The Missing-Mass Problem.

J.-P. PETIT (*)
Observatory of Marseille - Marseille, France

(ricevuto il 6 Luglio 1993; revisionato il 17 Novembre 1993; approvato il 12 Maggio 1994)

Summary. — A new field equation is proposed, associated to an $S_3 \times R_1$ topology. We introduce a differential involutive mapping A which links any point of space $\boldsymbol{\sigma}$ to the antipodal region $A(\boldsymbol{\sigma})$. According to this equation, the geometry of the manifold depends both on the energy-momentum tensor $\boldsymbol{T}$ and on the antipodal tensor $A(\boldsymbol{T})$. Considering time-independent metric with low fields and small velocities, we derive the associated Poisson equation, which provides cluster-like structures interacting with halo-like antipodal structures. The second structure helps the confinement of the first. It is suggested that this model could explain the missing-mass effect and the large-scale structure of the Universe.

PACS 04.90 – Other topics in relativity and gravitation.

1. – Introduction.

The equilibrium of a galaxy is studied through a certain set of non-relativistic equations, as, for example, Vlasov equation coupled to Poisson equation, which comes from the general Einstein field equation

$$\boldsymbol{S} = \chi \boldsymbol{T} \tag{1}$$

plus a steady-state hypothesis in which we take weak fields and small velocities. It is well known that the gravitational field due to the visible mass of our galaxy cannot balance the centrifugal and the pressure forces. Some people assume that some invisible-mass, dark matter may contribute to the field and balance the centrifugal force. In the following we are going to propose another model, based on a new field equation.

2. – A new field equation.

We assume that the Universe has the topology of $S_3 \times R_1$. The Gaussian coordinates are

$$\boldsymbol{x} = (x^0, \boldsymbol{\sigma}), \tag{2}$$

(*) For scientific correspondence: 9 tour d'Aygosi, 13100 Aix-en-Provence, France where x^0 is a time marker and the vector $\boldsymbol{\sigma}$ represents the spatial markers. Space-time is oriented. It is possible to define a differential involutive mapping linking a given point $\boldsymbol{\sigma}$ to the antipodal point $\boldsymbol{\sigma}^*$,

$$\boldsymbol{\sigma}^* = A(\boldsymbol{\sigma})\,. \tag{3}$$

Consider two tensor fields $\boldsymbol{S}$ and $\boldsymbol{T}$, defined on the manifold. Suppose that they are linked in the following field equation:

$$\boldsymbol{S} = \chi(\boldsymbol{T} - A(\boldsymbol{T})) \tag{4}$$

with

$$A(\boldsymbol{T}) = \boldsymbol{T}^* = \boldsymbol{T}(x^0, \boldsymbol{\sigma}^*)\,, \tag{5}$$

We assume that the light follows the geodesics of space-time. $\boldsymbol{g}$ is the metric tensor, $\boldsymbol{R}$ is the Ricci tensor, so that

$$\boldsymbol{g}^* = \boldsymbol{g}(x^0, \boldsymbol{\sigma}^*)\,, \qquad \boldsymbol{R}^* = \boldsymbol{R}(x^0, \boldsymbol{\sigma}^*)\,. \tag{6}$$

We can write the field equation in the more explicit form

$$\boldsymbol{R} - \frac{1}{2}\boldsymbol{g}R = \chi\left(\boldsymbol{T} - \frac{1}{2}\boldsymbol{g}T - \boldsymbol{T}^* + \frac{1}{2}\boldsymbol{g}^*T^*\right). \tag{7}$$

Let us write the tensors $\boldsymbol{T}$ and $\boldsymbol{T}^*$ as

$$\boldsymbol{T} = \begin{vmatrix} \rho & 0 & 0 & 0 \\ 0 & -\dfrac{p}{c^2} & 0 & 0 \\ 0 & 0 & -\dfrac{p}{c^2} & 0 \\ 0 & 0 & 0 & -\dfrac{p}{c^2} \end{vmatrix}, \tag{8}$$

$$\boldsymbol{T}^* = \begin{vmatrix} \rho^* & 0 & 0 & 0 \\ 0 & -\dfrac{p^*}{c^2} & 0 & 0 \\ 0 & 0 & -\dfrac{p^*}{c^2} & 0 \\ 0 & 0 & 0 & -\dfrac{p^*}{c^2} \end{vmatrix}, \tag{9}$$

with

$$\rho^* = \rho(x^0, \boldsymbol{\sigma}^*), \qquad p^* = p(x^0, \boldsymbol{\sigma}^*).$$

If we take the zero-divergence condition, the fluid obeys the following conservation equations:

$$\partial \boldsymbol{T} = 0. \tag{10}$$

3. – Time-independent conditions with weak fields and small velocities. The Poisson equation.

We can apply the classical method, taking a quasi-Lorentzian metric

$$\boldsymbol{g} = \boldsymbol{\eta} + \varepsilon\gamma, \tag{11}$$

where $\boldsymbol{\eta}$ is the Lorentzian metric and ε is a small parameter.

In three-dimensional notations

$$\frac{d^2\boldsymbol{x}}{dt^2} = -\frac{c^2}{2}\gamma_{00}|_i = -\frac{c^2}{2}\nabla\gamma_{00}. \tag{12}$$

The Newtonian law applies over all space. In addition, the gravitational potential is defined as follows:

$$\Psi = -\frac{c^2}{2}\varepsilon\gamma_{00}. \tag{13}$$

Conversely, given the gravitational potential Ψ, the motion of a particle will be along a four-dimensional geodesic if the g_{00} term of the metric tensor has the form

$$g_{00} = 1 + \frac{2\Psi}{c^2}; \tag{14}$$

we get

$$\varepsilon\sum_{\beta=1}^{3}\gamma_{00|\beta|\beta} = -\chi(\rho - \rho^*). \tag{15}$$

By identification we get the following Poisson equation:

$$\Delta\Psi = 4\pi G(\rho - \rho^*). \tag{16}$$

If we consider a spherically symmetric system

$$\frac{d^2\Psi}{dr^2} + \frac{2}{r}\frac{d\Psi}{dr} = 4\pi G(\rho - \rho^*), \tag{17}$$

where

$$\rho^* = \rho(\boldsymbol{\sigma}^*), \tag{18}$$

from (17)

$$\Psi^* = -\Psi . \tag{19}$$

4. – Spherically symmetric solution.

In 1916 Eddington derived a spherically symmetric steady-state solution, combining the Vlasov and the Poisson equations. He assumed that the ellipsoid of the velocities was spherically symmetric and pointed towards the centre of the system, see fig. 1.

Eddington derived the following relation between the mass density and the gravitational potential:

$$\rho = \rho_0 \frac{\exp[-m\Psi/kT]}{1 + r^2/r_0^2} \tag{20}$$

which represents a steady-state distribution of matter in a collision-free gas, in a gravitational potential Ψ, in which the gravitational force balances the pressure force. Let us take the same kind of solution for the antipodal region,

$$\rho^* = \rho_0 \frac{\exp[-m\Psi^*/kT]}{1 + r^2/r_0^2} = \rho_0 \frac{\exp[m\Psi/kT]}{1 + r^2/r_0^2} , \tag{21}$$

so that we have to solve the following equation:

$$\frac{d^2\Psi}{dr^2} + \frac{2}{r}\frac{d\Psi}{dr} = 4\pi G\rho_0 \left(\frac{\exp[-m\Psi/kT] - \exp[m\Psi/kT]}{1 + r^2/r_0^2} \right). \tag{22}$$

Take

$$r_0 \lambda \sqrt{\frac{kT}{4\pi G\rho_0 m}} . \tag{23}$$

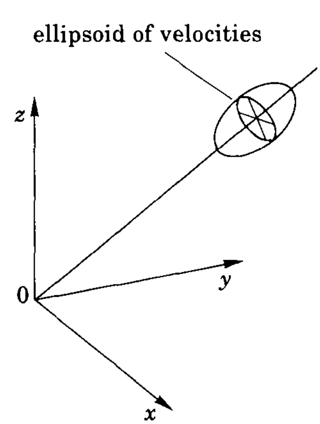

Fig. 1. – Ellipsoid of velocities corresponding to an Eddington-type solution.

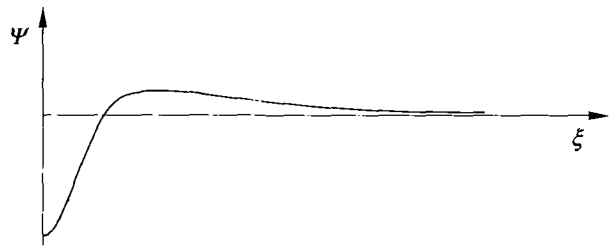

Fig. 2. – Spherically symmetric Eddington-type solution. The gravitational potential.

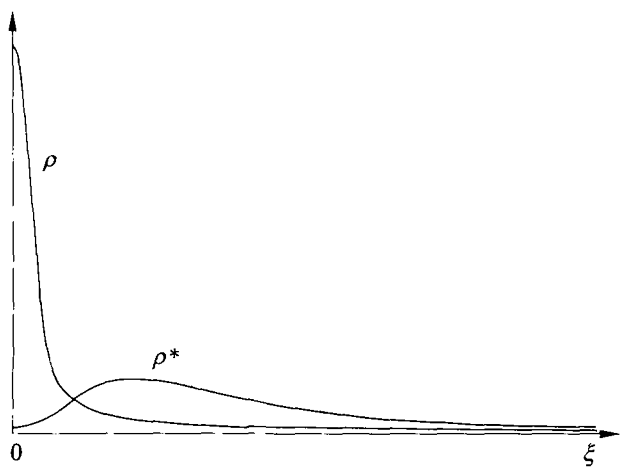

Fig. 3. – Spherically symmetric Eddington-type solution. Mass densities. If a cluster exists in one fold, an associated diffuse halo exists in the conjugated region of the second fold.

Introduce the following adimensional quantities:

$$r = \sqrt{\frac{kT_0}{4\pi G\rho_0 m}}\,\xi\,, \qquad \Psi = \frac{kT}{m}\,\varphi\,. \tag{24}$$

We get

$$\varphi'' + \frac{2}{\xi}\,\varphi' = \frac{\exp[-\varphi] - \exp[\varphi]}{1 + (\xi/\lambda)^2} \tag{24a}$$

which can be solved by numerical computation. We can take the following initial conditions:

$$\varphi_0' = 0\,, \qquad \varphi_0'' = 10\,, \qquad \lambda = 10\,,$$

$$\rho = \rho_0\,\frac{\exp[-\varphi]}{1 + (\xi/\lambda)^2}\,, \qquad \rho^* = \rho_0\,\frac{\exp[\varphi]}{1 + (\xi/\lambda)^2}\,.$$

See fig. 2 and 3.

5. – The large-size structure of the Universe.

From eq. (24a) we see that if a cluster exists in one fold, an associated diffuse halo structure exists in the conjugated region of the second fold. If this model is correct, we should find halo structures in our fold of the Universe. With the help of Dr. Pierre Midy, from the University of Orsay, France, we have performed numerical simulations, using a Cray-1 computer. We consider two distributions of 350 points. The first is represented by little circles and the second by small crosses. At the beginning the points are randomly distributed on the screen and are supposed to represent two uniform gazes. Each mass owns a random velocity corresponding to an isotropic Maxwellian distribution with an averaged thermal velocity $\langle V \rangle$. Call m_1 the elements of the first population and m_2 the elements of the second population. We apply the Newton law with

i) m_1 attracts m_1: gravitational effect;

ii) m_2 attracts m_2: gravitational effect;

iii) m_1 and m_2 repel each other: antigravitational effect.

We consider this two-dimensional system as periodic over space. In other terms, the upper boundary is linked to the lower one and the right to the left (Euclidean 2D torus). So that we can compute the sum of the mutual actions of the particles. For each interval of time Δt we compute the acceleration of each particle and determine the trajectory by Taylor expansion. Each particle that comes out through the right boundary reappears through the left one, and the same thing occurs for the upper and lower boundaries. This makes it possible to study the gravitational instability of these two coupled systems in a finite portion of space (with toroidal topology). The interval of time is determined in order to get significant computational results. In other terms, we demand the trajectory of a particle to be approximatively regular. The following figures show the typical behaviour of the system after 4000 intervals of time. In fig. 4, 5, and 6 we find both clusters and cellular patterns. This is enhanced in fig. 7 and 8.

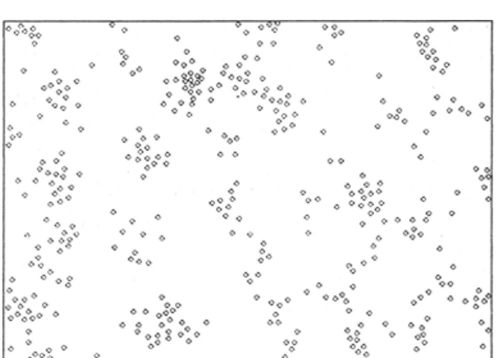

Fig. 4.

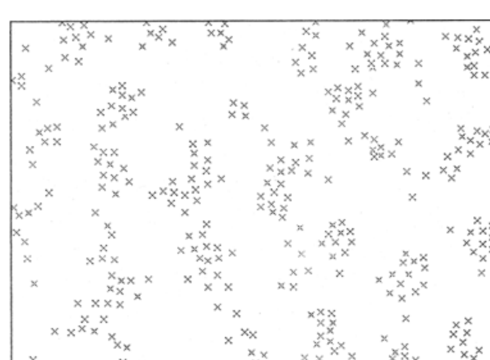

Fig. 5.

Fig. 4. – Effect of the gravitational instability on the system 1.

Fig. 5. – Effect of the gravitational instability on the system 2.

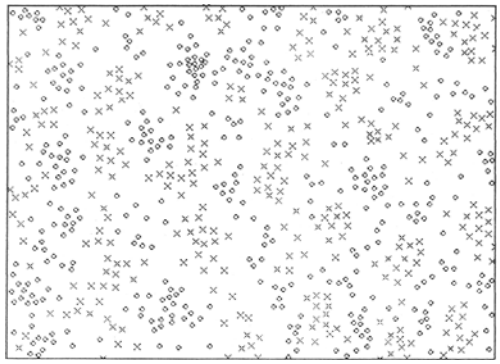

Fig. 6. – Superposition of the two systems 1 and 2.

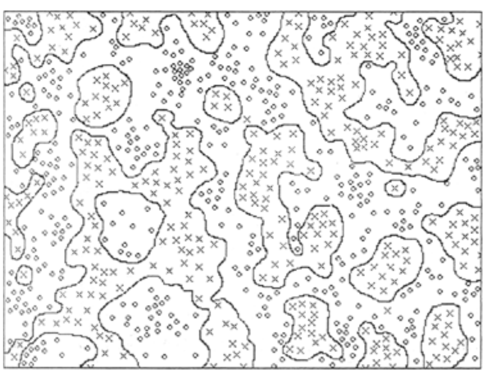

Fig. 7.

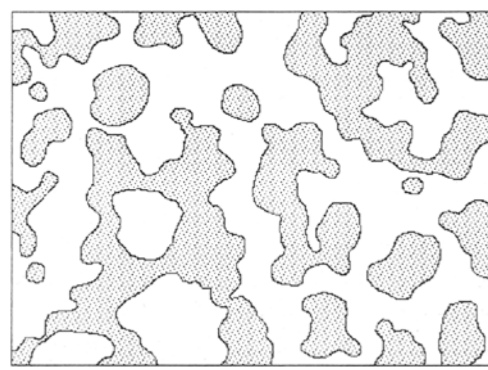

Fig. 8.

Fig. 7. – Enhanced spatial distribution of the two populations.

Fig. 8. – White: population 1. Grey: population 2.

We suggest that such a mechanism could explain the large-scale structure of the Universe and the observed distribution of galaxies. Suppose that our fold of the Universe corresponds to the population 1. In the right lower part of the screen this matter is arranged around large «empty» bubbles. These bubbles correspond to a cluster arrangement in the population 2, supposed to be located in the second fold of the Universe (in fact the antipodal region), according to our theory. But, as seen in fig. 8, for a given population, in some places the matter can be arranged as a Swiss «gruyère» cheese and in other places as an emulsion.

These first crude numerical simulations have to be developed with a larger number of points and in a three-dimensional representation. We know that the three-dimensional behaviour of a system can be somewhat different from the two-dimensional one. But we expect the conclusions to be similar. We think that with a larger number of points we could get a fractal system, as suggested in fig. 14, but we precise that this peculiar computation has not yet been done, it is under study though. According to this idea, the galaxies should be located in the holes of the associated anti-matter cloud, which would ensure their confinement, as suggested earlier.

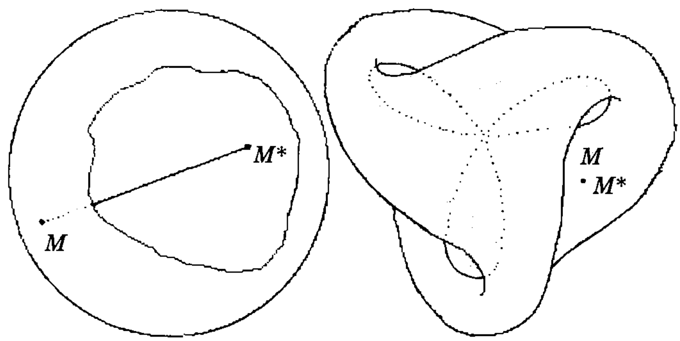

Fig. 9. – A couple of antipodal points on a sphere S_2 and the Boy surface, image of the projective space P_2.

5. – The interpretation of the solution.

From fig. 2 we see that the potential Ψ tends to a constant at infinity. In the classical Eddington solution the potential owns a logarithmic growth. Figure 3 shows the association of a cluster of matter, located in the region σ, sourrounded by a smooth hollow located in the region σ^*.

In both regions matter attracts matter. But the negative sign, from the field equation and the Poisson equation, makes the matter and the «antipodal matter» to repel each other. This helps the confinement of the cluster. For a given thermal velocity, the necessary quantity of matter to balance the pressure force is smaller. The smooth halo acts like a corset.

A field equation provides a macroscopic description of the Universe. It does not take account of the corpuscular nature of matter. The model implies that particles and antipodal particles live in very distant, antipodal portions of space. In fact their natures are identical. The physical meaning of the field equation is the following: the particles and antipodal particles interact by gravitational effect, but not by electromagnetic effect. We assume that the antipodal particles, clusters, rings are not observable with a telescope, or a radiotelescope. The observation of antipodal structures should require some sort of gravitational telescope.

From eq. (22) clusters can be located in the antipodal region. Then, associated

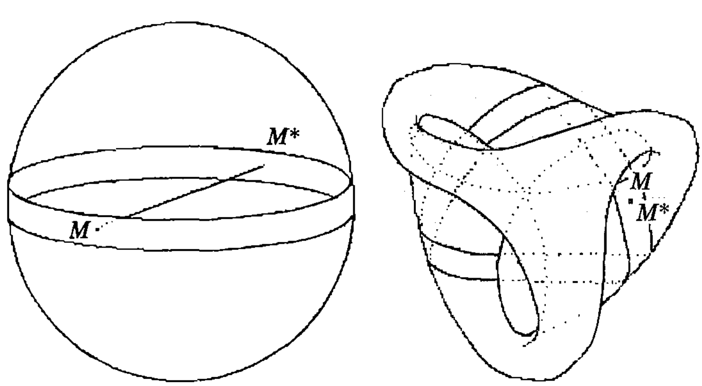

Fig. 10. – The vicinity of the equator of a 2-sphere and its location on a Boy surface.

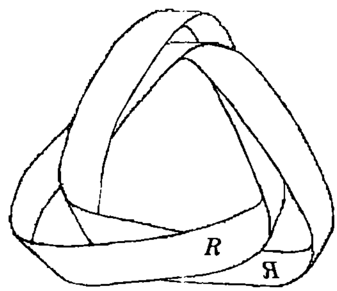

Fig. 11. – Enantiomorphic image corresponding to the cover of a Möbius belt.

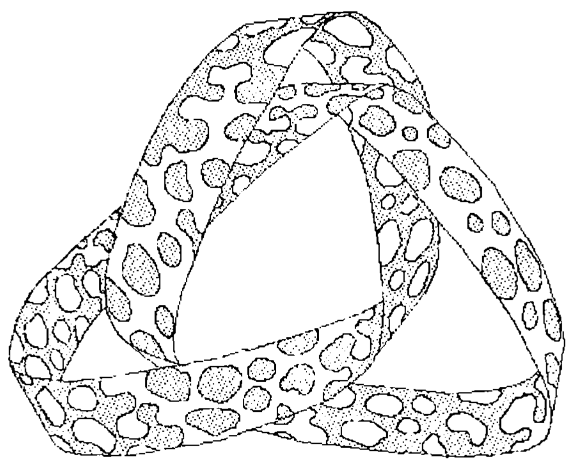

Fig. 12. – Two-dimensional image of the global large structure of the Universe.

large halos, sourrounding wide rarefied regions, should exist in the observable Universe too. In fact they do, for it corresponds, in our mind, to the observed large-scale structure of the Universe: the galaxies seem to be arranged around large rarefied bubbles. According to our model, large clouds of antipodal matter should exist in the corresponding associated antipodal regions.

The Universe was assumed to have an $S_3 \times R_1$ topology. The reader has probably some difficulties to understand this strange three-dimensional geometry. In fact, the sphere S_3 is simply shaped as the double cover of a projective space P_3. In such arrangement each point σ of the sphere is associated to its antipode $A(\sigma)$. The situation is similar for a sphere S_2 covering a projective space P_2, which can be represented in our space R_3 as the well-known Boy surface, see fig. 9.

In fig. 10 we have figured the equator of a sphere and its location on the Boy surface.

Figure 11 shows how the equator of an S_2 sphere can be glued on itself along a three half-turns Möbius belt. Locally, the surface can be assimilated to a bundled manifold whose bundle owns two values $+1$ and -1.

In a 3-sphere S_3, if one follows a geodesic, the antipodal point is half-way. If the 3-sphere is immersed in a four-dimensional space, it is possible to make any point and

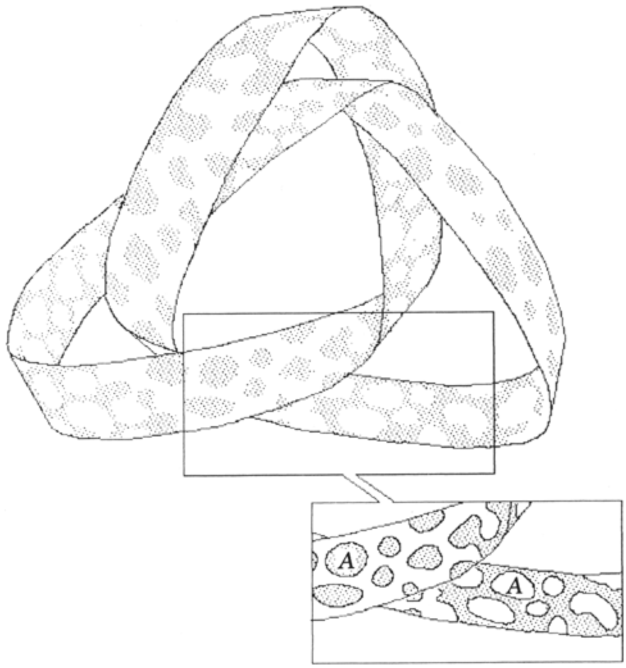

Fig. 13. – The interaction between two antipodal regions.

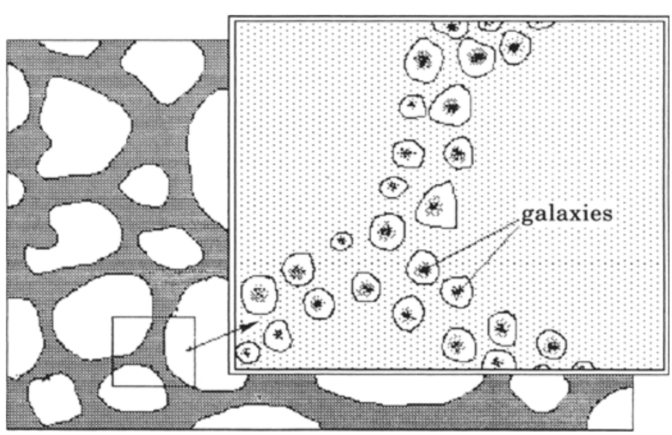

Fig. 14. – Smaller-size structure.

its antipode to coincide. These couples of points are associated through the antipodal diffential involutive mapping A, but not identified.

As shown in fig. 12, we can proceed continously from a «gruyère» structure to a cluster structure. This peculiar feature was illustrated before, through 2d numerical simulations. When a region of space is put «in front» of the antipodal region, as suggested in fig. 13, the clusters nest in the holes.

This effect could act at the level of the galactic structure, as suggested in fig. 14, each galaxy nesting in a «hole» of the conjugated antipodal region.

7. – Some comments about the axioms.

The classical General Relativity proposes a macroscopic description of the Universe, shaped by the gravitational field. But, basically, the electromagnetic phenomena is not taken into account. In order to link this classical model to the observations, one has to bring the following additional axioms:

i) The Universe is filled by particles: neutral particles with a mass equal to m, and photons. Both contribute to the field.

ii) These particles move along geodesics of space-time.

iii) A particle may send an electromagnetic signal.

iv) Another particle may receive this electromagnetic signal.

v) This electromagnetic signal, carried by photons, follows the null geodesics of space-time.

vi) A massive particle may send a gravitational signal, which is supposed to follow a null geodesic.

vii) A massive particle may receive this gravitational signal.

So that, for an observer composed by matter, the Universe becomes optically perceptible, according to these axioms. The photons are the go-betweens bringing an optical message from a massive particle to another.

In the present model, the Universe is to be considered as a cover of an S_3 sphere; locally we have a structure similar to a bundled manifold, whose bundle should be limited to two values, $+1$ and -1. Then we introduce the new following axioms:

i) The Universe is filled by particles: neutral particles whose mass is equal to m, and by photons. Both contribute to the field.

ii) The massive particles and the photons move along the geodesic of space-time and cannot cross from a region to the conjugated antipodal region of S_3.

iii) A massive particle may send electromagnetic and gravitational signals, which can be received by another massive particle.

iv) The gravitational signal travels along the geodesics of space-time, but also along the geodesics of the «adjacent folds of the Universe», «through the bundle structure» so that the gravitational signal owns some sort of ubiquity, because it acts both in a region of the manifold and in the antipodal region (or, in other terms, in the «adjacent region», if we choose the bundled-manifold image).

v) The structure of the new field equation brings the following features: If a gravitational signal is emitted and received by two particles which «belong to the same fold», the phenomenon identifies with the classical description. But a gravitational signal emitted by a massive particle can be received by another particle located in the adjacent region (the antipodal region); in other terms «through the bundle structure», the negative sign in the second member of the field equation changes the nature of the signal, as if it were emitted by a «negative mass».

vi) The electromagnetic signal follows the ordinary null geodesics of the manifold, but does not own this property of ubiquity. It cannot cross from a fold to the «adjacent fold through the bundle structure». To travel from a region of the manifold to the antipodal region, light has to do a complete half-turn of the S_3 sphere.

We confess that this proposed geometric description remains primitive and somewhat unclear. A correct description should imply a more refined model, including the gravitational and electromagnetic phenomena, *i.e.* a unified theory, which does not exist presently.

The bundled-manifold local description is similar to a 5d Kaluza model, in which the fifth dimension would be limited to two values +1 and −1, as suggested earlier by Alain Connes.

8. – Estimation of the «missing-mass effect».

Apply a perturbation method to the Euler equations:

$$\Psi = \Psi_0 + \delta\Psi , \qquad \Psi^* = \Psi_0^* + \delta\Psi^* \tag{25}$$

with the first-order solution

$$\rho_0^* = \rho_0 , \qquad \Delta\Psi_0 = 0 . \tag{26}$$

The Poisson equation gives

$$\begin{cases} \Delta\delta\Psi = 4\pi G\rho_0 \left(\exp\left[-\dfrac{m\,\delta\Psi}{kT} \right] - \exp\left[-\dfrac{m\,\delta\Psi^*}{kT} \right] \right), \\ \delta\rho = \rho_0 \exp\left[-\dfrac{m\,\delta\Psi}{kT} \right], \qquad \delta\rho^* = \rho_0 \exp\left[-\dfrac{m\,\delta\Psi}{kT} \right], \qquad \delta\Psi = -\,\delta\Psi^* , \end{cases} \tag{27}$$

$$\Delta\delta\Psi + \frac{8\pi G\rho_0}{kT}\,\delta\Psi = 0 . \tag{28}$$

L_J is the classical Jeans length

$$L_J = \sqrt{\frac{kT}{4\pi G\rho_0}} , \tag{29}$$

$$\Delta\delta\Psi + 2\,\frac{\delta\Psi}{L_J^2} = 0 . \tag{30}$$

This is the well-known Helmoltz equation.

In classical steady-state approach we had

$$\Delta\delta\Psi + \frac{\delta\Psi}{L_J^2} = 0 . \tag{31}$$

The interaction with the antipodal region shortens the Jeans length by a factor 1.414, so that we have a confinement effect. If we have a positive concentration of matter $\delta\rho$ in our space-time fold, we will find a negative $\delta\rho^*$ in the associated antipodal region, and vice versa. The confinement of the mass due to the action of the antipodal region should reduce the necessary mass to balance pressure or centrifugal force by a factor

$$\frac{1}{2^{3/2}} = 0.353 .$$

9. – Writing the equation in a complex form.

Write

$$\left\{\begin{aligned} &\boldsymbol{S}_1 = \boldsymbol{S}(x^0, \boldsymbol{\sigma}), \\ &\boldsymbol{S}_2 = \boldsymbol{S}(x^0, \boldsymbol{\sigma}^*), \\ &\boldsymbol{T}_1 = \boldsymbol{T}(x^0, \boldsymbol{\sigma}), \\ &\boldsymbol{T}_2 = \boldsymbol{T}(x^0, \boldsymbol{\sigma}^*), \\ &\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_1 + i\boldsymbol{\Sigma}_2 , \\ &\tau = \tau_1 + i\tau_2 , \\ &\tau^c = \tau_1 - i\tau_2 . \end{aligned}\right. \tag{32}$$

Equation (4) can be written as

$$\boldsymbol{\Sigma} = \chi(\tau - i\tau^c) . \tag{33}$$

As suggested previously by Penrose, the quantification of the gravitation could be due to the complex form of the field equation. Equation (33) could perhaps bring a new insight into the problem.

10. – Conclusion.

We propose a new field equation, from which, with the classical approximation: steady state, weak fields, low velocities, we derive the associated Poisson equation. Coupled Eddington solutions give a set of clusters, associated to interacting ring-like clouds, located in the antipodal region. The antipodal halo-like structure repels the cluster and helps its confinement. The reduction factor is roughly evaluated. It is suggested, through 2d numerical simulations, that this model could explain the large-scale structure of the Universe. In addition, the interaction between a cluster and its associated antipodal structure could provide spiral structure. A collision of a cluster with an anti-cluster could also explain the very irregular galaxies.

REFERENCES

[1] A. D. Sakharov: *Ž. Ėksp. Teor. Fiz. Pis'ma*, **5**, 32 (1967) (English translation *JETP Lett.*, **5** 24 (1967)), preprint R2-4267, JINR, Dubna.

[2] D. Novikov: *Ž. Ėksp. Teor. Fiz. Pis'ma*, **3**, 223 (1966) (English translation *JETP Lett.*, **3**, 142 (1966)); *Astr. Ž.*, **43**, 911 (1966) (English translation *Sov. Astr.*, **10**, 731 (1967)).

[3] J. P. Petit: *C.R. Acad. Sci.*, **285**, 1217 (1977).

[4] J. P. Petit: *C.R. Acad. Sci., Ser. A*, **284**, 1413 (1977).

[5] J. P. Petit: *Le topologicon* (Editions Belin, France, 1984).