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Modelling the flyby anomalies using a modification of inertia

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ABSTRACT

The flyby anomalies are unexplained velocity jumps of 3.9, −4.6, 13.5, −2, 1.8 and 0.02 mm s^{-1} observed near closest approach during the Earth flybys of six spacecraft. These flybys are modelled here using a theory that assumes that inertia is due to a form of Unruh radiation, and varies with acceleration due to a Hubble-scale Casimir effect. Considering the acceleration of the craft relative to every particle of the rotating Earth, the theory predicts that there is a slight reduction in inertial mass with increasing latitude for an unbound craft, since near the pole it sees a lower average relative acceleration. Applying this theory to the inbound and outbound flyby paths, with conservation of momentum, the predicted anomalies were 2.9, −0.9, 20.1, 0.9, 3.2 and −1.3 mm s^{-1}. Three of the flyby anomalies were reproduced within error bars, and the theory explains their recently observed dependence on the latitude difference between their incident and exit trajectories. The errors for the other three flybys were between 1 and 3 mm s^{-1}.

Key words: gravitation – celestial mechanics – Solar system: general – cosmology: theory.

1 INTRODUCTION

During six Earth gravity assist flybys, significant anomalous velocity increases of a few mm s^{-1} were observed (Antreasian & Guinn 1998; Anderson et al. 2008) using both doppler frequency data and ranging methods. These are known as the flyby anomalies, and, so far, no explanations have been found that can account for them. Explanations considered and rejected to date have included: anomalous thruster activity, computer software glitches, troposphere and ionosphere effects and others (see Antreasian & Guinn 1998; Lammerzähl, Preuss & Dittus 2006).

Anderson et al. (2008) analysed the available data, some of which are summarized in Table 1 (columns 1–4), and intriguingly managed to show that the six velocity anomalies observed so far (dv) fitted a formula, given, with slightly modified notation, by

$$\mathrm{d}v = 3.099 \times 10^{-6} \times v_\infty \times (\cos\phi_1 - \cos\phi_2), \quad (1)$$

where v_∞ is the hyperbolic excess velocity, ϕ_1 is the incident angle of the trajectory and ϕ_2 is the exit angle. Equation (1) shows that the anomalous velocity gain dv depends on the difference between the incident latitude and the latitude of the exit trajectory. For example, the *NEAR* (*Near Earth Asteroid Rendezvous*) probe approached at low latitude and left on a polar trajectory and its velocity jump was large, whereas *Messenger* approached and left on an equatorial trajectory and only a very small jump was seen. This led Anderson et al. (2008) to make the interesting suggestion that the cause may be somehow related to the Earth's rotation, although they did not suggest a cause. A possible cause is suggested in this Letter.

McCulloch (2007) proposed a model in which the inertial mass reduces slightly as the acceleration decreases: a modification of inertia due to a Hubble-scale Casimir effect (hereafter: MiHsC). This is interesting because if we take the unusual step of summing all the accelerations seen by *NEAR* on its flyby, then on its equatorial approach it would see high accelerations as the masses comprising the planet rotate towards and away from it, and many of the acceleration vectors would point at the craft, but on its polar exit trajectory, *NEAR* would see much less acceleration since the Earth's acceleration vectors, pointing at the spin axis, would not point at the craft. Therefore, MiHsC predicts a lower post-flyby inertial mass for *NEAR*, which, through conservation of momentum, implies an increase in its speed. In this Letter, it is shown that the increase in speed predicted by MiHsC agrees quite closely with the observed flyby anomalies.

2 METHOD AND RESULTS

Haisch, Rueda & Puthoff (1994) suggested that inertial mass could be caused by a form of Unruh radiation. Milgrom (1994, 1999) suggested that there could be an abrupt break in this effect for very low accelerations since the Unruh wavelengths would then exceed the Hubble distance, and the loss of inertia for low acceleration would be similar to the behaviour of his empirical Modified Newtonian Dynamics (MOND) scheme. The model of McCulloch (2007) builds on this suggestion, but uses a Hubble-scale Casimir effect instead so that inertia diminishes linearly as acceleration reduces, since fewer

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Table 1. Flybys: observed and predicted. The mission name (column 1), the initial and final flyby speeds (column 2), the initial and final latitudes (3) and the observed anomalous velocity jumps (4) (from Anderson et al. 2008). The error bars are also shown, where known. The predictions made in this Letter are shown in column 5.

Mission	Flyby speeds ($\mathrm{km\,s^{-1}}$)	Latitudes (°)	Observed dV ($\mathrm{mm\,s^{-1}}$)	Predicted ($\mathrm{mm\,s^{-1}}$)
Galileo-I	31,35	−12.5,−34.2	**3.92** ± 0.08	**2.9** ± 0.6
Galileo-II	34.5,38.5	−34.3,−4.9	**−4.6**± 0.08	**−0.9**± 0.2
NEAR	36.5,34	−20.8,−72	**13.46** ± 0.13	**20.1** ± 4.0
Cassini	35,39	−12.9,−5	**−2**	**0.9** ± 0.2
Rosetta	31,35	−2.8,−34.3	**1.8** ± 0.05	**3.2** ± 0.6
Messenger	29,25	31.4,−31.9	**0.02** ± 0.05	**−1.3**± 0.2

wavelengths fit within the Hubble diameter. This is a more gradual process than the abrupt break of Milgrom. This model could be called Modified Inertia due to a Hubble-scale Casimir effect (or MiHsC). In MiHsC, the equivalence principle ($m_\mathrm{i} = m_\mathrm{g}$) is changed slightly to

$$m_\mathrm{I} = m_\mathrm{g}\left(1 - \frac{\beta\pi^2 c^2}{a\Theta}\right), \tag{2}$$

where m_I is the modified inertial mass, m_g is the gravitational mass of the spacecraft, $\beta = 0.2$ (from the empirically derived Wien's constant), c is the speed of light, Θ is twice the Hubble distance $2c/H$ and a is the acceleration of the craft relative to the matter in its local environment. In McCulloch (2007), this was simplified to be the acceleration of the *Pioneer* craft relative to the Sun's centre of mass (Ignatiev 2007, in his version of modified inertia, uses an acceleration relative to the Galactic Centre). At most terrestrial values of acceleration, the difference from standard physics is small, but this model predicts the *Pioneer* anomaly correctly beyond 10 au from the Sun with no adjustable parameters (McCulloch 2007).

However, the model also predicted an anomaly within 10 au of the Sun (when the *Pioneer* craft were in bound orbits, and no anomaly was observed). The model is also not needed to explain the orbits of the planets, and its variation of inertial mass disagrees with precise Earth-bound tests of the equivalence principle undertaken by, for example, Carusotto, Cavasinni & Mordacci (1992) and Schlamminger et al. (2008). These results suggest that, if this model is correct, it only applies to unbound orbits. The reason for this is unknown. However, the work of Price (2005) is interesting because he showed that a bound system does not follow the cosmological expansion, whereas an unbound system does.

To analyse the trajectories of the flyby craft (which are not bound to the Earth), we first assume conservation of momentum so that

$$m_{1\mathrm{e}}v_{1\mathrm{e}} + m_1 v_1 = m_{2\mathrm{e}}v_{2\mathrm{e}} + m_2 v_2, \tag{3}$$

where the terms are: the initial momentum of the Earth, the initial momentum of the craft and the final momenta. We now replace the inertial masses of the unbound craft m_1 and m_2 with the modified inertia of McCulloch (2007) (equation 2) so that

$$m_{1\mathrm{e}}v_{1\mathrm{e}} + m_\mathrm{g}\left(1 - \frac{\beta\pi^2 c^2}{a_1\Theta}\right)v_1 = m_{2\mathrm{e}}v_{2\mathrm{e}} + m_\mathrm{g}\left(1 - \frac{\beta\pi^2 c^2}{a_2\Theta}\right)v_2, \tag{4}$$

where m_g is the gravitational mass of the craft, or the uncorrected inertial mass. Some algebra implies that

$$v_2 - v_1 = \mathrm{d}v = \frac{m_\mathrm{e}}{m_\mathrm{g}}(v_{1\mathrm{e}} - v_{2\mathrm{e}}) + \frac{\beta\pi^2 c^2}{\Theta}\left(\frac{v_2}{a_2} - \frac{v_1}{a_1}\right). \tag{5}$$

The first term on the right-hand side is well known. So we now look at the new velocity change due to modified inertia represented by the second term and call it $\mathrm{d}v'$:

$$\mathrm{d}v' = \frac{\beta\pi^2 c^2}{\Theta}\left(\frac{v_2}{a_2} - \frac{v_1}{a_1}\right). \tag{6}$$

We take the incoming and outgoing craft at a radius where the standard gravitational acceleration is equivalent, so by standard physics $a_1 = a_2$, but take the new step of assuming that the accelerations of the craft relative to each part of the Earth also contribute to a_1 and a_2. To picture these accelerations, one could imagine a line connecting the craft with every mass in the Earth and measure the acceleration of the length of each line to determine the inertial mass to use for each gravitational interaction. It can be shown that the average acceleration of particles in the x-direction (the assumed direction of the craft) within the solid Earth $a = 0.07v_\mathrm{e}^2/R$ (see the Appendix and Fig. 1 for a derivation), where v_e is the rotational velocity at the surface equator and R is the Earth's radius. The component of the acceleration seen by a flyby craft at a latitude ϕ (Fig. 1) is therefore $a = (0.07v_\mathrm{e}^2/R)\cos\phi$. We now use these accelerations in equation (6) and get

$$\mathrm{d}v' = \frac{\beta\pi^2 R c^2}{0.07 \times v_\mathrm{e}^2\Theta} \times \left(\frac{v_2\cos\phi_1 - v_1\cos\phi_2}{\cos\phi_1\cos\phi_2}\right). \tag{7}$$

Substituting values as follows $R = 6371\,\mathrm{km}, c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$, $v_\mathrm{e} = 465\,\mathrm{m\,s^{-1}}$ and the Hubble diameter $\Theta = 2c/H = 2.7 \times 10^{26}$ m, the same value used by McCulloch (2007) to reproduce the *Pioneer* anomaly:

$$\mathrm{d}v' = 2.8 \times 10^{-7} \times \left(\frac{v_2\cos\phi_1 - v_1\cos\phi_2}{\cos\phi_1\cos\phi_2}\right). \tag{8}$$

The derived equation (8) is similar to the empirical equation derived by Anderson et al. (2008) (equation 1) from their data, especially the dependence on the difference in the cosine of the incident and outgoing latitude (the denominator in the brackets has little effect on most of the flybys being a little less than 1 for most of them, and 0.3 for *NEAR*).

Equation (8) was used to predict the flyby anomalies and the results are shown in Table 1, column 5 and in Fig. 2. The errors were calculated assuming a 9 per cent error in the Hubble constant

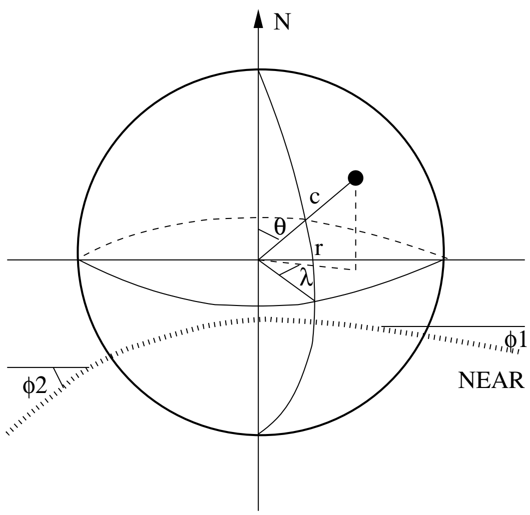

Figure 1. Schematic showing the *NEAR* flyby. This was incident at $\phi_1 = -21°$ latitude and left the Earth at $\phi_2 = -72°$ latitude. Also shown are some of the parameters used in the text and Appendix.

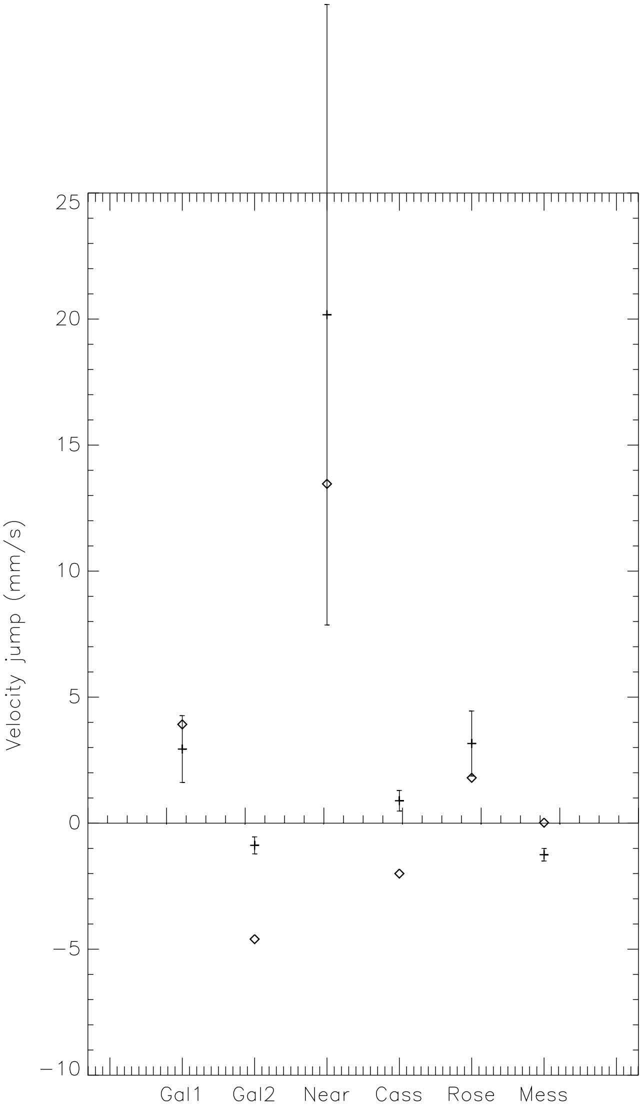

Figure 2. Observed (diamonds) and predicted (pluses) velocity jumps for all six flybys in mm s^{-1} with error bars shown for the predictions. The predictions agree for the *Galileo-I*, *NEAR* and *Rosetta* flybys, and the differences are about 1 mm s^{-1} for *Messenger*, 2 mm s^{-1} for *Cassini* and 3 mm s^{-1} for *Galileo-II*.

(Freedman 2001), and an error caused by the assumption in the Appendix that it is the x-component of the acceleration that matters: assuming that the craft is at an infinite distance from the Earth. It is, more properly, the component of acceleration pointing along the aforementioned lines between the craft and each point in the Earth that matters. The error, from this source, was calculated by assuming a distance from the Earth of 36 000 km (roughly the distance from which the post-encounter data were available). The average error in the acceleration vector's orientation by taking only the x-component is then about 3°. To calculate the errors, the ϕ_1 and ϕ_2 in equation (8) were each altered by 3°. The resulting variations in the predicted dv were 0.5, 0.5, 6.4, 0.2, 0.5 and −0.04. These were added to the errors due to the Hubble constant and a 10 per cent error in the assumed linear vertical density profile of the Earth (see the Appendix). The resulting error bars are shown in Fig. 2.

In Fig. 2, the flyby passes are shown one by one along the x-axis, the diamonds show the observed velocity jump (the observational error was assumed to be 0.1 mm s^{-1}) and the pluses show the predicted jump using equation (8), with error bars.

The predictions agree with the observations for the *Galileo-I*, *NEAR* and *Rosetta* flybys. In the other cases, there are differences. For *Galileo-II*, the difference is 3 mm s^{-1}, although it is possible that the observed jump in this case may have a larger error since this flyby grazed the atmosphere and had to be corrected for atmospheric drag.

Nevertheless, the dependence on change of latitude seen by Anderson et al. (2008) is reproduced. The predicted values are in proportion to those observed, and the correlation between the six observed and predicted values is 0.94 (although this is not significant at the 5 per cent level: the p value is 30). It would be useful to have a larger set of observations to assess the theory.

Flybys of other planets or moons would provide an interesting test of this model, since the planet's radius R and especially its equatorial velocity v_e should strongly affect the size of the anomaly (see equation 7). A large slowly rotating planet, or even a galaxy, could show a strong, and more detectable, velocity boost for polar exit trajectories. The recent (2007 February) *Rosetta* flyby of Mars could be useful, since the values of R and v_e for Mars imply that its flyby anomaly should be approximately double that of the Earth.

The predictions of equation (8) are not as close as the predictions of Anderson et al.'s suggestive equation (1), although their empirical formula was, of course, fitted to the data, whereas equation (8) was derived from a theory.

3 CONCLUSION

Six well-observed Earth flybys with unexplained velocity anomalies were modelled using a theory that assumes that inertia is due to a form of Unruh radiation, and varies with acceleration due to a Hubble-scale Casimir effect.

The theory reproduces three of the observed flyby anomalies within error bars, without the need for adjustable parameters, and explains the recently observed dependence of the anomalies on the latitude difference between the incident and exit trajectories. The errors for the other flybys were 1, 2 and 3 mm s^{-1}.

It should be stressed that the suggested model only seems to apply to unbound trajectories and the reason for this is unknown. Also, the definition of acceleration used here is different from that used in Milgrom (1994, 1999) and Ignatiev (2007). The method uses the same physics as McCulloch (2007) (e.g. equation 2), but differs in including more detailed information about accelerations. The results of McCulloch (2007) would be largely unchanged using this enhanced scheme since the *Pioneer* craft maintained low latitude with respect to the Sun (Parthasarathy & King 1991).

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APPENDIX

The magnitude of the acceleration of a point a distance r from the rotational axis of a uniform sphere is $a = v^2/r$. Expressing this in terms of the equatorial rotational velocity of the Earth v_{e} we get $a = {v_{\mathrm{e}}}^2 r/R^2$, where R is the Earth's equatorial radius. To calculate the average acceleration seen by the flyby craft, we assume they are far enough from the planet along the x-axis, and therefore see the x-coordinate of a which in terms of the longitude λ and angle from the north pole θ (see Fig. 1) is

$$a_x = \frac{{v_{\mathrm{e}}}^2 r}{R^2} \sin\lambda \sin\theta; \tag{A1}$$

we use $r = c \sin\theta$, where c is the distance from the centre of the Earth, to give

$$a_x = \frac{{v_{\mathrm{e}}}^2 c \sin\theta}{R^2} \sin\lambda \sin\theta. \tag{A2}$$

We integrate this over the sphere to find the average acceleration of all the mass in the Earth. Since the density of the core of the Earth is 12.8–13.1 $\mathrm{g\,cm^{-3}}$ and that of the crust is only 2.2–2.9 $\mathrm{g\,cm^{-3}}$ (Dzievonski & Anderson 1984) we need to weight the integral higher towards the centre. For simplicity, we assume a linear increase in density with depth which can be modelled as $\rho = (R - \alpha c)/R$ where $\alpha = 0.974$. In the integral, ϕ is the longitude and θ is the angle from the north pole. A volume element is therefore $\mathrm{d}c \times c \sin\theta \mathrm{d}\phi \times c\mathrm{d}\theta$

$$\overline{a_x} = \frac{1}{(4/3)\pi R^3} \times \int_{c=0}^{R} 2 \int_{\lambda=0}^{\pi} \int_{\theta=0}^{\pi} \frac{v_{\mathrm{e}}^2}{R^2} c^3 \frac{R - \alpha c}{R} (\sin\theta)^3 \times \sin\lambda \, \mathrm{d}c \, \mathrm{d}\lambda \, \mathrm{d}\theta. \tag{A3}$$

The result is

$$\overline{a_x} = \frac{4 v_{\mathrm{e}}^2}{\pi R} \left(\frac{1}{4} - \frac{\alpha}{5} \right) \sim 0.07 \times \frac{v_{\mathrm{e}}^2}{R}. \tag{A4}$$

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