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Shock wave annihilation by MHD action in supersonic flow. Quasi one dimensional steady analysis and thermal blockage

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ABSTRACT. — The *shock wave cancellation* by the action of a *Lorentz force field* is described by a *steady quasi one dimensional* and *isentropic model*. The shock structure is eliminated if the self crossing of the characteristic lines can be prevented. It is shown that ρ, T, *p* are constant along a streamline. A criterion which prevents the *thermal blockage* is established. Numerical results are given concerning the parameter range required for the design of a *shock tube* experiment.

1. Introduction

The phenomenon of shock wave cancellation has not been greatly studied. Shock waves appear in supersonic flows as soon as a wall induces even moderate direction variations in such flows. They could be considered equally well as a disturbing or an advantageous phenomenon. However, the action of a crossed field $\mathbf{J} \times \mathbf{B}$, in a flow may serve to guide the fluid in its passage around a obstacle. Our purpose here, is to show that in special conditions, these shock waves can be avoided. For the moment, this approach is purely theoretical, but should have some aeronautical applications.

A one-dimensional steady-state investigation has been undertaken by Sutton (1965), who studied a plasma flow in a magneto-hydro-dynamic channel. He has shown that, if the force field remains constant with respect to a flow parameter such as the velocity, the pressure or even the mass density, the fluid could pass through a convergent channel without being shocked. The case of a constant-area channel in which the fluid is decelerated by a crossed field has been carried out by Resler and Sears (1956). These authors showed that special conditions of velocity and load factor allow the flow to decelerate from a supersonic regime to a subsonique regime without the formation of a shock.

Experimental work has also been performed by Petit (1983) around a blunt body (cylinder) placed in a salt water flow. With an appropriate crossed field, he has obtained the annihilation of the front wave, and the reattachment of the boundary layer leading to a decrease in the turbulent wake. Furthermore, a decrease in the wave drag around the body was obtained. Even a propulsive action has been obtained with such a device.

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When a blunt body is placed in a supersonic gas flow, two domains can be distinguished, the supersonic one, and the subsonic one. First, to avoid the problems involved in this subsonic region, we will confine our attention to the flows around smooth bodies such as bumps.

Also, we have been led to study a quasi-onedimensional steady-state model. This model has been developed by taking into account the real gas effects and is based upon the theoretical work of Norman (1965).To describe shock tube experiments, the following conditions are assumed to be satisfied:

H1 : The plasma is considered as a non-viscous fluid;

H2 : The excitation and radiative processes can be neglected in the plasma;

H3 : The plasma is close to thermodynamic equilibrium ($T_e = T_g$):

H4 : The ionization can be derived from Saha's law.

We shall show that the Joule effect can be neglected too.

This work is based upon the following conditions which have been suggested by Petit:

a supersonic flow with a Mach number M is described by a System of equations (Euler's equations) which become in this case a hyperbolic system. From these equations, we can work out characteristic directions linked to the flow, which correspond to the Mach waves for the case of a bidimensional flow. We know [Carrière (1957)], [Courant et Friedrichs (1948)] that shock wave formation is theoretically associated with the self crossing of these lines in the real space of the real space of the fluid. On the other hand, if one could preserve the parallelism of these lines, the shock wave must disappear, near the body, and at infinity. Such a flow can be obtained by an appropriate force field, which is calculated with respect to this parallelism condition.

2. Overall equations, bidimensional steady-state approach

Fontaine (1973, 1971) and Forestier (1973), have studied the magneto-hydro-dynamic interaction on the supersonic argon flows produced by a shock tube ($p = 1$ bar, $T = 10{,}000$ K). According to their work, we can assume that the following conditions are fulfilled:

H5 : The plasma is a neutral medium;

H6 : The magnetic Reynolds number is low and the induced magnetic field negligible. This assumption is checked in Chapter 6 where it is shown that $R_m < 0.3$.

Using these conditions, for a steady-state flow, Maxwell's equations lead to simplied forms where the electric field, the magnetic field and the plasma flow are no longer linked:

$$\nabla . \mathbf{E} = 0 \tag{1}$$

$$\nabla \times \mathbf{E} = 0 \tag{2}$$

$$\nabla . \mathbf{B} = \mathbf{0} \tag{3}$$

$$\nabla \times \mathbf{B} = 0. \tag{4}$$

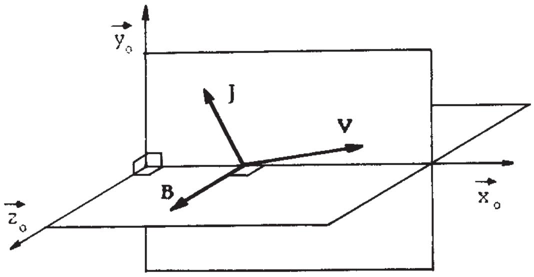

Fig. 1. — Crossed field.

We will consider the laboratory frame of reference $R_0(x_0, y_0, z_0)$ shown on Fig. 1 and a force field configuration in which the magnetic field is perpendicular to the plane of **J** and **V**.

A first formulation of the conservation equations is now carried out in this frame. The energy conservation is obtained from the internal energy expression proposed by Fontaine (1973). His own thermodynamic developments are based on the theoretical work of Norman (1965):

(5) $$\rho\left(\frac{\partial u_0}{\partial x_0}+\frac{\partial v_0}{\partial y_0}\right)+u_0\frac{\partial \rho}{\partial x_0}+v_0\frac{\partial \rho}{\partial y_0}=0$$

(6) $$u_0\frac{\partial u_0}{\partial x_0}+v_0\frac{\partial u_0}{\partial y_0}+\frac{1}{\rho}\frac{\partial p}{\partial x_0}=\frac{J_{y0}B}{\rho}$$

(7) $$u_0\frac{\partial v_0}{\partial x_0}+v_0\frac{\partial v_0}{\partial y_0}+\frac{1}{\rho}\frac{\partial p}{\partial y_0}=-\frac{J_{x0}B}{\rho}$$

(8) $$\frac{\partial u_0}{\partial x_0}(a^2-u_0^2)-u_0 v_0\left(\frac{\partial u_0}{\partial y_0}+\frac{\partial v_0}{\partial x_0}\right)+\frac{\partial v_0}{\partial y_0}(a^2-v_0^2)=\mathrm{TS}(u_0^2+v_0^2)^{3/2}.$$

In this frame of reference, the velocity vector has two components (u_0, v_0), and "a" represents the sound velocity in a real gas (relation 37). "TS" refers to the "Source Terms" which are due to the magneto-hydro-dynamic interaction. Contributions to "TS" arise from the Joule effect and the force field work, *i. e.*:

(9) $$\mathrm{TS}=\frac{\mathbf{E}^*.\mathbf{J}}{\rho V^3 A}-\frac{\mathbf{V}.(\mathbf{J}\times\mathbf{B})}{\rho V^3}=\mathcal{J}-\mathcal{F}.$$

Here, **E*** is the electric field seen by the electrons. The dimensions of $\mathcal{J}$ and $\mathcal{F}$ are the inverse of a length.

The expression, $\mathcal{J}$. L represents the ratio of the force field work to the kinetic energy of the flow;
and $\mathcal{F}$. L is the ratio of the Joule power converted into heat to the kinetic energy flux.

The real gas effects are taken into account in the expression "A" which is written (N., 1965):

$$A=\frac{1+Z_t}{\gamma-1} \quad \text{with} \quad Z_t=\frac{\alpha_i}{2}(1-\alpha_i)\left(\frac{5}{2}+\frac{E_i}{kT}\right). \tag{10}$$

The parameter α_i is the degree of ionization, γ the ratio of specific heats, and E_i the ionization energy of the considered element ($E_i/k=182{,}900$ for Argon). Z_t is defined as the compressibility ratio at constant temperature (N., 1965).

Under the assumption of steady flow, the Lagrangian co-ordinates $R(x, y, z)$ and the streamlines are linked together. Let φ be the angle between x_0 and x. Using these conditions:

$$\begin{cases} u=V, & du=dV, \\ v=0, & dv=V\,d\varphi \end{cases} \tag{11}$$

and the conservation equations are as follows:

$$\rho\left(\frac{\partial V}{\partial x}+V\frac{\partial \varphi}{\partial y}\right)+V\frac{\partial \rho}{\partial x}=0 \tag{12}$$

$$\rho V\frac{\partial V}{\partial x}+\frac{\partial p}{\partial x}=J_y B \tag{13}$$

$$\rho V^2\frac{\partial \varphi}{\partial x}+\frac{\partial p}{\partial y}=-J_x B \tag{14}$$

$$\frac{\partial V}{\partial x}(a^2-V^2)+V\frac{\partial \varphi}{\partial y}a^2=TS\,V^3. \tag{15}$$

Now, these relations can be expressed in a frame associated to the characteristic directions η and ξ, which respectively are the ascending and the descending characteristic

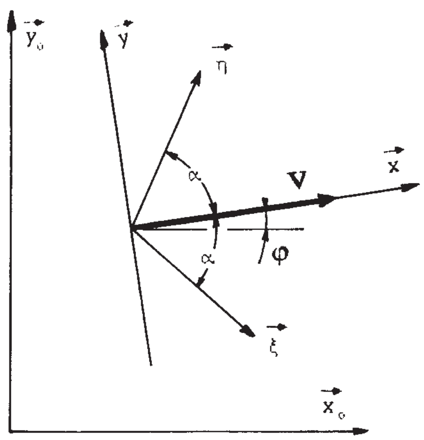

Fig. 2. — Different coordinate systems.

lines defined in Fig. 2. This change of co-ordinate system can be achieved by noting that for any given parameter H, we get:

$$(16)\qquad \begin{aligned} dx&=\cos\alpha(d\eta+d\xi) \quad \text{and} \quad \frac{\partial H}{\partial \eta}=\cos\alpha\frac{\partial H}{\partial x}+\sin\alpha\frac{\partial H}{\partial y} \\ dy&=\sin\alpha(d\eta-d\xi) \quad \text{and} \quad \frac{\partial H}{\partial \xi}=\cos\alpha\frac{\partial H}{\partial x}-\text{son}\,\alpha\frac{\partial H}{\partial y} \end{aligned}$$

where α is the Mach number defined as

$$(17) \text{ and } (18) \qquad \sin\alpha=\frac{1}{M} \quad \text{with} \quad M=\frac{V}{a} \quad \text{being the Mach number.}$$

It is now possible to build a quasi-one dimensional model from the system [(12) to (16)].

3. Quasi-onedimensional steady state model

The quasi-onedimensional condition is that all thermodynamic parameters (p.T.ρ) and flow parameters (V.φ.M) should remain constant along an ascending characteristic η, i. e.:

$$(19)\qquad \frac{\partial H}{\partial \eta}=0 \quad \text{hence} \quad dH=\frac{\partial H}{\partial \xi}d\xi.$$

In this case, the ascending characteristic lines are straight.

Then, the energy conservation Eq. (15) gives :

$$(20)\qquad \frac{1}{V}\frac{\partial V}{\partial \xi}=-\frac{2Ts}{\cos\alpha}-\text{tg}\,\alpha\frac{\partial \varphi}{\partial \xi}$$

and the continuity equation (12) becomes:

$$(21)\qquad \frac{1}{\rho}\frac{\partial \rho}{\partial \xi}=-\frac{1}{V}\frac{\partial V}{\partial \xi}+\frac{1}{\text{tg}\,\alpha}\frac{\partial \varphi}{\partial \xi}.$$

From the momentum equations, the pressure variations imply, first, a condition on the direction of the electric current density **J**:

$$(22)\qquad \frac{\partial p}{\partial \eta}=0 \quad \Rightarrow \quad \frac{J_x^2}{\sigma_0 VA\cos\alpha}-J_x B\sin\alpha-J_y B\frac{\sin^2\alpha}{\cos\alpha}+\frac{J_y^2}{\sigma_0 VA\cos\alpha}=0.$$

The quantity σ_0 represents the scalar electric conductivity. Just one physical solution is admitted by this equation of second degree:

$$(23)\qquad J_x=(-B\sin\alpha+\sqrt{\Delta})\frac{\sigma_0 VA\cos\alpha}{2}=f(J_y)$$

with:

(24) $$\Delta = B^2 \sin\alpha + 4\frac{J_y B \sin^2\alpha}{\sigma_0 VA \cos\alpha} - 4\frac{J_y^2}{(\sigma_0 VA \cos\alpha)^2} > 0.$$

These equations being established, we suppose the Joule effect to be negligible. Therefore, we consider the MHD interaction as quasi-isentropic. The Source Terms "TS" of the equation (9) then simplify to:

(25) $$TS = -\frac{J_y B}{\rho V^2}$$

and the relation (22) becomes:

(26) $$\frac{\partial p}{\partial \eta} = 0 \quad \Rightarrow \quad \frac{J_y}{J_x} = -\frac{1}{tg\,\alpha} = -\sqrt{M^2 - 1}.$$

This amounts to requiring the current density vector to be perpendicular to the ascending characteristics η: the force field must be parallel to this direction. In the case of a non-negligible Joule effect, the relation (23) indicates that the force field must balance the pressure gradient due to the temperature increase caused by the Joule effect. So the F_x component of the force field must be stronger than it is when the Joule effect is negligible, as shown in Fig. 3.

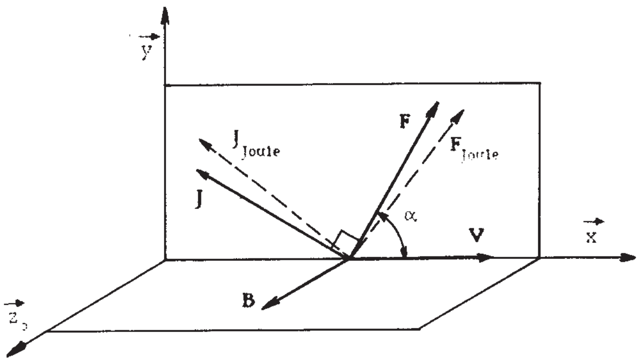

Fig. 3. — The orientation of the current density vector J and the force field F for MHD interactions with and without the Joule effect.

In the work that follows, we shall assume that the Joule effect is negligible.

There are two important remarks we must make concerning the pressure condition:

— The first one concerns the force field action. It must accelerate and push out the flow in the convergent channels, and must decelerate and attract the flow in the divergent channels. The bidimensionel studies developed in Chapter 4 lead to the same result, while explaining the role of each force field component.

— The second remark concerns the achievement of such a force field. To obtain a current which is not perpendicular to the electrode wall, *i. e.* not colinear with the electric field, the Hall effect must be considered. This phenomenon is induced by the rolling up of the free electrons in the magnetic lines and it is characterized by the Hall factor:

(27) $$\beta=\frac{e\mathrm{B}}{m_e \mathrm{V}_e}=\mu_e \mathrm{B}.$$

The electron collision frequency is represented by V_e, "e" means the electron charge, m_e the electron mass, and μ_e is a measure of the electron mobility. The orientation of

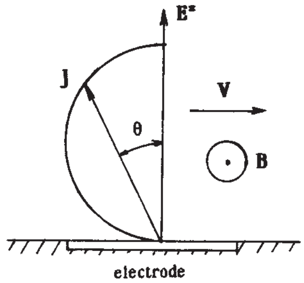

Fig. 4. — The action of the magnetic field upon the direction of the current density: the Hall effect.

the current density with respect to the electric field makes the angle θ such that $\mathrm{tg}(\theta)=\beta$ (see *Fig.* 4). To obtain such a current density direction, the magnetic field **B** must be taken as:

(28) $$\mathrm{tg}\,\alpha=\beta \quad \Rightarrow \quad \mathrm{B}=\frac{m_e \mathrm{V}_e}{e\sqrt{\mathrm{M}^2-1}}=\frac{\mathrm{tg}\,\alpha}{\mu_e}.$$

The magnetic intensity also depends on the parameters of the plasma and the flow.

Following these developments, the pressure along the ascending characteristics can be expressed in the form:

(29) $$\frac{\partial p}{\partial \xi}=\mathrm{J}_y \mathrm{B} \cos \alpha+\mathrm{J}_x \mathrm{B} \sin \alpha+\frac{\rho \mathrm{V}^2}{\cos \alpha} \mathrm{T}_s+\rho \mathrm{V}^2 \,\mathrm{tg}\,\alpha \frac{\partial \varphi}{\partial \xi}.$$

The relations (20), (21), (26) and (29), associated with a displacement along the x-axis such as:

(30) $$(d\xi)_{y=\mathrm{ost}}=\frac{dx}{\cos \alpha}$$

lead to the final expressions for the thermodynamic parameter variations.

$$\frac{dV}{V}=\frac{1}{\sqrt{M^2-1}}\left(\frac{2M^2}{\sqrt{M^2-1}}\frac{J_y B}{\rho V^2}dx-d\varphi\right) \tag{31}$$

$$\frac{d\rho}{\rho}=\frac{M^2}{\sqrt{M^2-1}}\left(-\frac{2}{\sqrt{M^2-1}}\frac{J_y B}{\rho V^2}dx+d\varphi\right) \tag{32}$$

$$\frac{dp}{p}=\frac{\gamma M^2}{(1+Z_p)\sqrt{M^2-1}}\left(-\frac{2}{\sqrt{M^2-1}}\frac{J_y B}{\rho V^2}dx+d\varphi\right). \tag{33}$$

According to Norman (1965), the equation of state for a real gas can be written:

$$\frac{dT}{T}=\frac{dp}{p}\frac{1+Z_p}{1+Z_t}-\frac{d\rho}{\rho}\frac{1}{1+Z_t} \tag{34}$$

with $Z_p=\alpha_i(1-\alpha_i)/2$, representing the compressibility factor at constant pressure.

The expression for the temperature variation becomes:

$$\frac{dT}{T}=\frac{M^2(\gamma-1)}{(1+Z_t)\sqrt{M^2-1}}\left(-\frac{2}{\sqrt{M^2-1}}\frac{J_y B}{\rho V^2}dx+d\varphi\right). \tag{25}$$

The system (31), (32), (33) and (35) allows us to describe the behaviour of the gas flow when we apply both a weak variation of the wall $d\varphi$, and a crossed field $\mathbf{J}\times\mathbf{B}$. When the force field does not exist, we recover the classical expressions of a Prandlt-Mayer fan (Carrière, 1957).

The shock wave cancellation is achieved if the parallelism of the characteristics originating from the wall is maintained, and this requires $d\alpha$ to be equal to $-d\varphi$, as shown in Fig. 5.

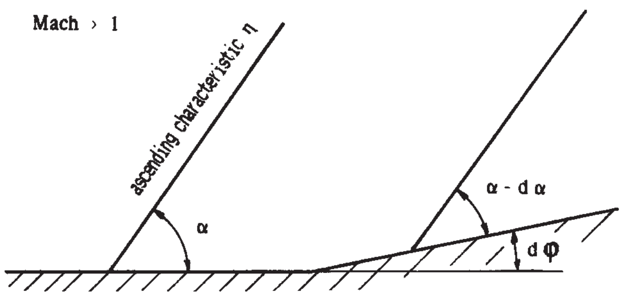

Fig. 5. — The parallelism of characteristics originating from a weatily-varying wall.

From the derivation of equation (17), we have:

$$\frac{dM}{M}=\sqrt{M^2-1}\,d\varphi. \tag{36}$$

Furthermore, the sound velocity in a real gas is, according to Norman (1965):

$$a=\sqrt{\frac{\gamma R T}{1+Z_p}}. \tag{37}$$

The real gas effects appear through the variations of γ and Z_p, and together with the differential relations (18) and (19), they lead to:

$$\frac{dM}{M}=\frac{dV}{V}-\frac{1}{2}\left(\frac{dT}{T}+\frac{d\gamma}{\gamma}-\frac{dZ_p}{1+Z_p}\right) \tag{38}$$

or:

$$\frac{dM}{M}=\frac{M^2(\gamma+2Z_t+1)J_y B}{(M^2-1)(1+Z_t)\rho V^2}dx-\left(\frac{M^2(\gamma-1)}{2(Z_t+1)}+1\right)\frac{d\varphi}{\sqrt{M^2-1}}-\frac{1}{2}\left(\frac{d\gamma}{\gamma}-\frac{dZ_p}{1+Z_p}\right). \tag{39}$$

The relative variations of $Z_p (0<Z_p<0.02$ for an Argon temperature smaller then 12,000 K) are neglected with respect to those of γ $(5/3<\gamma<1.275)$. The parallelism condition Eq. (35) leads to the force field criterion:

$$\frac{J_v B}{\rho V^2}dx=\frac{\sqrt{M^2-1}}{2}d\varphi+\frac{\sqrt{M^2-1}}{M^2(\gamma+2Z_t+1)}\frac{d\gamma}{\gamma}. \tag{40}$$

Remarking that:

$$J=\sqrt{J_y^2+J_x^2}=J_y\frac{M}{\sqrt{M^2-1}} \tag{41}$$

and that $d\varphi/dx$ is the inverse of the curvature radius of the wall R, the relation (40) becomes:

$$\frac{JBR}{\rho V^2}=\frac{M}{2}+\frac{R\sqrt{M^2-1}(1+Z_t)}{\gamma M(\gamma+2Z_t+1)}\frac{d\gamma}{dx}. \tag{42}$$

In the case of a perfect gas, the force field value must be such that:

$$\frac{JBR}{\rho V^2}=\frac{M}{2}. \tag{43}$$

Finally, the variations of the thermodynamic parameters are zero:

$$\frac{dT}{T}=\frac{dP}{p}=\frac{d\rho}{\rho}=0 \tag{44}$$

so that:

$$\frac{dV}{V}=\frac{dM}{M}=\sqrt{M^2-1}\,d\varphi. \tag{45}$$

In this way, the assumptions proposed by Sutton (1965) are found again to be true. That is, the shock wave cancellation requires one of the flow parameters to be constant. In this first study, the quasi-isentropic assumption for a perfect gas involves the parameters p, T and ρ to be constant. We note that in this case the condition is not an assumption but the result of the characteristic parallelism. The simplicity of these results is remarkable.

The possible variations of these parameters must occur to balance the variations in the middle caused by the Joule effect and the real-gas effect. For the conditions considered in Chapter 6, these effects remain moderate.

One of these results which is of some significance, is *the nullification of the wave drag.*

4. Bidimensional steady isentropic model for a perfect gas

In the quasi-onedimensional study, we have shown that the force field consists of two components F_x and F_y. In this following bidimensional study, the action of each component is considered.

In a flow where the force field $\mathbf{J} \times \mathbf{B}$ keeps the pressure constant, the shock waves, which correspond to the focussing of the pressure waves, can't appear. The isentropic assumption, justified by the negligible Joule effect, also involves the constancy of both pressure and density. Thus, once again we obtain the results of the quasi-onedimensional analysis.

The conservation equations can be written in the form:

(46) $$\frac{\partial \rho V}{\partial x} + \rho V \frac{\partial \varphi}{\partial y} = 0$$

(47) $$\rho V \frac{\partial V}{\partial x} + \frac{\partial p}{\partial x} = J_y B = F_x$$

(48) $$\rho V^2 \frac{\partial \varphi}{\partial x} + \frac{\partial p}{\partial y} = -J_x B = F_y$$

(49) $$\frac{\partial s}{\partial x} = 0.$$

The entropy equation takes the place of the energy one. Notice that:

(50) $$V^2 \frac{\partial \rho}{\partial x} = M^2 \frac{\partial p}{\partial x}$$

which together with (46) and (47) for a constant pressure, yields:

(51) $$J_x B = -\frac{\gamma p}{\sin^2 \alpha} \frac{\partial \varphi}{\partial x}$$

(52) $$J_y B = -\frac{\gamma p}{\sin^2 \alpha} \frac{\partial \varphi}{\partial y}.$$

So, it appears that the force field is directly linked to the shape of the current tubes, which are defined by their curvature $\partial\varphi/\partial x$, and their area variations, corresponding to $\partial\varphi/\partial y$.

Two simple analytic studies allow an analysis of the flow and the associated force field:

— The first study considers a flow where the stream lines build a network of convergent lines. This theoretical flow is shown in Fig. 6. The focal point O is a singular point

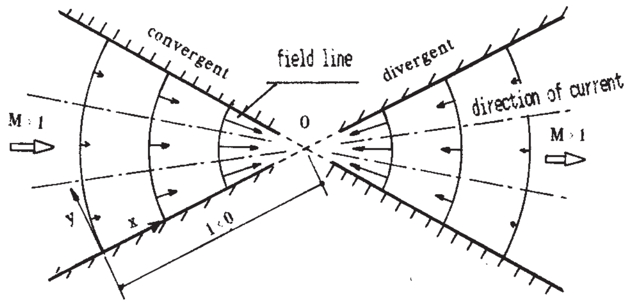

Fig. 6. — Force field for convergent-divergent flow at constant pressure.

where no real solution exists. The force field reduces to:

$$F_x = -\frac{\rho V^2}{1} \qquad (53)$$
$$F_y = 0$$

and the force field is oriented along the stream lines.

— In the second study, the flow is no longer subjected to area variations, only to direction changes. The stream lines are concentric circles of radius "r", shown in Fig. 7. The force field which must be applied is such that:

$$F_x = 0 \qquad (54)$$
$$F_y = -\frac{\rho V^2}{r}.$$

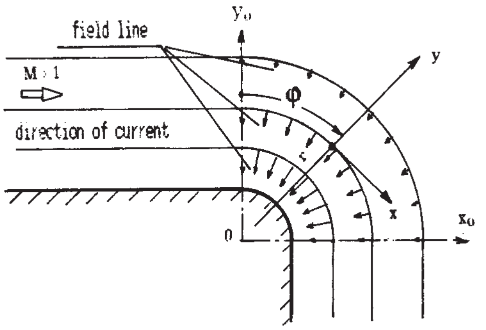

Fig. 7. — Force-field for circules flow at constant pressure.

and is in a direction perpendicular to the stream lines.

These bidimensional studies allow us to define, according to the quasi-onedimensional analysis, the two components of the force field. We consider a volume element "*dv*", subjected to three different forces (without taking into account the gravitation force) the inertial force, the pressure force, and the Lorentz force. To avoid the pressure variations, and in fact, shock wave formation, the force field $\mathbf{J} \times \mathbf{B}$ must exactly balance the inertial force, as shown in (53) and (54). The force field component F_x must balance the effect of the linear acceleration, and F_y must balance the centripetal acceleration due to the variations in the flow direction.

5. Thermal blockage analysis

A thermal blockage appears when the plasma is subjected to very strong electric currents which induce a very important Joule Effect. Fontaine (1973) achieved some discharges with an electric intensity close to 10^7 amperes/m^2 in the interaction channel of an Argon shock tube. The electrode section was about 5×10^{-3} m^2. These conditions allowed him to bring the thermal blockage to the fore.

From the above theoretical studies, it is possible to establish a simple criterion for there to be no thermal blockage. To obtain a non negligible acceleration of the flow with the force field, $\mathscr{F}.L$ must be close to unity. On the other hand, a thermal blockage should be avoided if the increase in temperature is not large *i.e.* $\mathscr{J}.L$ remains small compared with unity. For the case of a moderate but non negligible Hall effect, we can write:

$$J_y = J \cos \theta \tag{55}$$

$$\mathscr{F}.L = \frac{J_y B L}{\rho V^2} \approx 1 \tag{56}$$

$$\mathscr{J}.L = \frac{J^2 L}{\sigma_0 \rho V^3 A} \ll 1. \tag{57}$$

Ohm's law is:

$$J = \frac{\sigma_0}{1+\beta^2} \begin{vmatrix} 1 & -\beta \\ \beta & 1 \end{vmatrix} (\mathbf{E} + \mathbf{V} \times \mathbf{B}). \tag{58}$$

For a force field configuration such as is shown in figure 1, and for the case where $\mathbf{E}$ is perpendicular to $\mathbf{B}$, the current density becomes:

$$|J| = \frac{\sigma_0 VB}{\sqrt{1+\beta^2}}(K-1) = \sigma_0 VB(K-1) \cos \theta \tag{59}$$

where K represents the load factor. This parameter is smaller than unity in the case of the conversion and larger than unity for an accelerating assembly.

In substituting the expressions for J obtained from (56) and (59), into (57), we obtain a criterion on the load factor characterizing the avoidance of a thermal blockage:

(60) $$K \ll A+1=K_L.$$

This criterion does not depend on the Hall effect. On the other hand, we can introduce into the relation (57) twice the expression of J. from (56). That leads to a criterion in which the various parameters for the non-thermal blockage come to the fore:

(61) $$N=\frac{\sigma_0 L(1+Z_t)}{\rho V(\gamma-1)}\frac{B^2}{(1+\beta^2)} \gg 1.$$

To avoid the thermal blockage, the electric conductivity must be high, the characteristic length important, and γ low. The Hall effect and the magnetic field act in opposite

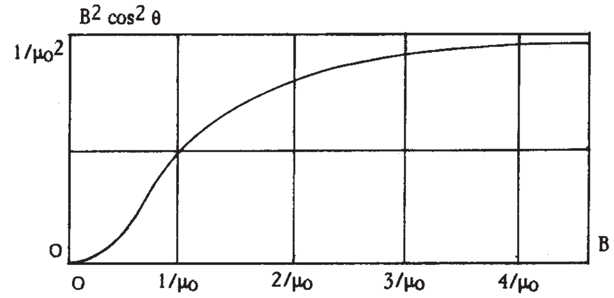

Fig. 8. — Behaviour of $B^2/(1+\beta^2)$ as a function of B.

directions. Figure 8 shows the behaviour of $(B^2/(1+\beta^2))$ as a function of B. We notice that this term tends asymptotically to $(1/\mu_e)^2$ for the important values of B and to zero for low B. So the increasing of B will not help avoid a thermal blockage.

Also, we notice that for important values of B, the criterion N is varying like $N_e . V_e$. In the case of a Coulomb plasma. *i.e.* fully ionized, N shall vary like N_e^2, and in the case of a weakly ionized plasma, it will vary like N_e. Because the variation of N_e as a function of temperature is exponential for weakly ionized plasma (S. & S., 1965) ($T<6{,}000$ K for Argon pressure 1 bar), we can deduce that the thermal blockage is very sensitive to the temperature variations in such plasmas.

The real gas effects, which are characterized by a decreasing of γ, favour an avoidance of a thermal blockage and the force field acts better if it points along the stream lines ($J_x=0$).

It is interesting to consider two particular cases:

— J perpendicular to the flow direction ($\theta=0$).

(62) Here, $$K \ll \frac{\gamma+Z_t}{\gamma-1}=K_L \quad \text{and} \quad N=\frac{\sigma_0 B^2 L(1+Z_t)}{\rho V(\gamma-1)} \gg 1$$

and J perpendicular to the ascending characteristic lines ($\theta=\alpha$):

(63) where $$K \ll \frac{\gamma+Z_t}{\gamma-1}=K_L \quad \text{and} \quad N=\frac{M^2-1}{M^2}\frac{\sigma_0 B^2 L(1+Z_t)}{\rho V(\gamma-1)} \gg 1.$$

The second case corresponds to the theoretical development of shock wave annihilation.

Our results are confirmed by the works of Forestier (1973) who calculated differently the limit value of the load factor K_L. He obtained $K_L = 4.5$ for $\gamma = 1.275$, while the criterion (60) gives $K_L = 4.6$.

6. Experimental study proposed

The MHD interaction leading to the shock wave cancellation needs strong electric currents to be present in the gas, and that requires the use of ionized gases which are good electric conductors. Since it is relatively simple to obtain such supersonic plasma bursts, in a shock tube, the first experiments should use such a device.

At first, it is better to consider familiar experimental conditions so we can use the existing knowledge and limit the number of new unknown parameters. For this, the shock tube conditions developed by Fontaine (1973) and Forestier (1973) have been used again (Argon flow at 1 bar).

TABLE 1. — Flow characteristics obtained by shock tube for argon at pressure 1 bar.

Mach's number	1.4	1.6	1.8	2
T(K)	8,200	9,500	10,450	11,400
V (m/s)	2,240	2,575	2,986	3,614
ρ (kg/m^3)	0.058	0.050	0.044	0.039
ν_e (/s $\times 10^{-10}$)	4.9	16.1	31	45
σ (Mhos/m)	1,879	2,810	3,520	4,250
γ	1.50	1.31	1.24	1.22
α_i	0.0021	0.012	0.032	0.077
Z_t	0.025	0.12	0.31	0.60
β (Hall's parameter)	1	0.80	0.66	0.58
B (Tesla)	0.28	0.73	1.2	1.6
J (A/m^2 $\times 10^{-6}$)	4.8	2.4	2	2.1
K	4.6	1.4	1.16	1.1
K_L	3	4.6	6.5	7.8
$\mathcal{F}$.L	0.4	0.53	0.60	0.66
$\mathcal{J}$.L	0.80	0.055	0.018	0.01

Table I indicates the flow conditions obtained by shock tube for argon plasmas. These conditions, determined for different values of the Mach's number, all correspond to a pressure of 1 bar.

First, we consider the problem of the Hall effect. To obtain a force field parallel to the ascending characteristic lines, we must have $\theta = \alpha$ as seen earlier. Also, the magnetic field is determined according to the relation (28) and the plasma properties.

Following this, the force field intensity can be calculated for a curvature of the wall equal to 0.2 m. J is determined using the relation (43). The argon flow is a fully ionized plasma flow.

First, we notice that a magnetic field, close to one tesla, and also an electrical density, close to 2×10^6 A/m^2, can be easily achieved. Such intensities were obtained by Fontaine (1973) and Forestier (1973) about twenty years ago.

Furthermore, we remark that, the low values of B at weak Mach number imply that J must be important, involving a strong Joule effect. The thermal blockage could even appear for a flow at Mach 1.4, as indicated by the values of the Hall factor ($K = J/\sigma VB + 1$) compared with K_L. On the other hand, for the higher Mach number flows, the Joule effect becomes weak and the possibility of a thermal blockage is no longer feared. We notice also that the Joule energy introduced into the flow is negligible in comparison with the work of the volume forces i. e. ($\mathcal{J}.L \ll \mathcal{F}.L$).

The relative increase in temperature caused by the Joule effect is very weak. At Mach 1.6, it is about:

$$\frac{\Delta T}{T} = \frac{J^2 L}{\sigma_0 \rho V C_v T} \approx 0.05. \tag{64}$$

So, the assumption of a negligible Joule effect is verified *a posteriori*. The experiments could be carried out for Mach numbers close to 1.6, and for such conditions, we can also verify that the magnetic Reynolds number which is written:

$$R_m = \mu_0 \sigma VL \tag{65}$$

is close to 0.3 and that the induced magnetic field is effectively negligible (μ_0: permittivity of the vacum; $L = 0.03$ m length of the magnetic field according to Fig. 9).

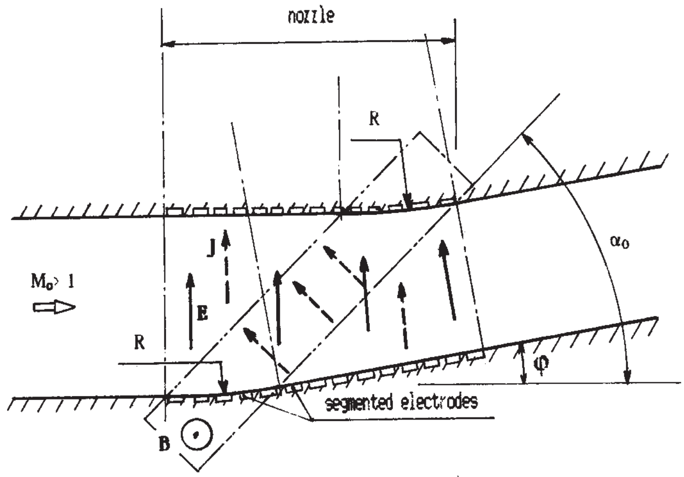

Fig. 9. — Experimental apparatus for schock wave annihilation in a shock tube.

All these conditions allow us to design an experiment of shock-wave annihilation to be carried out in a shock tube. Figure 9 shows the tube to have a simple bend in it.

Because of the short duration of the burst, transient problems involving variations of the flow features may occur, but for the moment they are not taken into account in the theoretical developments.

7. Conclusion

This approach has allowed us to demonstrate that shock-wave cancellation should be possible around a parallel profile of weak relative depth. We notice that this analysis involves the *nullification of the wave drag*. Determining experimental conditions is our first interest: it is better to realize conditions where the gas is a good electric conductor and presents important real gas effects. This involves the use of a shock tube, with the same experimental conditions as those used at the Institut de Mécanique des Fluides de Marseille (I.M.F.M.) by Fontaine (1973) and Forestier (1973).

As part of this program, the french M.R.T. has financed an experiment which is being carried out at the Thermodynamic Laboratory of Rouen (L.A. No. 230), under the management of C. Thenard. The first experimental results should be produced during 1989. To the best of our knowledge, it is the first time that this problem has been approached theoretically and experimentally.

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