can make the plasma approximation (that the electron concentration equals the ion concentration). We shall not consider plasma oscillations. We shall also exclude electromagnetic wave propagation from consideration and assume that the electric field has a potential $\phi$ so that

$$\mathbf{E} = -\text{grad} \phi$$  \hspace{1cm} (7)

We shall also assume that the ion conduction current is small compared to the electronic current. It then follows from the plasma approximation that

$$\text{div} \mathbf{J} = \text{div} [\sigma(n, \nu) \mathbf{E} + (kT/e) \text{grad} \ln n] = 0$$  \hspace{1cm} (8)

Our formulation will be completed by a relationship between the electron concentration $n$ and the mean thermal energy $\epsilon$. It is clear that this relationship may be quite complex; indeed, the specification of $\epsilon$ will not determine $n_0$ in general. The complete electronic distribution function is required. However, since we treat only the energy integral of the Boltzmann equation, we must assume some form for the distribution function of the free electrons. We shall assume that it is Maxwellian, although the effects of inelastic collisions (radiant energy loss) and the Ramzauer effect in certain cross sections (e.g., argon) are clearly such that they depress the distribution function at high electron energies.

It was further assumed in Ref. 1 that the electron concentration was in equilibrium with the electron temperature. An argument will be given below which indicates that this assumption should be valid for all conditions of interest in MHD applications, provided that the free electrons are Maxwellian.

Thus, we shall assume, as in Ref. 1, that in the steady state $n$ is related to $\epsilon$ by the Saha equation. If the ionization is by electron-atom impact and the recombination involves two electrons and an ion, the ionization rate equation is then

$$Dn/dt = k_s[K_s n_{en} - n^2]$$  \hspace{1cm} (9)

where $k_s$ is the usual recombination coefficient and

$$K_s = \frac{4\pi m_e^{3/2}}{3e} \left( \frac{e^2}{2\epsilon} \right)^{1/2} \epsilon^{1/2}$$  \hspace{1cm} (10)

Equation (9) completes the description of the two-temperature gas. Before considering the consequences of the model that has been established, it will be well to elaborate upon the principal assumptions.

**Significance of $\delta$**

As stated previously, the term $\delta(m/m_0)n(\epsilon - \epsilon_i)$ in Eq. (1) represents all energy loss from the free electrons except that by radiation. The electrons can transfer energy to translational, rotational, and vibrational degrees of freedom, as well as the electronic degrees of freedom. In general, the strength of the coupling increases in the order translational, rotational, and vibrational, electronic.

Considering first the electronic degrees of freedom, we note that the cross section for interaction of a free electron with a valence electron is of the same order of magnitude as that for interaction of two free electrons. Thus, for the present purpose, it is convenient to think of the valence and free electrons together comprising a single gas, which may lose energy by collision with the atoms and molecules or by radiation. But an inelastic collision of a free electron with a valence electron does not constitute a loss of energy to the electron gas, unless a photon escapes in the process. We shall consider this possibility in the next section.

On the other hand, the electrons can transfer energy to the other degrees of freedom of the gas. For a monatomic gas with a simple force law, this process is susceptible to analysis. The results are that, for a Maxwellian force law, $\delta = 2$, whereas for hard sphere collisions it is 1.85. In more complicated situations, it may be best to regard $\delta$ as an empirically determined parameter.

Traditionally, this viewpoint has been followed in defining an equivalent $\delta$ for diatomic and polyatomic gases to include energy transfer to rotational and vibrational degrees of freedom. Values of this equivalent $\delta$ are tabulated by Massey and Burhop. However, it should be noted that
Fig 1 The ratio of electron thermal energy $\epsilon$ to atomic thermal energy $\epsilon_0$ as a function of the current density parameter $\chi$, and the ratio of ionization energy $\epsilon_i$ to atomic thermal energy $\epsilon_0$. The parameter $\chi$ is the value of ($\epsilon/\epsilon_0 - 1$) which would exist in the plasma for a given $j$ if $n$ did not increase from the equilibrium value with increasing $\epsilon_0$.

these values of $\delta$ are usually taken from swarm experiments with very low electron concentration, where the distribution function may be strongly non-Maxwellian. They should be applied to gases with high electron densities only with caution.

The experimental work to be reported in Part II of this paper will serve to determine the effective value of $\delta$ for alkali metal-noble gas mixtures. As we shall see, there is some difficulty in separating the collisional and radiative contributions to $\Delta \epsilon$, so that only an upper bound to the value of $\delta$ can be determined.

Equilibrium of Electrons

As noted previously, we may think of the free and valence electrons as comprising a single gas. Electron-electron collisions tend to force this gas toward a Boltzmann distribution, whereas collisions of electrons with atoms and radiation, in general, tend to perturb it. Considering first the free electrons, let $S$ be the coulomb cross section for interaction of two electrons, and $S_N$ the electron-atom cross section. As Ben Daniel and Tamor$^2$ have observed, we expect Maxwellization when

$$n \delta S \gg \delta(m/\mu_e)n n_y S_N$$

Evaluating $\sigma$ for an electron energy corresponding to the first excited state of potassium, $S \approx 10^{-13}$ cm$^2$. Typically, $S_N \approx 10^{-16}$ cm$^2$, $\delta = 10$, and $m/\mu_e = 10^{-2}$, so that we have $n_y/\mu \delta > 10^{-8}$ as the condition for the electrons to attain a Maxwellian distribution.

At atmospheric pressure and 2000$^\circ$K, $n_y = 3.6 \times 10^{18}$ cm$^{-3}$, so that this criterion implies an electron concentration of about $4 \times 10^{12}$ cm$^{-3}$. This concentration of electrons gives a conductivity of about 0.05 mho-cm$^{-1}$ in argon. It appears that conductivities on the order of 1 mho-cm$^{-1}$ are necessary for high-performance MHD generators, so that the plasma in such generators should satisfy the criterion for Maxwellization of the free electrons rather well. These rough estimates are verified experimentally in Part II.

Next, consider the equilibrium of the valence electrons with the free electrons. The equilibrium can be upset by radiative transitions, provided that the radiation escapes from the gas and thereby depletes the population of an excited state faster than it can be replenished by electronic excitation from lower states. Ben Daniel and Tamor$^2$ have considered this balance and have concluded that resonance radiation would indeed reduce the free electron population by depleting the first excited state in cesium. However, this conclusion depends critically on the resonance escape probability and on the magnitude of the excitation cross section.

Denoting the excitation cross section by $\Sigma$, and the radiation lifetime of the excited state (for radiations which escapes the gas) by $\tau$, we expect the effect of radiation on the population of the excited state to be small if

$$n \delta \Sigma \gg 1/\tau$$

From Byron's calculations,$^6$ using Gryzinski's classical atomic model, $S \approx 10^{-13}$ cm$^2$ for the resonance transition in potassium, with $T = 2000^\circ$K.

An a priori estimate of $\tau$ can be obtained by properly accounting for line broadening and the geometry of the plasma. This has been done by Lutz$^4$ for a plane parallel geometry. Alternatively, we may obtain a lower limit for $\tau$ directly from experimental evidence by noting that the power radiated by the resonance lines must be less than the Joule heating rate of the plasma. Thus, if the radiated power per unit volume is $n q$ as in Eq. (1), $\tau \geq n e \exp(-\epsilon/2kT)/nq$, where $\epsilon$ is the energy of the first excited state.

For a typical potassium-seeded plasma at atmospheric pressure, the Joule heating is about 1 W cm$^{-3}$, $n_e \approx 10^{16}$ cm$^{-3}$, $n_y \approx 10^{12}$ cm$^{-3}$, and $T \approx 2000^\circ$K. Thus we find $\tau \approx 0.6 \times 10^6$ s$^{-1}$, whereas $n \delta \Sigma \approx 10^8$ s$^{-1}$. Thus, we conclude that radiation should not perturb the electronic distribution for such a plasma. This conclusion is in agreement with Ben Daniel and Tamor$^2$, provided that their resonance escape probability is assigned a value of about 0.01.

The fact remains that, in experiments conducted to date, the conductivity (electron concentration) has been lower by about a factor of 2 than the simple theory of Ref 1 predicts it to be. It seems very likely that this is due to the lowering of the electron temperature, for a given electric field or current density, which occurs as a result of energy loss from the electrons by resonance radiation.

Two-Temperature Conduction Law

In the steady state for a uniform gas, Eqs. (13) reduce to simply

$$j^2/\sigma(n, \nu) = \delta(m/\mu_p)n \epsilon - \epsilon_y + n Q$$

whereas Eq. (9) becomes

$$n \delta/\mu = K_s(\epsilon)$$

Taking $\sigma = \sigma^2 n /m_N, \nu$, Eq. (13) becomes

$$\epsilon - \epsilon_a = m_e \left(\frac{j}{e n}\right) \frac{d}{d \mu} \frac{Q}{n}$$

If $Q$ is neglected, Eqs. (14) and (15) can be solved for $\epsilon$ and $n$ for given the gas composition, pressure, temperature, and the current density $j$. It is clear that $\sigma(n, \nu)$ can then be represented as a function of $j$ and the composition, pressure, and temperature of the gas.

Thus, we may write a conduction law of the form

$$j = \sigma(j, \mu, \mathbf{E})$$

Because $\sigma$ depends on the magnitude $j$ of the current vector, Eq. (16) can be very complicated if we attempt to give the form of $\sigma(j)$ explicitly. However, we may note that, if $\mathbf{E}^f$ is the component of $\mathbf{E}$ in the direction of $j$, then we have

$$j = \sigma(j_E)$$

It was shown in Ref 1 that, if $n \ll n_r$, the ratio $\epsilon /\epsilon_r$ is obtained from the expression (neglecting $Q$)

$$\epsilon /\epsilon_r = 1 = \chi \left(\frac{\epsilon_r}{\epsilon_r/\epsilon - 1}\right)^{1/2} \exp \left(\frac{3 \epsilon}{2 \epsilon_r} - 1\right)$$

where $n_r \ll 1$ (18)
\begin{equation}
X = m_j e^2 / 5e_n^2 \kappa_x (\epsilon_e)
\end{equation}

This relationship is shown in Fig. 1, which is taken from Ref. 1.

At the opposite extreme of \( n = n_e \), where \( n_e \) is the original concentration of ionizable species (i.e., \( n_e = n_a + n_i \)), we have

\begin{equation}
(\epsilon_e / \epsilon_s) - 1 = m_j e^2 / 5e_n^2 n_a / n_e \approx \epsilon_s / \epsilon_e
\end{equation}

Comparing Eqs (18) and (20), we see that \( \epsilon / \epsilon_s - 1 \) increases much more rapidly with increasing \( j \) when the seed is completely ionized than it does when it is partially ionized. This rapid increase of \( \epsilon / \epsilon_s - 1 \) can lead to an instability. This point will be elaborated later.

Neutral Collisions Dominant

If the ion concentration is small enough so that the resistivity is determined by electron-atom collisions, \( \sigma \propto n / \epsilon_n^{1/2} \).

Consider first the situation where \( n / n_a \ll 1 \) and Eq. (18) applies. It was shown in Ref. 1 that, in this case, \( \sigma \) is well represented by

\begin{equation}
\log \sigma = \gamma \log T_a + \alpha \log j + \text{const}
\end{equation}

\begin{equation}
\alpha = \frac{\partial \sigma}{\partial \log T} \left[ \frac{1}{2} \right] \left( \frac{(\epsilon_e / \epsilon_s) - 1}{(\epsilon / \epsilon_s) + \mu (\epsilon / \epsilon_s) - 1} \right)^{1/2}
\end{equation}

\begin{equation}
\gamma = \frac{\partial \log \sigma}{\partial \log T} = \eta = \left[ \frac{1}{1 + 3(\epsilon / \epsilon_e)^2} \right] \left[ 1 + (\epsilon / \epsilon_e) \right] \frac{n_s}{n e} \ll 1
\end{equation}

Thus both \( \alpha \) and \( \gamma \) depend only on \( \chi \) and \( \epsilon / \epsilon_e \). Values of \( \alpha \) are given in Fig. 2, which is taken from Ref. 1.

Combining Eqs (17) and (21), we can write a dimensionless collision law of the form

\begin{equation}
j / j^* = \left( T_d / T_e \right)^{\gamma / (1 - \alpha)} \left( E'_o / E_o \right)^{1 / (1 - \alpha)}
\end{equation}

where the asterisks indicate an arbitrary reference value. From Eqs (22) and (23), it can be shown that \( \gamma (1 - \alpha) = 3 \epsilon / \epsilon_e \), whereas if we have ionization equilibrium at the gas temperature, \( \alpha = 0 \) and \( \gamma \) becomes \( 3 \epsilon / 4 \epsilon_e \). Thus, the dependence of \( j \) on \( T_s \) is not altered greatly by nonequilibrium effects unless \( T > T_s \).

This two-temperature collision law, for example, indicates that, if \( j \) is held constant while \( T_s \) varies (as in the boundary layer on an electrode), the resulting change of \( E'_o \) is quite small if \( \alpha \) is near unity. This situation was considered in Refs 1 and 10. On the other hand, if \( j \) is not constrained, as in the boundary layer on the insulating wall of an MHD channel, extreme variations of \( j \) can occur due to variation of both \( T_s \) and \( E'_o \). This situation was considered in Ref 11.

For the limit of complete ionization of the ionizable species [Eq. (20) applicable], we find

\begin{equation}
\alpha = - [1 - (\epsilon_e / \epsilon_s)] \quad n_i = n_s
\end{equation}

\begin{equation}
\gamma = [1 - (\epsilon_e / 2 \epsilon_e)] \quad n = n_e
\end{equation}

For this case, then, \( \sigma \) is quite insensitive to both \( T_s \) and \( j \).

Coulomb Collisions Dominant

If the ion concentration is so large that coulomb collisions dominate the behavior of the conductivity, then it is given by

\begin{equation}
\frac{1}{\sigma} = 0.0176 \left( \frac{e^2}{m} \right) \left( \frac{3 \epsilon_n}{4 \epsilon_e} \right)^{1/2} \log \Lambda
\end{equation}

This expression is applicable to all ionizable species when \( \epsilon_i / \epsilon_e \rightarrow \infty \), and the region in

Fig. 2 The coefficient \( \alpha \) in the two-temperature conduction law: \( j = f(T_e) (E'_o)^{1 - \alpha} \). Where \( \alpha \) is not zero, the conventional Ohm's law is applicable. As \( \alpha \) approaches unity, the conduction law becomes highly nonlinear.

\begin{equation}
\alpha = \frac{3 [1 - (\epsilon_e / \epsilon_s)] [1 + \frac{1}{2} (\epsilon_e / \epsilon_s) - \frac{3}{4} [1 / \log \Lambda]]}{1 + \frac{3}{2} [1 + (\epsilon / \epsilon_e)] [1 - (\epsilon_e / \epsilon_s)]}
\end{equation}

and

\begin{equation}
\alpha = \frac{3 [1 - (1 / \log \Lambda)] [1 - (\epsilon_e / \epsilon_s)]}{n_s / n_e} \quad n = n_s
\end{equation}

The difference between these two results lies in the variation of \( \epsilon_e \) with \( j \), which is much more rapid for the case of \( n = n_s \).

In either case, \( \gamma \) is relatively small, since \( T_s \) influences the conductivity only through \( T \).

We may summarize these results by saying that, for a seeded gas in which the electron-neutral cross section dominates with a low degree of ionization of the seed species, the effect of nonequilibrium is to increase the sensitivity of the current to the electric field without greatly changing its dependence on the gas temperature. If the seed material is completely ionized, the current is nearly proportional to the electric field and quite insensitive to gas temperature.

When coulomb collisions dominate, \( \alpha \) is small if the degree of ionization of the seed gas is small, but it can be large if the seed is nearly completely ionized [Eq. (28)]

This last conclusion is of some importance, since, from the form of Eq. (24), we see that the conduction process is unstable if \( \alpha \geq 1 \), in the sense that an arbitrarily small \( E'_o \) will lead to a large \( j \). This will occur for the coulomb-limited gas if \( n = n_s \) and \( \epsilon / \epsilon_e > \frac{3}{4} \) (for \( \log \Lambda \) large) The instability is a result of the decrease of the coulomb cross section with increasing electron energy. If an increase in current increases \( \epsilon \) more than a certain amount (given by the condition \( \epsilon / \epsilon_e > \frac{3}{4} \)), the resulting increase in \( \sigma \) is enough to cause a current concentration. Of course, such an instability is possible for any completely ionized plasma, if a situation exists such that the electron temperature is greater than about \( \frac{3}{4} \) times the gas temperature. The author is not aware of such an instability having occurred in practice, probably because the conductivities are so high in fully ionized plasmas that \( \epsilon = \epsilon_e \) at experimentally attained current densities.

From Fig. 2, we see that the slightly ionized gas approaches instability of this type as \( \epsilon_i / \epsilon_e \rightarrow \infty \), that is, for small gas temperatures.

The variations of \( \alpha \) and \( \gamma \) with \( j \) and \( T_s \) are depicted in Fig. 3 for potassium at 001 atm pressure.
which the classical Ohm's law is valid is below the solid line, where \( \alpha \) is small. Above it, there is appreciable electron heating, but the value of \( \alpha \) depends on the extent of ionization of the gas. If the gas is only slightly ionized, \( \alpha = 0 \), as in the lower left portion of the diagram. If the gas becomes coulomb-limited while still slightly ionized (as \( j \) increases for fixed \( T_0 \)), \( \alpha \) becomes small. For larger values of \( j, \alpha \) will then increase again as the degree of ionization approaches unity, and \( \varepsilon/\varepsilon_a \) increases, until the region of instability is reached. In this region, \( j \) would increase without limit for fixed \( E'_0 \).

If we add an inert gas to the potassium, the behavior is not qualitatively altered in the lower portions of Fig. 3. However, if the concentration of inert gas is sufficiently large that the mixture is coulomb-limited, even when the potassium is completely ionized, the region of instability at large \( j \) is removed, since then Eqs. (25) and (26) apply for large \( j \). Naturally, this conclusion only follows if the electron temperature remains below that required to ionize the inert gas.

**Inlet Relaxation**

The two-temperature conduction law described in the last section is strictly applicable only to a steady, uniform plasma. We may then ask how far a plasma initially in equilibrium must flow in an MHD device before relaxing to the non-equilibrium state.

Consider the two extreme configurations sketched in Fig. 4. In configuration a, the electric field is held constant in \( z \), so that \( j \) increases as the conductivity rises. In configuration b, the current is held constant, so that \( E \) decreases with increasing conductivity as the gas flows down the channel.

For either case, Eq. (2) reduces to

\[
\frac{d(n/\varepsilon)}{dz} = jE = \delta(m/\varepsilon_a)n(\varepsilon - \varepsilon_a) \tag{29}
\]

where we have neglected div \( J \), radiation, and the effects of nonuniform \( n_0 \). The left side of the equation represents the increase of energy invested in ionization and electronic thermal energy. This energy must be supplied by \( jE \), with the result that \( \varepsilon \) is initially depressed.

From Eq. (9), the rate of ionization is

\[
u(dn/dz) = \frac{k_o}{\varepsilon_0} [K_n n_0 n - n_i^A] \tag{30}
\]

We should integrate Eqs. (29) and (30) simultaneously to obtain \( n(z) \) and \( \varepsilon(z) \).

To obtain a first estimate of the relaxation length, we shall consider the case of \( k_o \to \infty \), wherein Eq. (30) is replaced by simply \( n_i^A = n_0 K_n(\varepsilon) \). Thus, we obtain the rate as limited only by energy input. Since the electron concentration, and therefore the current density, has at all positions in the channel the maximum possible value consistent with the energy that has been added by the electric field up to that point, the computed rate should be an upper limit.

Smith has considered the effect of finite \( k_o \) for an assumed constant \( \varepsilon \) without accounting for the energy that must be invested in ionization. Clearly, if Smith's calculation predicts a markedly different relaxation length than does the present calculation, then the correct length is near the longer of the two.

Equation (20) may then be put in dimensionless form as follows:

**Configuration a**

\[
\frac{d(\varepsilon/\varepsilon_a)}{d\xi} = \frac{\varepsilon_i}{\varepsilon_a} \left( \frac{\varepsilon_i}{\varepsilon_a} \right)^{1/2} - \frac{\varepsilon_i}{\varepsilon_a} \left[ \frac{\varepsilon_i}{\varepsilon_a} - 1 \right] \tag{31}
\]

**Configuration b**

\[
\frac{d(\varepsilon/\varepsilon_a)}{d\xi} = \frac{\varepsilon_i}{\varepsilon_a} \left( \frac{\varepsilon_i}{\varepsilon_a} \right)^{1/2} \exp \left\{ \frac{3}{2} \left( \frac{\varepsilon_i}{\varepsilon_a} - \frac{\varepsilon_i}{\varepsilon_a} \right)^{1/2} \right\} - \frac{(\varepsilon_i/\varepsilon_a)^{1/2}}{(\varepsilon_i/\varepsilon_a)_{\infty}} \left[ \frac{\varepsilon_i}{\varepsilon_a} - 1 \right] \tag{32}
\]

In both cases,

\[
\xi = \frac{x}{L} = \frac{4}{3} \frac{1}{\varepsilon_a} \left( \frac{\varepsilon}{\varepsilon_a} \right)^{1/2} \left( \frac{\varepsilon_i}{\varepsilon_a} \right)^{1/2} \frac{(\varepsilon_i/\varepsilon_a)_{\infty}}{(\varepsilon_i/\varepsilon_a)_{\infty}} - \frac{1}{\varepsilon_i/\varepsilon_a} \frac{x}{(\varepsilon_i/\varepsilon_a)_{\infty}} \right] \tag{33}
\]

where \( \xi \) is the electron mean free path.

Solutions of Eqs. (31) and (32) are given in Fig. 5 for several values of \( \varepsilon/\varepsilon_a \). The solution for case (b) depends also on \( \varepsilon_i/\varepsilon_a \), which is taken as 22.3 in Fig. 5.

We note first of all that the electron temperature relaxes much faster when \( j \) is held constant than when \( E \) is held constant. This is because \( E \) increases at the entrance in case (b) to drive the current through the incoming gas of low conductivity.

The excess voltage drop associated with the accelerated relaxation in case (b) is of interest in that it would tend to appear as an electrode drop in the geometry sketched in Fig. 4b. It can be computed as

\[
\delta V = \int_0^1 [E - E_w]dx
\]
This voltage drop is shown in dimensionless form in Fig 6. We see that the excess voltage drop is a small fraction of the voltage drop in length $L$ in the asymptotic fluid, unless $(\epsilon/\epsilon_a)_0$ is quite large. This simply reflects the fact that the investment in ionization energy is small.

**Magnitudes**

For a mixture of argon and potassium at 2000°K, if $\delta = 2$, 

$$L = 95 \frac{M}{(\epsilon/\epsilon_a)_0[(\epsilon/\epsilon_a)_0 - 1]} \text{ cm}$$

From Fig 5, we see that the actual relaxation length is about $0.1L$ for case (b) and from $0.1L$ to $L$ for case (a) as $(\epsilon/\epsilon_a)_0$ increases from 1 to 2. Thus, for case (a), the actual relaxation length is nearly independent of $(\epsilon/\epsilon_a)_0$ and about $95M$ cm.

This relaxation process is then of some practical importance in MHD generators, particularly if they operate supersonically. It may be that a set of electrodes, connected as in case (b) at the entrance to the channel, will be necessary in order to reduce the relaxation length to a small fraction of the generator length.

**Electrothermal Waves**

A close examination of Eqs (2, 8, and 9) indicates the possibility of a wave-like disturbance propagating in the non-equilibrium plasma. The mechanism may be seen as follows. Suppose (Fig 7) that a sinusoidal disturbance of electric field exists in the plasma. Electrons diffusing to the right in the gas will gain energy more rapidly where $E$ is smaller, with a resultant perturbation of $\epsilon$, shifted in phase by $\pi/2$. Then, by Eq (9), a disturbance of $\omega$ will occur which lags that of $\epsilon$ by an amount depending on the magnitude of $k_0$. From Eq (8), we see that the concentration gradient is such as to produce an effective force on the electrons which opposes that due to the original electric field disturbance. Thus, the criterion for an oscillation is satisfied, and there is a possibility that the disturbance will propagate in the fluid and grow in amplitude. We shall examine this possibility, making the assumption that the gas temperature and pressure remain uniform in the presence of the disturbances of electron energy and concentration.

As is usual in stability analyses, we shall consider an arbitrary disturbance to have been analyzed at time $t = 0$ into its Fourier components and ask which, if any, of these components will then grow in time. Expanding about the nonequilibrium steady state, we write

$$\begin{align*}
\omega &= n_0 + \eta(x,t) \\
\epsilon &= \epsilon_0 + \epsilon(x,t) \\
E &= E_0' + \text{grad} \varphi(x,t) \\
j &= j_0 + i(x,t) \\
\sigma &= \sigma_0 + s(x,t)
\end{align*}$$

To zeroth order, we then have

$$j_0 E_0' = \delta(m/m_0)v_0(\epsilon_0 - \epsilon_0)$$

$$j_0 = \sigma_0 ME_0' \quad n_0^3 = n_0 K_n(\epsilon_0)$$

These equations, of course, yield the steady-state nonequilibrium condition described previously.

To first order, the energy balance is

$$\begin{align*}
\langle \epsilon_0 + \epsilon_0 \rangle \frac{D\eta}{Dt} + m_0 \frac{D\epsilon}{Dt} &= 2 \frac{\lambda}{3k} \text{ div}(M \text{ grad} \varphi) + \frac{\epsilon}{3c^2} \text{ grad} \epsilon - \\
\text{div}E &= j_0 \text{ grad} \varphi + \sigma_0 \text{ E}_0' \left\{ M \left[ \text{ grad} \varphi - \frac{2\epsilon_0}{3c^2} \text{ grad} \eta \right] \right\} + \\
\frac{j_0}{\sigma_0} \text{ E}_0' &= \delta \frac{m_0}{m_0} \nu [(\epsilon_0 - \epsilon_0) \eta + n_0 \epsilon]
\end{align*}$$

where

$$D/\text{Dt} = (\partial/\partial t) + \nu (\partial/\partial x)$$

The condition that div $j = 0$ becomes div $\text{ grad} j = 0$, since $j_0$ is constant. Thus,

$$\text{ div} \left\{ M \left[ \text{ grad} \varphi - \frac{2\epsilon_0}{3c^2} \text{ grad} \eta \right] \right\} + \frac{1}{\sigma_0} j_0 \text{ grad} \epsilon = 0$$

**Fig 5** The variation of electron energy with the nondimensional coordinate $\xi$ in the region of relaxation. $\xi$ is proportional to the ratio of (Bouc-Mach number/electron mean free path) For argon at atmospheric pressure and 2000°K, the actual relaxation length is about 95 cm for a Mach number of unity.

**Fig 6** The excess voltage drop which occurs in the entrance region of the channel for case (b) of Figs 4 and 5. $\delta V$ is small compared to the voltage drop in one relaxation length $L$ in the asymptotic fluid.

**Fig 7** Illustrating the mechanism of oscillation in electrothermal waves. The oscillation results from the interaction of the perturbed electric field with the electron partial pressure gradient which it induces by perturbing the electron energy and concentration.
Finally, the ionization rate equation is

$$\frac{D\eta}{Dt} = k_e n_s^3 \left[ \frac{3}{2} \left( \frac{1}{1 + \epsilon_i} - \frac{\epsilon_i}{\epsilon_0} \right) \eta - \frac{2}{n_0} \right]$$  \hspace{1cm} (36)

For the sake of simplicity, we shall assume throughout that $n_s$ is constant. Our considerations are thereby limited to the case of slight ionization of the ionizable species. For the same reason, we take $\nu$ constant, so that $t / \alpha = \eta / \eta_0$.

Taking a typical Fourier component of the disturbance as

$$\eta = N_0 e^{i(K x + \omega t)}$$
$$\epsilon = \epsilon_0 e^{i(K x + \omega t)}$$
$$\varphi = \varphi_0 e^{i(K x + \omega t)}$$

we compute a dispersion relation from the condition for existence of a solution for $N, \epsilon, \varphi$ and $P$.

$$\frac{\omega}{\nu} = -\left( \frac{2\lambda/3k_B\nu}{(K^2/1 + \beta^2)} + \delta(m_c/m_e) \left\{ \frac{2}{1 + \frac{3}{2} \left[ 1 + (\epsilon_i/\epsilon_0) \right] \left[ 1 - (\epsilon_i/\epsilon_0) \right] (K_x/K_t)^4 (1 - \beta) (K_x/K_s) \right\} \right)$$

For present purposes, it is sufficiently general to consider the case where all electric field lies in a plane normal to $B$. We shall also take $u$ normal to $B$ and for the sake of definiteness suppose that $B$ lies along the $x$ axis and $u$ along the $x$ axis.

**Infinite Ionization Rate**

Even in this case, the dispersion relation is quite complicated. However, in the limit of $k_x \to \infty$, that is, of complete ionization equilibrium, we find for $\omega = \omega - i \omega_i$.

$$\omega + i \omega_i = \left( \frac{2\lambda/3k_B\nu}{(K^2/1 + \beta^2)} + \delta(m_c/m_e) \left\{ \frac{2}{1 + \frac{3}{2} \left[ 1 + (\epsilon_i/\epsilon_0) \right] \left[ 1 - (\epsilon_i/\epsilon_0) \right] (K_x/K_t)^4 (1 - \beta) (K_x/K_s) \right\} \right)$$

where $K^2 = K_x^2 + K_s^2$.

The magnitude of the velocity of propagation is best seen by noting that Eq. (19) may be written as $\epsilon / \epsilon_s - 1 = m_c u / \delta \epsilon_s$, so that

$$j_0/\eta_0 = u = \left[ 3k_B T_s / (\epsilon_i/\epsilon_s - 1) / 2m_a \right]^{1/2}$$

and we have

$$\omega + i \omega_i = \left[ (\epsilon_i/\epsilon_s) - \frac{1}{2} \right] \left\{ \frac{3}{2} \frac{k_B T_s}{m_a} \left( \frac{\epsilon_i}{\epsilon_s} - 1 \right) \right\}^{1/2}$$

Since the ratio $\epsilon_i/\epsilon_s = 1$ for cases where $\eta / \eta_0 \ll 1$, as assumed here, the wave propagates at a speed that is $(2\lambda/3k_B\nu) / \epsilon_i$ times sonic speed in the gas if $\epsilon_i/\epsilon_s - 1 = 1$. Since $\epsilon_i/\epsilon_s \ll 1$, this speed is considerably less than sonic speed in the gas unless $\delta$ is very large. As the electron temperature approaches the gas temperature, the wave speed vanishes

From another point of view, $u$ is the electron diffusion velocity, so that the wave propagates at a speed that is proportional to the diffusion velocity. Thus, the disturbance is, in fact, carried through the plasma by the diffusing electrons and is a "kinematic" rather than a "dynamic" wave.

If $\omega / \nu$ is positive, the wave amplifies. Since the first two terms in the numerator of Eq. (37) are inherently positive, we see that conduction and collisional energy losses are damping effects, as expected. On the other hand, the last term can be negative. This is most easily seen for special cases, as follows.

**Special Cases:** $J_x = 0$ or $E_x = 0$ ($k_x \to \infty$)

The first of these cases is that where the Hall current is prevented from flowing, whereas the latter is the case where it is unimpeded. In either case, Eq. (37) for $\omega / \nu$ becomes

$$\omega / \nu = \left( \frac{2\lambda/3k_B\nu}{(K^2/1 + \beta^2)} \right)$$

If the factor $(K_x/K_t)^4 (1 - \beta)$ is sufficiently negative, the disturbance will amplify. Since $K_x/K_t$ is negative and large for wave vectors at angles slightly greater than $\pi/2$ and $3\pi/2$ from the $x$ axis, there is for any non-zero value of $\beta$ a range of directions in which the region of amplification increases as $\beta$ increases. Note that, in the equilibrium state, the instability disappears, since $\epsilon_i = \epsilon_s$.

It is, at first, somewhat surprising that the wave switches from maximum attenuation to maximum amplification as the wave angle passes through $\pi/2$ and $3\pi/2$. This can be understood qualitatively by reference to Eq. 8, where a wave is indicated, propagating at an angle slightly greater than $\pi/2$. Comparison with Fig. 7 shows that the perturbations of electric field are such as to produce Hall current components as indicated. For the case of $J_x = 0$, $E_x = 0$ is positive, so that the Hall currents increase the dissipation where $e$ is increasing and decrease it where $e$ is decreasing, thus amplifying the wave. On the other hand, if the wave angle is below $\pi/2$, the directions of the Hall currents are reversed (and $E_x$ is, of course, unchanged), so that they reduce the dissipation where $e$ is increasing and increase it where $e$ is decreasing, thus damping the wave.

The same type of argument can be made for the case of $E_x = 0$. In this case the Hall current perturbations add and subtract from $J_x$.

**Magnitudes**

The electronic heat conductivity $\lambda$ is in first approximation

$$\lambda = \frac{2k_B T_e / m_e}{1 + \beta^2}$$

where $\nu$ is the electron-electron collision frequency, and so we have

$$\lambda = \frac{2k_B T_e / m_e}{1 + \beta^2}$$

As a typical numerical example, we may take a mixture of argon and potassium with $T_e = 2500^0K$, $T_x = 1500^0K$, $\nu = 2 \times 10^6$ cm$,^1$, $m_e / m_e = 2.7 \times 10^4$, $\beta = 2$, $\epsilon_i/\epsilon_s = 22.3$, and $K = 2x10^3$. Then, $\omega / \nu$ varies with $\beta$ and $\theta$ (the angle of $K_x$ to the $x$ axis), as shown in Fig.

We see that, for a typical amplified wave, the characteristic growth time is about $10^{-4}$ sec. For this example,

$$\omega_0 + K_x / \nu = 20 (J_x K_x / j_0 K)$$

so that the velocity of propagation of the wave relative
to the fluid is $20.6 \left( j_0 K / j_0 K \right) \text{msec}^{-1}$. Sonic speed in this case is $720 \text{msec}^{-1}$.

A standing wave can exist in a channel provided that the wave stands at such an angle that the component of flow velocity normal to the wave equals the wave velocity, that is, provided $\omega(\omega) = 0 \text{ at } \left( K_S / K \right) u = 20.6 \left( j_0 K / j_0 K \right)$.

In terms of $\theta$, the condition for a standing wave is that $\tan \theta = -35M$, where $M$ is the flow Mach number. The wave will stand at a $45^\circ$ angle for $M = 0.029$.

We conclude, then, that this type of disturbance will normally be convected with the flow in magnetogasdynamic devices. However, since the size of the time small compared to a typical fluid residence time in such devices, the waves may be of some practical importance.

A special case of these waves has been analyzed by Wight, who included the effects of electron-gas thermal coupling for a wave propagating in the flow direction. He concluded that the disturbance would be damped and would not propagate. This agrees with the present analysis, since for his case $j_0 K = 0$.

### Finite Ionization Rate

With the restrictions on $B$ and the electric field previously given, $\omega$, satisfies the quadratic relation

$$\left( \omega + K_s u \right)^2 + \frac{5 j_0 K}{\epsilon_0 n_0} \left[ \frac{2}{3} \ln \frac{1}{\epsilon_0} - e_0 n_0 \lambda + 2 \eta K_0 + \frac{2}{3} \ln \frac{1}{\epsilon_0} - e_0 n_0 \lambda + 2 \eta K_0 \right] - K_S K_0 = 0$$

where

$$D = \left[ \frac{1}{1 + \beta^2} \right] \left[ E_{\alpha} \left( K_S - B K_0 \right) + E_{\beta} \left( \beta K_S + K_0 \right) \right]$$

Unfortunately, this relation is too complicated to be very useful. To obtain an idea of the effect of finite ionization rate, we can expand in $\nu/n_0 K_0$. This gives

$$\omega = \omega(\nu) = \frac{\omega(0)}{\nu} \left[ 1 - \frac{\nu}{n_0 K_0} \left( \frac{2}{3} \ln \frac{1}{\epsilon_0} - e_0 n_0 \lambda + 2 \eta K_0 \right) \frac{\omega(0) + K_s u \nu}{2 \left[ 1 + \beta^2 \right]} - \frac{2 \ln \frac{1}{\epsilon_0} - e_0 n_0 \lambda + 2 \eta K_0}{D \left[ 1 + \beta^2 \right]} \frac{\omega(0) + K_s u \nu}{2 \left[ 1 + \beta^2 \right]} \frac{\omega(0) + K_s u \nu}{2 \left[ 1 + \beta^2 \right]} \right]$$

Here $\omega(0)$ and $\omega(\nu)$ are the approximations to $\omega$ and $\omega$ for $\nu/n_0 K_0 = 0$, that is, the values given by Eqs (37) and (39).

We see first that, if $\omega(\nu) > 0$, that is, if the wave is amplified for infinite ionization rate, the last term in the numerator of Eq (43) tend to reduce the rate of growth, whereas the first increases it. Secondly, the effect of finite ionization rate is to reduce the wave speed, as expected.

For the example just cited, if $\beta = 10$ and $\omega(0) + K_s u \nu = 10^{-5}$, we find

$$\omega = \omega(\nu) \left[ 1 - 11 \times 10^{-5} \left( \epsilon_0 n_0 K_0 \right) \right]$$

There is some uncertainty concerning the magnitude of $K_s$. Taking Hinnow and Hirschberg's estimate for large electron densities,

$$K_s = 11 \times 10^{-5} T_e^{-1/2} \text{cm}^2 \text{sec}^{-1}$$

and we find

$$\nu/n_0 K_0 = 9.1 \times 10^9 \left( \nu T_e^{1/2} / n_0 \right) \text{ cm}^{-1}$$

For the foregoing example, $\nu/n_0 K_0 = 0.62 \times 10^9$, and the corrections to $\omega(0)/\nu$ and $\omega(\nu) + K_s u$ are 6.8 and 10.5%, respectively.

The linear approximation [Eq (43)] to the effect of $\nu$ predicts that $\omega(0)/\nu$ would become zero for an electron concentration about $\frac{1}{3}$ of that of the foregoing example, or about $2 \times 10^6$. Therefore, we should expect amplification of the electrothermal ionization rate only for electron concentrations greater than about $10^5 \text{ cm}^{-2}$. Unfortunately, this concentration corresponds roughly to the lower limit of electron concentrations of interest in nonequilibrium MHD generators.

### Concluding Remarks

From the foregoing considerations, we may conclude that the two-temperature conduction law proposed in Ref 1 and elaborated upon in the present paper should be applicable to plasmas where the electron temperature is raised by joule heating, provided the electron concentration is above about $10^{12} \text{ cm}^{-3}$ and the dominant ionization process is electron-atom collisions. Such conditions are likely to occur in MHD generators using monatomic working fluids.

Under these conditions, the behavior of the plasma may be very different from what it would be under conditions of thermal equilibrium. It is suggested that, as a first approximation, the two-temperature conduction law should be substituted for the classical Ohm's law in analytical studies of magnetohydrodynamic phenomena where electron heating may occur. Some efforts along these lines are reported in Refs 1, 10, and 11, but more work is needed.

The important question of the stability of the two-temperature plasma has been answered only partially in the present paper. Certainly there is a possibility of disastrous instability if the gas is slightly ionized and not coulomb-collision-dominated, as assumed in the last section. On the other hand, it appears that the wave instability would not exist in a gas mixture with the seed almost completely ionized.

It is clear that the question of instability at high Hall parameters must be investigated experimentally. Such work is in progress.

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Fig 9 Amplification diagram for a typical argon-potassium plasma. The perturbations of electron concentration, etc., vary as $\exp \left[ i(\lambda X + \omega T/2) \right]$ is the electronic collision frequency. The diagram is periodic in $\theta$ with period $\pi$. 

For the foregoing example, $\nu/n_0 K_0 = 0.62 \times 10^9$, and the corrections to $\omega(0)/\nu$ and $\omega(\nu) + K_s u$ are 6.8 and 10.5%, respectively. 

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$K_s = 2 \times 10^5 \text{ cm}^{-1}$

$T_s = 2500 \text{ K}$

$S = 223$

$\nu = 2.5 \times 10^9 \text{ cm}^{-1}$

$\frac{m_0}{m_0} = 2.7 \times 10^{-5}$

Damped waves
Nonequilibrium Ionization Due to Electron Heating: II. Experiments

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An experimental study has been conducted of nonequilibrium conductivity of mixtures of argon and potassium. A pure plasma is produced in thermal equilibrium at temperatures up to 2500°K by means of a steady-flow tantalum heat exchanger. Conductivity measurements made with probes between the electrodes indicate that the two-temperature conductivity law proposed is valid for electron concentrations above about 10^{20} cm^{-3}. The effective energy loss factor δ depends on the size of the apparatus and was found to be 10 for the test sections used in these experiments. At lower electron concentrations, the theory is in error, probably because the electron energy distribution is very non-Maxwellian. In this region, the electric field goes through a maximum with increasing current density, suggesting that it may be difficult to make the transition to the higher electron density nonequilibrium state from the initial low electron density equilibrium state in an MHD generator.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>electric field, v/cm</td>
</tr>
<tr>
<td>E'</td>
<td>electric field as seen in the moving gas frame of reference</td>
</tr>
<tr>
<td>j</td>
<td>current density, amp/cm²</td>
</tr>
<tr>
<td>k</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>δ</td>
<td>molecular weight</td>
</tr>
<tr>
<td>ni</td>
<td>mass flow rate, g/sec</td>
</tr>
<tr>
<td>S</td>
<td>collision cross section, cm²</td>
</tr>
<tr>
<td>T</td>
<td>temperature, °K</td>
</tr>
<tr>
<td>V</td>
<td>voltage, v</td>
</tr>
<tr>
<td>α</td>
<td>slope, (d loge/ d log T)na (in two-temperature conduction law)</td>
</tr>
<tr>
<td>γ</td>
<td>slope, (d loge/ d log T)na (in two-temperature conduction law)</td>
</tr>
</tbody>
</table>

δ = energy loss parameter
σ = electrical conductivity, mho/cm

Subscripts

A = argon
a = gas (atom)
c = coulomb
e = electron
ex = excitation
K = potassium

Introduction

The theory of nonequilibrium conduction proposed in Ref. 1, and further developed in Part I of this paper, was tentatively confirmed by experiments reported in Ref. 1. Experiments reported in Ref. 2, also, indicated reasonable agreement with the theory. There were various uncertainties in these experiments, including purity and composition of the test gas, the uniformity of the current density, and electrode effects.

The experiment to be described here was designed to eliminate these uncertainties and to determine the range of applicability of the proposed two-temperature conduction...
law of Ref 1:
\[ j^*/j^0 = (T_a/T^*0)^{\gamma(1-\alpha)}(E^*/E^0)^{(1-\alpha)} \]

where \( j^*, E^*, \) and \( T^* \) are reference conditions and where it is shown in Part I that \( \gamma \) and \( \alpha \) are, in general, functions of both \( T_a \) and \( T \) for a given gas.

It is of particular interest to determine that region over which \( \gamma \) and \( \alpha \) can be assumed to be independent of \( T \) and, hence, functions only of the gas temperature \( T_a \). This leads to an especially simple and useful form of the two-temperature conduction law. In addition, it is desired to determine what, if any, empirical corrections to the conduction law are necessary to permit it to be used for quantitative engineering calculations. Therefore, the emphasis in the experimental work has been on the measurement and control of the thermo-dynamic condition and composition of the plasma to make it possible to determine accurately the relationship between its bulk conductivity and the current density.

To insure the control of the composition and temperature of the plasma, it is produced in a continuous-flow heat exchanger, where it comes in contact only with stainless steel at the lower temperatures and tantalum at the higher temperatures. This is particularly important, since a very slight contamination with polyatomic impurities could greatly influence the electronic energy loss. The heat exchanger also insures the absence of extrathermal ionization in the plasma prior to its entrance to the test section. Such residual ionization may be troublesome with arc heaters. Finally, by designing the heat exchanger carefully, it has been possible to obtain a direct and accurate measurement of the plasma temperature.

The conductivity has been determined by probe measurements between the electrodes, so that uncertainties due to electrode effects are eliminated. Visual observation of the plasma has indicated that the current is quite uniformly distributed.

In addition to these basic measurements, preliminary determinations of the excitation temperature of the first excited states of a trace impurity of sodium have been made by the classical line-reversal technique. No great accuracy is claimed for these measurements; however, they do show a reasonable agreement with theory at high electron concentrations, where Maxwellization of the electrons is expected (see Part I).

All of the work to be reported here has been done with mixtures of argon and potassium. The behavior of the conductivity, as predicted by the two-temperature conduction law, is shown in Fig. 1 for the range of temperatures and current densities attained in the experiments. One point particularly should be noted: The theory predicts that the slope of the lines of \( \log \sigma \) vs \( \log j \) (denoted \( \alpha \) in Ref 1 and Part I of this paper) should be dependent only on the ratios of gas temperature to ionization energy and gas temperature to electron temperature. If the electron temperature exceeds the gas temperature by more than about 10%, the predicted value of \( \alpha \) is almost constant at about 0.8. This provides a rather sensitive check of the theory.

**Description of Test Facility**

The test gas heater (Fig 2) is basically a counterflow heat exchanger. A mixture of \( N_2 \) and \( A \) is heated by an arc source to several thousand degrees Kelvin and then flows down the annular region between a graphite outer tube and a \( \frac{1}{2} \)-in.-diam tantalum inner tube and is exhausted as shown. The graphite assembly is fixed only at the upper end, where it attaches to the arc, and is free to expand downward. The tantalum tube is supported by a graphite sleeve at the top and is free to expand independently both above and below this sleeve. With these provisions, the tantalum tube can be used for many runs, even though it becomes carburized and very brittle after the first run.

A very pure test gas (welding grade argon with impurities of less than 30 ppm in the test to be described) is first put through a preheater consisting of a molten lead bath in a stainless-steel pot. A stainless-steel potassium boiler (Fig 3) is immersed in a second pot, shown on the right in Fig 2. The temperature of this lead pot is carefully controlled at a constant value of about 850°C throughout the experiment. This yields a vapor pressure of the liquid potassium of over 2 atm, resulting in choked flow through a 0.004- to 0.006-in.-diam orifice at the top of the boiler. The very low potassium flow rates, ranging from about 0.002 to 0.005 g/sec, required to obtain the desired seeding concentrations are obtained in this way.

The argon mixes with the potassium vapor in the top of the potassium boiler, and the mixture flows into the tantalum tube at the base of the heat exchanger. In order to vary the seed fraction during a run, the argon flow rate is varied. To avoid disturbing the potassium boiler temperature, the argon preheater is separated from the boiler heater, as shown in Fig 2. Typical argon mass flow rates range from about 0.1 to 1.0 g/sec.

The seeded test gas flows up through the tantalum tube in the counterflow heat exchanger and enters a test section.
at any desired temperature between 1000° and 2500°K (depending primarily on the arc power). This results in flow velocities of at most about 100 m/sec in the test section.

The entire heat exchanger is packed with graphite-felt insulation and supported in a steel pipe and frame assembly. The insulation and the test section are purged with N₂ before and during each run to prevent oxidation of the high-temperature parts. A pyrometer views the interior of a cavity which is essentially isothermal and through which the tantalum tube passes. In this way, the gas temperature is measured within limits that will be given later.

The test facility was designed to operate at up to 2500°K. As many as five runs of about 3-hr duration each have been made at about 1500°K before any maintenance of the facility was required. The normal maintenance required at this stage consists of replacement of the tantalum tube and possibly some of the smaller graphite parts connecting the arc assembly to the heat exchanger.

This test facility can accommodate a wide variety of experiments with the minimum modification. To date, five different test-section configurations have been employed on conductivity, electrode, and heat-transfer tests.

**Fig 3** Stainless-steel boiler used for injecting very small, precise flows of alkali metal seed gases into noble gas flows.

**Fig 4** Test-section configuration 3A used to collect accurate bulk conductivity data for noble gas-alkali metal plasmas at gas temperatures from 1000° to 2200°K.

**Fig 5** Wiring diagram showing test section, power supply, and electrical connections.

### Test Sections

The test-section configuration used to obtain the experimental results discussed in the next section is the result of considerable experience gained with other configurations in early tests. The three basic test-section configurations which have been tried in the conductivity experiments will be described briefly in order to bring out the principal features of the final configuration.

All tests but those with the second configuration were run with the test section mounted on top of the heat-exchanger assembly as shown in Fig. 2. The second configuration was mounted in the upper part of the heat exchanger in the open region near the pyrometer viewpoint.

The first test-section configuration consisted of two circular tantalum electrodes mounted in a boron nitride insulator assembly. Heater elements were incorporated in this assembly to ensure that the gas would not cool excessively and that the potassium would not condense on the insulator walls. The basic measurements included the voltage across the electrodes, the current (which flowed up or down the channel), and the various temperatures and gas flow rates.

The results with this test section were very promising. However, it was felt desirable to run a series of tests with the entire test section at the gas temperature, and so in the second configuration the insulator heaters were eliminated and the entire test assembly installed in the very hot upper part of the heat exchanger. Circular tantalum electrodes were once again used, but, in addition, three tungsten probe wires were inserted across the test section to allow voltage measurements away from the electrode drop regions. Two thermocouples were added to allow direct measurement of the gas and insulator temperatures.

Several complicating factors arose with this arrangement. The experimental results showed strange behavior up to current densities of about 1 amp-cm⁻² which could possibly have been a result of some electrical conduction paths through the very hot boron nitride insulators (about 2000°K in some runs).

Therefore, the third test section, shown in Fig. 4, was once again mounted on top of the heat exchanger, and nichrome wire heating elements were installed at the top and bottom. The test-section insulator temperature was monitored continuously with a thermocouple.

This test section incorporated all of the refinements evolved up to this point plus a gap in the boron nitride insulator. This was inserted for two reasons. First, it eliminates any insulator or wall-conduction paths across the gap. Second, it permits direct observation of the uniformity of the plasma. In addition, it allows direct measurements of electron "temperature" using the sodium line-reversal technique.

This third test section configuration and mounting arrangement was the one used, with minor modifications of spacing and electrode configuration, in all of the tests to be discussed in this paper. Each electrode consisted of four 0.015-in.-diam tungsten wires arranged in a star or cross pattern across the channel. Each probe consisted of a single 0.010-in.-diam tungsten wire strung across the diameter of the test section at the appropriate location. Three different arrangements of probe wires and gap location were tried. All three variations gave essentially the same test results.
**Experimental Results**

The basic experimental results are presented in this section, along with the general conclusions regarding their agreement with the two-temperature conduction law.

Almost two dozen runs have been made with argon seeded with various mole fractions of potassium from about 0.1% to about 1% and over a range of gas temperatures from 1400\(^\circ\)K to about 2000\(^\circ\)K. Several of these runs are under almost identical conditions of temperature and seeding concentration and show excellent reproducibility. Four representative runs at about 1500\(^\circ\)K gas temperature and four different seed fractions have been selected for detailed quantitative analysis in this paper. The conclusions reached, however, are valid for all of the test results obtained to date.

The basic electrical measurements in these experiments consisted of the voltages between the electrodes and each probe wire and the current flowing in the test section. Direct plots of the voltages vs electrode and probe wire locations, such as those shown in Fig. 6, yield the complete voltage variation in the test section for each current density.

Using these results, it is possible to separate clearly the volume characteristics of the plasma from the electrode surface and electrode drop effects. All conductivity calculations presented in this paper are based on the average electric field in the gas well away from the electrodes.

It should be noted that the electric field between probe wires in the boron nitride insulator is essentially the same as that in the gap, except at the lowest current densities. It is observed that the electric field tends to increase, and hence the electrical conductivity tends to decrease, as the gas flows from the bottom to the top of the test section for the lower current densities. This is true for both polarities of the bottom electrode and is most probably due to a slight cooling of the plasma by radiation and conduction. At the higher current densities, Joule heating tends to counter these losses, and the electric field tends to decrease less with distance along the test section.

The important conclusion to be drawn from these voltage profiles is that the conductivities calculated from the electric field in the insulators and in the gap are essentially the same. In addition, they are clearly free of electrode effects.

The electrode drops can be seen to range from almost zero up to about 10 v for the higher current densities. Note that straight lines have been drawn between data points, but that some dotted lines have been included to suggest that the electrode drops may actually occur in a shorter distance. No further interpretation of the electrode effects will be included in this paper, since the electrode geometry in these test sections was not well suited to analysis. Special electrode investigations are underway with more suitable geometries and will be reported in the near future.

The experimental electric field vs current density characteristics for four representative runs are shown in Fig. 7. These characteristics definitely have the shape predicted by the two-temperature conduction law at the higher current densities, but they fall above the theoretical curves for \(\delta = 2\). A transition region is apparent at the low current densities, where a peak in required electric field occurs. A possible explanation of this will be given in the next section.

The corresponding conductivity curves are shown in Fig. 8 for the same gas temperatures and seeding fractions.

![Fig 6 Typical experimental voltage variations along the test section both in the insulators and in the gap for two seeding fractions of 0.65 and 0.15% Note different polarities relative to the gas flow direction](image)

![Fig 7 Experimental electric field vs current density characteristics for four different seeding fractions but all at about 1500\(^\circ\)K gas temperature using an argon gas seeded with potassium at 1 atm total pressure](image)
represent runs made over a period of many months during which time excellent reproducibility of the results was obtained above about 0.3 amp/cm². Below this level, the transition region corresponding to the hump in the voltage-current characteristics of Fig. 7 varied somewhat from run to run. It should be noted that the peaks of the voltage-current characteristics of Fig. 7 correspond to the region near the minimum conductivity points of Fig 8.

From the practical viewpoint, the low current density data are very important in one particular respect. They indicate that the electric field in the plasma must rise above its final value as the plasma passes from the equilibrium to the nonequilibrium state. Since, in some practical MHD devices such as generators, the electric field is limited by the product of flow velocity and magnetic field, it may be difficult to attain the desired operating point.

Comparison with Theory

In order to facilitate comparison of the theory and the experimental runs at different potassium seeding fractions, it is useful to change variables so as to eliminate dependence on \( n_K/n_A \). Based on the theory, this can be achieved by using normalized values of \( j \) and \( \sigma \) defined as follows:

\[
\tilde{j} = j \left[ \frac{(n_K/n_A)_{ref}}{(n_K/n_A)} \right]^{1/2} \]

\[
\tilde{\sigma} = \sigma \left[ \frac{(n_K/n_A)_{ref}}{(n_K/n_A)} \right]^{1/2} \left[ 1 + \frac{(n_K/n_A)(S_K/S_A)}{1 + (n_K/n_A) \epsilon(S_K/S_A)} \right]
\]

A convenient and meaningful reference value for \( n_K/n_A \) is the optimum seeding fraction at a constant electron temperature. In order to find the optimum value of the seeding fraction, the conductivity equation can be differentiated with respect to seeding fraction holding \( T \) constant. This leads to the result that \( (n_K/n_A)_{opt} = S_A/S_K \).

As a first estimate to facilitate normalization, constant values of \( S_A = 0.7 \times 10^{-8} \text{ cm}² \) and \( S_K = 0.25 \times 10^{-13} \text{ cm}² \) have been selected, giving \( (n_K/n_A)_{opt} = 0.0028 \).

### Table 1. Estimated errors in the basic variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimated Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductivity ( \sigma )</td>
<td>( \pm 9% )</td>
</tr>
<tr>
<td>Potassium mole fraction ( n_K/n_A )</td>
<td>( \pm 12% )</td>
</tr>
<tr>
<td>Gas temperature ( T_A )</td>
<td>( \pm 60\°C )</td>
</tr>
<tr>
<td>Potassium excitation temperature ( T_x )</td>
<td>( \pm 100\°C )</td>
</tr>
</tbody>
</table>

The normalized \( j \) is theoretically a function only of the gas temperature and pressure, the seed gas ionization energy, the energy loss parameter \( \delta \), and the electron temperature in regions where hard sphere collisions dominate. For the same experimental conditions of gas temperature and pressure, all experimental points at the same electron temperature should fall at the same value of \( j \).

Similarly, the normalized \( \sigma \) is a function only of the gas pressure, the argon collision cross section, the seed gas ionization energy, and the electron temperature. Therefore, all experimental points at the same electron temperature should fall at the same value of \( \sigma \). As a result, on a plot of log(\( \sigma \)) vs log(\( j \)), all experimental points should fall on a single curve, in the region where hard sphere collisions dominate.

The normalized conductivity curves are shown in Fig. 9. The theoretical curves all fall exactly on a single curve in the region shown.

The experimental curves fall remarkably close to a single curve at the higher current densities, leading to several important general conclusions. First is that the theory predicts the correct variation of conductivity with current density above about 0.3 amp/cm². In this region, it can be seen that the experimental slopes \( \alpha \) are very close to those predicted theoretically, providing strong support for the two-temperature conduction law. In addition, these results indicate that the average collision cross sections selected for argon and potassium are reasonably valid, and that the measured values of \( (n_K/n_A) \) are quite accurate.

A detailed error analysis is given in the Appendix. The estimated errors in the basic variables are collected in Table 1. Comparison of these estimates with the experimental results in Fig. 9 indicates that the overall accuracy is, in fact, better than these estimates would indicate.

The major discrepancy between the theory and the experimental results for the higher current density region is that the experimental conductivities are consistently about one-half the theoretical values. If we adjust \( \delta \) to a value of about 10, the agreement is then excellent, as shown in Fig. 9.

There are several possible explanations for this apparent increase in the electronic energy loss. The most obvious is impurities in the test gas. This is ruled out on the grounds that the impurity level of the welding-grade argon is too low to be significant, whereas the excellent reproducibility of the results virtually eliminates the possibility that contamination is significant.

The most probable explanation of the increase in \( \delta \) is that the electron gas loss energy by radiation. Estimates of this energy loss have been made by Byron and by Lutz. The most that can be said at the present time is that the pre-

![Fig 9 Normalized conductivity vs current density curves for the four experimental runs of Fig 8. Note that a theoretical curve for \( \delta = 10 \) is the best fit to the data.](image-url)
dicted energy losses are of the right order to explain a δ of 10 in apparatus of the present size. If this is the correct explanation, then the effective δ should approach 2 as the size of the apparatus increases or as the optical density of the plasma increases.

Close examination of Fig. 9 shows that, indeed, the curve for \( n_0/n_a = 0.01 \) does have a higher slope than the curves for lower potassium concentrations. It is tempting to attribute this to the greater optical density that results from the larger potassium concentration; however, there are other factors that could explain the trend. Among these are joule heating of the gas and the variation of collision cross sections with electron energy (e.g., Ref 7). The question could be resolved by a direct measurement of the intensity of the emitted radiation.

For lower current densities, the experimental results depart substantially from the theory. As the electric field is raised from zero to a few tenths of a volt per centimeter, an unexpectedly large current flows in the region near the origin of Fig. 7. This corresponds to the anomalously high conductivities of Fig. 8 at very low current densities of about 0.01 to 0.10 amp/cm². No explanation can be offered for this behavior at the present time.

As the electric field is raised further to values of one or more volts per centimeter, the current density does not increase as rapidly as the theory predicts. As a result, in the region from 0.1 to about 0.3 amp/cm², the conductivity is depressed below the theoretical values. It is very likely that, in this region, the electrons have not yet achieved a Maxwell-Boltzmann distribution. As the current density is raised from zero, the high-energy tail is depleted faster than it can be filled because ionization must be done by these more energetic electrons. The low electron-electron collision frequencies at these low current densities prevent the electron density from building up as fast as the theory predicts. Thus, the electric field required to pass a given current density would rise above the theoretical values, as indicated by the hump in Fig. 7.

We expect a Maxwellian distribution when

\[ n^* \gg \delta(m/m_a)n(n_aS_a + n_aS_e) \]

The electron energies most important for ionization are about 2 eV, where \( S = 10^{-13}, S_a = 10^{-13}, \) and \( S_e = 3 \times 10^{-14} \text{ cm}^3 \). Using \( \delta = 10 \) and \( m/m_a = 10^{-2} \) leads to the requirement that \( n/n_a \gg 10^{-2} \) for Maxwellization of the electrons. For typical seeding fractions, \( n_0/n_a \approx 0.005 \), and at atmospheric pressure and 1500°C gas temperature, the criterion implies a required electron concentration, \( n \gg 5 \times 10^{13} \text{ cm}^{-3} \).

Calculation of the electron densities using the experimental results yields lines of constant electron density as indicated in Fig. 8. The line of electron density equal to 10¹² cm⁻³ falls very close to \( j = 0.3 \text{ amp/cm}^2 \), where the transition can be seen to occur.

This close correspondence between the theoretical predictions of the transition of Part I of this paper and the experimental results lends weight to the theory that the non-Maxwellian distribution of the electrons contributes to the anomalous behavior observed at low current densities.

Initial attempts to measure electron temperature directly employed the sodium line-reversal technique using the standard arrangement shown in Fig. 10. The temperature at which the sodium D line is reversed is assumed to be a measure of the excitation temperature of the first excited state of the sodium atoms. This will be equal to the electron temperature if the electron-electron collision frequency is high enough to reach equilibrium for the entire electron gas. The line reversal was obtained only up to excitation temperatures of about 2600°C, since the tungsten strip-lamp light source used limited the maximum effective source tempera-

![Fig 10 Line reversal apparatus used to measure excitation temperature in the test-section gaps of a trace of sodium added to the argon-potassium flow](image)

![Fig 11 Preliminary sodium excitation temperature measurements compared to theoretical electron temperatures as a function of the normalized current density](image)
Conclusions

It is concluded from the previously given evidence that the two-temperature conduction law provides a satisfactory description of the behavior of a noble gas-alkali metal plasma at electron concentrations greater than about $10^{13}$ cm$^{-3}$, in the absence of a magnetic field.

For argon and potassium mixtures in apparatus with a 1-cm-long length, the effective energy loss parameter $\delta$ is about 10 rather than the ideal value of 2. It seems likely that this increase is due to radiative energy loss, so that the effective $\delta$ should be smaller in larger apparatus.

At low electron concentrations, the simple theory is inadequate, probably because the electrons have a highly non-Maxwellian energy distribution. The electric field required to produce a current in the plasma displays a maximum value as the current is increased from very low values. This suggests that, in applications such as the MHD generator, there may be difficulty in passing from the equilibrium state of the plasma at entry to the desired nonequilibrium state by the generator.

As noted in Part I, there seems to be no reason to doubt that the relationship between conductivity and current density verified here will also apply in magnetic fields. However, the question of instabilities does arise at large values of the Hall parameter. These questions are currently being studied experimentally in a modified version of the apparatus just described.

Appendix: Analysis of Measurement Techniques

The basic electrical measurements consisted of the voltages between the electrodes and each probe wire, and the current through the test section. These were measured using Hewlett-Packard 412A vacuum tube voltmeters, which have a full-scale accuracy of ±3%. The electrode and probe wire locations were known to within about ±0.1 mm or within ±2-3%.

The electric field $E$ used in Fig 7 is based on the average value in the gap between probes $B$ and $E$. Based on the foregoing figures, this average electric field is estimated to have an inaccuracy of about ±6%, attributable primarily to the Hewlett-Packard 412A. The current density $j$ is computed from the ratio of the total current to the cross-sectional area of the test-section flow passage. It was observed in all runs that a diffuse plasma flowed through the gap in the test section. Hence, the current density based on the area of the flow passage represents a good average value. When the expected radial temperature variation for laminar pipe flow is taken into account, the average value differs from the maximum value of current density in the center of the flow by at most a factor of 2.

Ignoring the radial variation of the current density across the test section, the estimated inaccuracy in the average value of the current density is about ±6%. This is due primarily to the stated inaccuracy in the Hewlett-Packard 412A used as an ammeter.

It can be seen that the combined inaccuracies in both $E$ and $j$ are not large enough to change the basic shape of the voltage-current characteristics of Fig 7. In fact, the scatter in the data points is probably a good indication of the effect of varying inaccuracies on different scales of the vacuum tube voltmeters. The scatter is approximately equal to the inaccuracy estimates previously mentioned.

The electrical conductivity $\sigma$ is calculated from Ohm's law and is equal to the ratio of the average current density $j$ and the average field $E$. The inaccuracies in $E$ and $j$ carry over in the calculation of $\sigma$. Random inaccuracies in $E$ would tend to show up as scatter of the data points. This amount of scatter does show up in Fig 8. However, random inaccuracies in $j$ would tend to shift the data points up or down along the experimental curve at the higher current densities, since the slope of the curve is close to unity. This would tend to cancel out the effect of random $j$ inaccuracies on the loge vs logj plot above about 0.3 amp cm$^{-2}$.

Although the random inaccuracies in $E$ and $j$ can be seen to have only a second-order effect on the experimental conductivity curves, errors in the estimation of either the gas temperature $T_e$ or the seeding fraction $n_k/n_A$ can have a first-order effect. That is, errors in $T_e$ or $n_k/n_A$ will be manifested as a systematic shift or deviation of the experimental curve from the theory. It is, therefore, vitally important to have an accurate estimate of these inaccuracies.

The gas temperatures were inferred from measurements of the temperature of the cavity at the top end of the heat exchanger (see Fig 2) using a Leeds and Northrop optical pyrometer. Earlier tests using a tungsten-rhenium thermocouple inserted directly into the plasma in the test section revealed that the thermocouple temperature was about 50°C below the pyrometer temperature. This empirical correction was included in the gas temperature value quoted.

The error in this empirical correction is difficult to estimate, since the emissivity of the thermocouple is not known. The calibration of the tungsten-rhenium thermocouple was given to ±2% or ±30°-40°C. Thus it is felt that the gas temperatures are within ±50°C of the quoted values.

Based on the two-temperature conduction law, a systematic error of 50°C in the gas temperature would shift the conductivity curves of Fig 8 by about 6-10% at the low current densities and by about 4-6% at the high current densities. This would be a small but noticeable shift in the loge vs logj curves.

The mole fraction of potassium is calculated from the following equation:

$$n_K = \frac{n_k}{n_A} = \frac{3kA}{3kA} = \frac{n_k}{n_A}$$

since the molecular weights of argon and potassium are essentially the same.

The potassium mass flow rate estimate was based primarily on the average value computed from the total weight of potassium loaded in the boiler and the total run duration. The potassium boiler was maintained at constant temperature throughout the run to within ±4°C. This should result in about ±3% variation in mass flow rate through the choked orifice of the boiler. The major inaccuracy is due to the uncertainty in the estimate of the run duration which was about ±15 min or about ±10% of the average run duration.

The argon flow rate was measured with a Fisher-Porter flowmeter of the spherical float-in-a-tapered tube type. This has an accuracy of ±3% of full scale, resulting in average errors around ±6% for flows around half scale.

The over-all inaccuracy in the calculation of the potassium mole fraction is taken as the square root of the sum of the squares of the inaccuracies of each of the independent error sources. This results in an estimated inaccuracy of ±12% in the mole fraction estimate. Based on the theory, a systematic error in $n_k/n_A$ of ±12% would shift the conductivity curves of Fig 8 by about ±6%.

Comparing this shift in the conductivity curves with the shift due to gas temperature errors, it can be seen that an error in mole fraction of 12% has approximately the same effect on the conductivity curves as an error in gas temperature of about 50°C. These two sources of error are, therefore, of about equal importance in causing small but noticeable shifts in the conductivity curves (Fig 8). Thus, it is not possible to separate the effects of these two error sources at this time. However, the total effect is seen to be well within the previously given inaccuracy estimates from Figs 8 and 9.

The excitation temperature was measured using the standard sodium line-reversal technique employing the optical arrangement of Fig 10. The tungsten strip lamp tempera-
titure was calibrated using an optical pyrometer, and appropriate color and optical path corrections were made (see, e.g., Refs 8-10) The optical path correction was determined by first calibrating the lamp alone and then recalibrating through the first lens and glass jar This was a very important contribution to the optical corrections, amounting to over 5% reduction in effective lamp temperature due to the low optical quality of the bell jar and lens The over-all optical corrections ranged from about 100°C at the low electron temperatures up to over 200°C at the high electron temperatures The corrected values are the solid curves shown in Fig 11

There were several uncertainties in the excitation temperature measurements The principal one was the range of lamp temperatures over which reversal occurred. The sodium D line would start reversing at the top of the image (corresponding to the top of the gap in the test section) The line would not be completely reversed, however, until the temperature of the lamp was raised about 200°C higher The data points plotted are average values corresponding to half the line reversed This leads to an uncertainty on order of ±100°C for the excitation temperature in the gap Other errors due to pyrometer calibration and reading uncertainties were much smaller than this figure

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