BOSE CONDENSATION OF STRONGLY EXCITED LONGITUDINAL ELECTRIC MODES

H. FRÖHLICH
Department of Theoretical Physics, University of Liverpool, UK

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If the energy fed into the branch of longitudinal electric modes exceeds a critical rate then, under stationary conditions, the excitation energy is channelled into the mode with lowest frequency in a manner typical for Bose condensation.

In the introductory talk to the recent conference on Theoretical Physics and Biology [1] I have conjectured that in living systems certain coherent longitudinal electric modes are strongly excited, and stabilized by deformations arising from non linear effects. This conjecture was stimulated by the extraordinary dielectric properties of cells and of some of the relevant giant molecules in conjecture with the requirement of a non-obvious long-range order (coherence) in systems which are relatively stable but not in all respects in thermal equilibrium. It is the purpose of the present note to show that under very general circumstances the energy fed into the branch of longitudinal electric modes gets channelled into a single mode in a manner which closely resembles the condensation phenomenon in a Bose gas.

Assume the system to consist of Z components suspended in a heat bath, and consider each component capable of dipolar electric oscillations. Such a component may be a repeated dipolar part of a giant molecule (e.g. H-bond) or it may be a larger unit. The long range interaction of these components then leads to the establishment of Z modes, usually in a relatively narrow frequency range, say

$$\omega_1 \leqslant \omega \leqslant \omega_2 . \tag{1}$$

Assume that energy is supplied locally to each component at a rate s, so that it may be considered supplied to each mode of the branch at the same rate s. Interaction with the heat bath then leads to emission of energy until a steady state has been reached. This interaction with the heat bath must depend on the excitation of the modes; processes of first and second order in the number of excited quanta will be considered. The condition for stationarity then requires for each l,

$$s = L_l , \tag{2}$$

where the rate of loss of energy L_l from mode ω_l (containing n_l quanta) is assumed of the form

$$L_l = \phi(T)(n_l \exp(\hbar\omega_l/kT) - (1+n_l)) + \tag{3}$$
$$+ \chi(T) \sum_j (n_l(1+n_j) \exp\{\hbar(\omega_l - \omega_j)/kT\} - n_j(1+n_l)) .$$

The interaction terms $\phi(T) > 0$ and $\chi(T) > 0$ with the heat bath will in general depend on temperature T. Clearly L_l vanishes in thermal equilibrium when n_l is a Planck distribution n_{lT}. With the notation $N = \sum n_j$ one finds from eqs. (2) and (3)

$$n_l = \left(1 + \frac{s}{\phi + \chi N}\right) \frac{1}{\exp\{(\hbar\omega_l - \mu)/kT\} - 1} , \tag{4}$$

which requires $\hbar\omega_1 > \mu \geqslant 0$ to satisfy $n_l \geqslant 0$. Also

$$\exp(-\mu/kT) = \frac{\phi + \chi \sum (1+n_j) \exp(-\hbar\omega_j/kT)}{\phi + \chi N} = \tag{5}$$
$$= 1 - \frac{\chi}{\phi + N\chi} \frac{S(T)}{\phi} > 0 ,$$

where

$$S(T) = \phi \sum \{n_j - (1+n_j) \exp(-\hbar\omega_j/kT)\} . \tag{6}$$

Clearly Bose condensation into the lowest mode ω_1 will take place when μ approaches $\hbar\omega_1$ very closely. One would expect this to occur when s becomes sufficiently large. We shall demonstrate this explicitly for the high temperature case,

$$kT \gg \hbar\omega_1 . \tag{7}$$

In this approximation one finds from (4)

$$n_l = \left(1 + \frac{s}{\phi + \chi N}\right) \frac{kT}{\hbar\omega_l - \mu} + \ldots . \tag{8}$$

Let $D(\omega)d\omega$ be the number of modes in a range $d\omega$. Then if no condensation has taken place $D(\omega_l)\, n_l$ can be considered a continuous function and one finds (N_n denotes the total number of quanta in this case)

$$N_n = \left(1 + \frac{s}{\phi + \chi N_n}\right) kT \int_{\omega_1}^{\omega_2} \frac{D d\omega}{\hbar\omega - \mu} < \left(1 + \frac{s}{\phi + \chi N_n}\right) N_0 \qquad (9)$$

and hence in the absence of condensation when $N = N_n$ one has

$$\frac{N(\phi + \chi N)}{\phi + \chi N + s} < N_0 = \frac{kT}{\hbar} \int_{\omega_1}^{\omega_2} \frac{D d\omega}{\omega - \omega_1} \,. \qquad (10)$$

Now N can also be estimated from the generally valid eq. (6), which yields

$$S(T) = \phi \frac{\hbar\bar{\omega}}{kT} (N - N_T) \,, \qquad (11)$$

where $\omega_1 \leqslant \bar{\omega} \leqslant \omega_2$, and N_T is the number of quanta in thermal equilibrium. Also in this approximation $S(T) \approx Zs$. It follows from eq. (11) that N is a linear function of s because $\bar{\omega}$ depends only weakly on s. Hence a value of s_0 of s exists above which (10) can no longer be fulfilled. The situation then closely resembles that in the Bose condensation of ideal gases. This implies that a continuous distribution $n_l D(\omega_l)$ can no longer accommodate the total number of quanta. $\mu/\hbar$ then takes a value below, but so close to the lowest frequency ω_1 that condensation into ω_1 takes place. The total number of quanta in this case becomes $N = N_n + kT/(\hbar\omega_1 - \mu)$.

Details depend, of course, strongly on the particular model. It can be seen, however, that in general N_0 will be larger than the number of thermal quanta by a factor of the order $\omega_1/(\omega_2 - \omega_1)$, but from (10) it follows that the critical number of quanta at which condensation starts is less than N_0. Finally it should be mentioned that N, according to eq. (11) is independent of χ, but χ is very relevant in the condensation condition.

References

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CRYSTALLINE FIELD EFFECTS AND FIRST ORDER PHASE TRANSITION IN $ErCo_2$

G. PETRICH and R. L. MÖSSBAUER
Physics Department, Technische Hochschule München, Germany

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The temperature dependence of the nuclear hyperfine splitting of ^{166}Er has been measured in the magnetically ordered $ErCo_2$ compound, showing a first order magnetic phase transition. The measurements indicate a crystalline field interaction which is large compared to the exchange interaction.

Measurements of the dependence upon temperature of the nuclear magnetic and quadrupole hyperfine splitting provide information on the influence of the crystalline electric field on the energy levels of ions embedded in a magnetically ordered metal. We report here the observation of a crystalline field interaction within the $J = \frac{15}{2}$ ground multiplet of Er^{3+} in the intermetallic Laves compound $ErCo_2$, which produces an over-all splitting of the multiplet which is large compared to the exchange splitting of about $100^{\circ}K$ of the lowest $|\pm\frac{15}{2}\rangle$ multiplet.

The hyperfine splitting was observed in the recoilless gamma resonance spectrum of the 80.6 keV transition of ^{166}Er in $ErCo_2$ as absorber material where the Er environment has cubic symmetry. The measurements employed sources of $HoAl_2$ which were kept at T =