

THE HOLOGRAPHIC SCENARIO, THE MODIFIED INERTIA AND THE DYNAMICS OF THE UNIVERSE

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This paper attempts to connect two new gravitational mechanisms: the Verlinde's holographic model of gravity and the modification of inertia resulting from a Hubble-scale Casimir effect (MiHsC) of McCulloch. First we give a short survey about how the holographic scenario can give the correct dynamics of the universe. The introduction of a two-holographic screens one comparable to the Hubble horizon and a second screen that takes into account the contribution of all the matter between the test particle and the observer gives directly the modified Friedmann acceleration equation for the dynamical evolution of the universe. Improvements of this equation using the quantum corrections will realize the inflation at high energy scales and the late-time acceleration (i.e. the accelerated expansion of the universe nowadays) obviating the dark energy. From both models we can derive a version of Modified Newtonian Dynamics (MOND) observed in the dynamics of the astronomical objects obviating the dark matter and explaining other astronomical anomalies. A first connection between both theories is given at the end of the paper.

Keywords: Gravitation theory; holographic model of gravity; modified inertia; cosmology; dark energy; dark matter.

1. Introduction

In the dynamics of the universe the most important influence is the gravity. Einstein's classical general relativity is the accepted theory of gravitational interactions and is the foundation of the modern cosmology. In this theory gravity is intimately connected with the structure of the spacetime. At high energy scale, i.e. at early time of cosmological evolution the quantum effects also become important. However, a satisfactory version of the quantization of Einstein gravity is still not found. String theory solves some of the problems that appear in the quantization of Einstein gravity, but not all.

In the '70s the black hole thermodynamics was developed mainly by Bekenstein¹ and Hawking³ and opened the possibility that Einstein gravity may be related with

thermodynamics giving a possible explanation of the nature of gravity. These developments brought other problems as the Hawking's information paradox. Black hole complementarity was conjectured the solution to the black hole information paradox, proposed by Susskind²⁹ and 't Hooft.^{30,31} As a consequence of these studies, 't Hooft proposed the holographic principle which states that the description of a volume space is encoded on a boundary to the region, preferably a light-like boundary like a gravitational horizon, see Ref. 32. The holographic principle resolves the black hole information paradox within the framework of string theory. The cosmological consequences suggest that the entire universe can be seen as a two-dimensional information structure encoded on the cosmological horizon, such that the three dimensions we observe are only an effective description at macroscopic scales and at low energies, see for instance Ref. 2 and references therein.

Verlinde conjecture in a holographic scenario that the Newton and Einstein gravities are originated from an entropic force arising from the thermodynamics on a holographic screen, see Ref. 36. In fact early studies in this direction can be seen in Refs. 14 and 25. The controversy if gravity is fundamental or emergent is still open. In this sense the most recent works choose one way or the other. Furthermore, in Ref. 12 it is showed that both formulations of Newtonian gravity and the thermodynamical one, are actually equivalent, but still there does not have a completely thermodynamical description equivalent to Einstein gravity. However, in Ref. 37 from the fact that Newtonian gravity is described by a conservative force Visser establishes some constraints on the form of the entropy and temperature functions that are not satisfied by the Verlinde's entropic gravity proposal. Therefore there are also serious concerns regarding the theoretical viability of Verlinde's model.

In Ref. 4 the general relativity is still considered a fundamental theory but including a boundary term and the entropic force arises from the contribution of this boundary term. The model developed in Ref. 4 leads to the current acceleration of the universe and the inflationary period at early universe, see also Ref. 5. Nevertheless the results of these models do not go beyond those obtained previously with the addition of a source or boundary term, see Ref. 6 and references therein where the introduction of the source terms are not completely justified.

In Ref. 38 it was suggested that the universe is accelerating because our CMB radiation (at temperature $T_{\text{CMB}} = 2.73$ K) and thus our universe and the holographic screen at the comparable Hubble horizon (at temperature $T_{\text{H}} = \frac{\hbar H}{2\pi k_{\text{B}}} \sim 3 \times 10^{-30}$ K) are not in thermal equilibrium and this causes a heat transfer. Moreover, the temperature gap is always very large at early times up to Planck scale. To obtain the result, in Ref. 38 it is conjectured that the existence of two holographic screens, one being the approximate Hubble horizon which is similar to the de Sitter (dS) horizon, while the other is a Schwarzschild horizon. The introduction of this Schwarzschild horizon is not completely justified in Ref. 38. The classical dynamics of such cosmological system can be fixed to obtain thermal equilibrium that corresponds to the standard FRW universe and the model is able to explain the thermal history

of our universe. Moreover, the quantum corrections give the entropic inflation at early universe and the late-time acceleration in a unified approach.

In Sec. 5, we will see a justification for the introduction of the second Schwarzschild screen. We can also consider the case where gravity is emergent from basic thermodynamic on a holographic screen (not necessary corresponding to a Schwarzschild horizon), see Ref. 26. Moreover, with the introduction of a second screen corresponding to a comparable Hubble horizon, it permits one to arrive at the same modified Friedmann equations found in Ref. 4 where gravity is considered fundamental. Consequently the holographic model with these two screens is equivalent to consider gravity as fundamental and use the Einstein equations of the general relativity but including a boundary term.

2. Verlinde Holographic Scenario

Verlinde propose a model where the second Newton law and Newton's law of gravitation arise from basic thermodynamic mechanisms. In the context of Verline's holographic model, the response of a body to the force may be understood in terms of the first law of thermodynamics. We consider a holographic screen in the plane yz that intersects the x -axis at $x + \Delta x$, where Δx is a small increment distance. As the body approaches the screen, its descriptive information becomes encoded holographically on the screen. The entropy of the screen increases by some amount ΔS . In a similar way in which a particle approaching the event horizon of a Schwarzschild black hole increases the entropy of the horizon, in Ref. 36 it is proposed that

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x. \quad (1)$$

When the body traverse the distance Δx , its energy changes by an amount $\Delta E = F\Delta x$, which is the incremental work done by the force F . Using the first law of thermodynamics, the model sets that

$$F\Delta x = T\Delta S. \quad (2)$$

An observer in an accelerated frame experiences the associated Unruh³⁵ temperature

$$T = \frac{1}{2\pi} \frac{\hbar a}{k_B c}. \quad (3)$$

The second law of Newton $F = ma$ follows from substituting Eqs. (1) and (3) in (2). Now it is assumed that the boundary is a closed surface, it is assumed that is an sphere. Assuming that the holographic principle holds, the maximal storage space, or the total number of bits, is proportional to the area of the boundary

$$N = \frac{Ac^3}{G\hbar} = \frac{4\pi R^2 c^3}{G\hbar}, \quad (4)$$

where a new constant G is introduced. The total energy is given by the equipartition rule

$$E = \frac{1}{2}Nk_{\text{B}}T. \tag{5}$$

Now we consider the total energy enclosed by the screen which is given by a mass M , i.e. it is satisfied $E = Mc^2$. Now equating this equation with Eq. (5) and substituting Eqs. (4) and (1) we obtain the Newton’s law of gravitation

$$F = G \frac{mM}{R^2}, \tag{6}$$

and the constant G is the universal gravitational constant. From this arguments it is stated in Ref. 36 the entropic origin of gravity because the acceleration is related with an entropy gradient.

The derivation of the Einstein equations in the holographic scenario is analogous to previous works, in particular, Ref. 14. Verlinde in his paper opens the question about if gravity is still a fundamental force or not. Gravity and spacetime are emergent phenomena and consequence of the statistical averaged random dynamics at the microscopic level, an old dream of some physicists as Sakharov, see Ref. 27. These revolutionary ideas have found many detractors and, in general, the further works treat the question if gravity is fundamental or not. Other works just studied the consequences of the proposed model. In Ref. 11 the cosmological consequences of the holographic scenario in relation with the McCulloch’s modified inertia theory are analyzed.

3. McCulloch’s Modified Inertia

In 2007, McCulloch proposed a model for inertia that could be called a modification of inertia resulting from a Hubble-scale Casimir effect (MiHsC) or in the next works also called Quantized Inertia. MiHsC assumes that the inertial mass of an object is caused by Unruh radiation (see Ref. 35) resulting from its acceleration with respect to surrounding matter, and that this radiation is subject to a MiHsC. This means that only Unruh waves that fit exactly into twice the Hubble diameter are allowed, so that an increasingly greater proportion of the Unruh waves are disallowed as accelerations decrease and these waves get longer, leading to a new gradual loss of inertia as acceleration reduces, see Ref. 15. This loss of inertia is far more gradual than Milgrom’s in Modified Newtonian Dynamics (MOND), see Ref. 22. In MiHsC the inertial mass becomes

$$m_I = m_i \left(1 - \frac{\beta \pi^2 c^2}{|a| \Theta} \right) \sim m_i \left(1 - \frac{2c^2}{|a| \Theta} \right), \tag{7}$$

where a is the total acceleration of the test particle, β appear in the Wien’s constant and has the value $\beta = 0.2$, and Θ is the Hubble diameter $\Theta = \frac{2c}{H_0} = 2R_U$. The modified inertia is applied to explain the Pioneer anomaly from the equation

$$m_I a = \frac{GM_\odot m_g}{r^2}, \tag{8}$$

where M_\odot is the solar mass and r is the distance from the Sun. In particular, in Ref. 15, it is found that the acceleration of the Pioneer craft is given by

$$a = \frac{GM_\odot}{r^2} + \frac{\beta\pi^2 c^2}{\Theta}. \tag{9}$$

The second term can be rearranged to give

$$a = \frac{GM_\odot}{r^2} + \frac{1}{2}\beta\pi^2 cH_0 \sim \frac{GM_\odot}{r^2} + 0.99 \times cH_0. \tag{10}$$

In a series of works McCulloch applied with success the modified inertia by a Hubble scale Casimir effect (MiHsC) to several problems and anomalies, see Refs. 16–20. Moreover, in Ref. 21 the connection between the MiHsC and the MOND is established and it is proved that the MiHsC predicts a Tully–Fisher relation³³ (in a similar form of MOND $v^4 = GMa_0$ with a_0 the constant acceleration) of the form

$$v^4 = GM \frac{2c^2}{\Theta} = GMcH_0, \tag{11}$$

which is in agreement with the observed data taking into account the errors bars.

4. Cosmology Consequences of the Verlinde’s Model

In Ref. 4, taking into account the entropy and temperature intrinsic to the horizon of the universe due to the information holographically encoded there, it is shown that we can obviate the dark energy and the accelerated expansion of the universe is due to an entropic force. The reasoning is the following: At this horizon there is a horizon temperature given by

$$T_H = \frac{\hbar H}{2\pi k_B} \sim 3 \times 10^{-30} \text{ K}, \tag{12}$$

and this temperature has associated the acceleration a_H given by the Unruh³⁵ relationship

$$a_H = \frac{2\pi c k_B T_H}{\hbar} \tag{13}$$

and substituting the value of T_H we arrive at $a_H = cH \sim 10^{-9} \text{ m/s}^2$ in agreement with the observation. The entropic force pulls outward towards the horizon apparently creating a dark energy component. In Ref. 4 it is stated that the possibility that the cosmic acceleration can be described by an entropic force should be distinguished from the idea that gravity itself is an entropic force; although the two ideas are not *prima facie* incompatible. However, the deduction of the acceleration is directly applying the Verlinde arguments to the screen given by the horizon of the

universe. This can be seen when the principles of entropy and thermodynamics are applied to show that the model also gives the present fraction of the critical energy associated with the accelerated expansion (before attributed to the dark energy). The entropy on the comparable Hubble horizon is given by

$$S_H = \frac{k_B c^3 A}{4G\hbar} = \frac{k_B c^3 \pi R_H^2}{G\hbar}. \tag{14}$$

The incremental ratio respect to r (the radial variable) is

$$\frac{\Delta S_H}{\Delta r} = \frac{k_B c^3 2\pi R_H}{G\hbar}. \tag{15}$$

Using Eq. (15) and taking into account that $R_H = \frac{c}{H}$ the entropic force is

$$F = -\frac{dE}{dr} = -T_H \frac{dS}{dr} = -\frac{c^4}{G}, \tag{16}$$

and the pressure exerted is $P = \frac{F}{A} = \frac{-2\rho_c c^2}{3}$ where ρ_c is the critical energy density $\rho_c = \frac{3h^2}{8\pi G}$. Close to the value of the currently measured dark energy in relation with ρ_c . However, Eqs. (1) and (15) are similar. To see this we use the following equality that links the mass of the universe and its radius

$$\frac{GM_U}{c^2 R_H} = 1, \tag{17}$$

obtained in different contexts, see Ref. 8. Using equality (17), Eq. (15) takes the form

$$\Delta S_H = \frac{2\pi k_B M_U c}{\hbar} \Delta r, \tag{18}$$

and Eq. (18) is similar to Eq. (1). In fact in one case, Eq. (1), it is being measured how the entropy increases when the mass m is approaching the screen and in the other case, Eq. (18), how the entropy increases when the total mass of the universe M_U is approaching the comparable Hubble horizon. Hence the model in both works is the same but applied to different contexts.

From here the work of Easson, Frampton and Smoot⁴ takes a different track. For them the gravitation is still a fundamental force and therefore Einstein’s equation is modified to take into account the boundary terms that reflect the entropic force towards the comparable Hubble horizon. This point of view, as we will see, is also followed by the authors of Ref. 38.

5. The Model of the Dynamics of the Universe

In fact when we talk about the model of the dynamics of our universe we are talking about the causal connected region of the universe around an observer. Therefore each observer is talking about its causal connected region which is different from other observer located in another place. In order to have a simplified notation in this section we take the convention $c = k_B = \hbar = 1$.

The dynamics of the universe refers to the dynamics of its content (matter and radiation) in the bulk. In the case of matter, in order to analyze its dynamics we consider a test particle located at some place p' of the universe at distance R of some observed located at p . We consider the causal connected region of the universe around this observer at p . Then we want to study the forces that act to the test particle at p' . One entropic force is the F_H toward the comparable Hubble horizon. Now we are going to find the others.

The equations of motion of the dynamic of the universe come from the variational principle where boundary terms play an important role of the background evolution. Hence, the Einstein–Hilbert action is

$$\mathcal{I} = \int_M (R + \mathcal{L}_m) + \oint_{\delta M} \mathcal{L}_b, \tag{19}$$

where R is the Ricci scalar of the whole spacetime, \mathcal{L}_m is the Lagrangian of matter fields, and \mathcal{L}_b is the corresponding Lagrangian describing the physics of the boundary. The application of variational procedures gives the usual Einstein field equations for general relativity with and additional surface energy term

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = 8\pi GT_m^{\mu\nu} + J_b^{\mu\nu}, \tag{20}$$

where the term $J_b^{\mu\nu}$ describes the exchange of energy and momentum between the bulk and the boundary. Now we consider a homogeneous and isotropic flat Friedmann–Robertson–Walker (FRW) universe described by the metric

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2), \tag{21}$$

where $a(t)$ is the scale factor and the Einstein field equation gives the following equation for the acceleration of the scale factor

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{a_H}{L_b}, \tag{22}$$

where L_b is a length scale relevant to the location of the holographic screen. If we put the values $a_H = H$ (recall that we have taken $c = 1$) and $L_b = \frac{1}{H}$ we obtain (see Ref. 4) that

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + H^2. \tag{23}$$

From this equation we obtain that the evolution of our universe would be changed too much and is hardly able to recover the usual form in radiation and matter dominated periods. In Ref. 4 the authors suggest to modify *ad hoc* some coefficients to better fit the data and introduce an alternative entropic acceleration a_H . In Ref. 38, the value of L_b is taken equal $L_b = \frac{1}{\beta H}$ where β is an undetermined coefficient which will be constrained by cosmological observations.

In Sec. 5.1 we will present a justification of the introduction of the second screen.

5.1. The justification of the second screen

First we consider that all the matter–energy content M_S in the spherical bulk around the observer and of radius R is concentrate at the point p (applying the Gauss’s law) acting to the test particle. Hence, we can consider that in the place of the observer as if there were a black hole which creates a Schwarzschild horizon at some radius $r_S = 2GM_S$ and its corresponding temperature is

$$T_S = \frac{1}{8\pi GM_S} . \tag{24}$$

Hence, on the test particle acts a second entropic force F_S toward this Schwarzschild horizon created by the spherical bulk of radius R between the test particle and the observer and given by

$$a_S = 2\pi T_S = \frac{1}{4GM_S} . \tag{25}$$

The conclusion is that the force is different in each particle because it depends on its distance R to the observer. For instance consider two particles at distances R_1 and R_2 from the observer, respectively, with $R_1 < R_2$ then we have that $M_{S_1} < M_{S_2}$ and therefore $a_{S_1} > a_{S_2}$. The consequence is that as further away the test particle, the lower the a_S and the entropic force F_S of the Schwarzschild horizon. This is what is observed in the universe. The a_H is the same for all the objects but a_S decreases with the distance. The conclusion is that any particle is going far away in an accelerated movement with $a_e = a_H - a_S$. For a test particle close to the comparable Hubble horizon, the event horizon has the radius

$$r_S = 2GM_U = 2G \int_{M_U} \rho dV = \frac{8\pi G\rho R_H^3}{3} = \frac{8\pi G\rho}{3\beta^3 H^3} , \tag{26}$$

because $R_H = \frac{1}{\beta H}$. In this case its corresponding temperature is given by

$$T_S = \frac{3\beta^3 H^3}{32\pi^2 G\rho} \tag{27}$$

and the modified entropic acceleration is

$$a_e = 2\pi(T_H - T_S) = \beta H - \frac{3\beta^3 H^3}{16\pi G\rho} . \tag{28}$$

The modified Friedmann acceleration equation is obtained by substituting a_H in (22) by a_e and is given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \beta^2 H^2 - \frac{3\beta^4 H^4}{16\pi G\rho} , \tag{29}$$

which is consistent with the data points of SN Ia at low redshift and a good approximation for the high redshift. In Ref. 38 it is considered the case $\beta = \sqrt{2}$ to have thermal equilibrium with $T_H = T_S$ and recover the exact form of the traditional Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho \Big|_{\text{equilibrium}} . \tag{30}$$

Moreover, in Ref. 38 the higher order quantum corrections for the holographic entropy and consequently for the entropic force that may rise to an implement of holographic inflation with an improved accelerate equation (29) are considered. The quantum corrections to the process of horizon evaporations could bring to a realization of late-time acceleration. In order to let this acceleration, there is fine-tuning of the coefficients of higher order. Finally the model is tested with the SN Ia observations.

The introduction of the second screen, the Schwarzschild horizon is introduced in order to find a thermal equilibrium. In fact the introduction of this second screen, in some sense, follows the Verlinde conjecture of the origin of the gravity and accounts for the entropic force of black holes. If we impose the thermal equilibrium (choosing $\beta = \sqrt{2}$), then we must use the quantum corrections to obtain the late-time acceleration. In fact the nonexistence of thermal equilibrium can give directly the late-time acceleration via Eq. (23) as is showed in Ref. 4 and also commented in Ref. 38 but it is hardly able to recover the usual form in radiation and matter dominated periods. Consequently, Eq. (23) is a first approximation to the correct equations of the dynamics of the universe. Moreover, the quantum corrections must be taken into account at low energies to give a better improvement of Eq. (23) to obtain a better approximation of the late-time acceleration, see Ref. 4. This quantum correction must also be taken into account at high energies in the early time of cosmological evolution to implement the holographic inflation, see Ref. 5.

In Sec. 5.2 we consider the case where gravity emerges from basic thermodynamic on a holographic screen (not necessary corresponding to a Schwarzschild horizon), see Ref. 26. This holographic screen takes into account the contribution of all the matter between the test particle and observer and gives directly the classical Friedmann equation in general relativity (see Eq. (35) below). Moreover, the introduction of a second screen corresponding to a comparable Hubble horizon permits to arrive at the same modified Friedmann equations found in Ref. 4, i.e. Eq. (23).

5.2. The Friedmann equation from entropic force

In this section we are going to derive the Friedmann equation governing the dynamical evolution of the FRW universe from the existence of an entropic force together with the equipartition law of energy and the Unruh temperature. For this purpose we follow the reasonings given in Ref. 26. As before we consider a test particle at the point p' and the observer at point p . We consider the compact spatial region \mathcal{V} given by spherical bulk of radius R between the test particle and the observer, with boundary $\partial\mathcal{V}$. This sphere of physical radius $R = a(t)r$ where $a(t)$ is the scale factor. Of course we are considering the FRW universe with metric (21). The compact boundary $\partial\mathcal{V}$ acts as the holographic screen. The number of bits on the screen is given by (4). Note that there is a factor difference $\frac{1}{4}$ from the Berkenstein–Hawking area entropy formula of black hole. We assume that the temperature on the screen

is T_ν , and according to the equipartition law energy, the total energy on the screen is given by (5). On the other hand, we have $E = Mc^2$ where M is the mass in the compact spatial region \mathcal{V} . If we assume that the matter source in the FRW universe is a perfect fluid stress–energy tensor $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$. The total mass $M = \rho V$ in the spatial region \mathcal{V} can be expressed as

$$M = \int_{\mathcal{V}} dV (T_{\mu\nu} u_\mu u_\nu), \quad (31)$$

where $T_{\mu\nu} u_\mu u_\nu$ is the energy density. The acceleration in the test particle at p' (in fact in any place of the screen) is

$$a_r = -\frac{d^2 R}{dt^2} = -\ddot{a}(t)r, \quad (32)$$

where the negative sign arises because the force is toward the screen. Now according to Unruh formula this acceleration corresponds to a temperature given by (3). From here it is straightforward to derive the equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho, \quad (33)$$

from Eqs. (3)–(5) and (31). In order to produce the Friedmann equation of FRW universe in general relativity, we must use the so-called active gravitational mass \mathcal{M} (see Ref. 24), rather than the total mass M . The active gravitational mass is the called Tolman–Komar mass defined as

$$\mathcal{M} = 2 \int_{\mathcal{V}} dV \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) u_\mu u_\nu. \quad (34)$$

Now replacing M by \mathcal{M} we obtain the acceleration equation for the dynamical evolution of the FRW universe given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \quad (35)$$

from which we can obtain the traditional Friedmann equation (30). Now the introduction of the screen in the comparable Hubble horizon permits to arrive unavoidably at Eq. (23) the first approximation to the correct equation for the dynamical evolution of the universe. In Fig. 1 we see the two screens influencing the test particle p' . However, the quantum corrections must be taken into account at low energies to give a better improvement of Eq. (23). These corrections are introduced to obtain a better approximation of the late-time acceleration and also to implement the holographic inflation, see Refs. 4, 5 and 38.

6. Entropic Dynamics, Modified Inertia and MOND

As we have mentioned, in Ref. 21 the connection between the MiHsC of McCulloch and the MOND has been established. Verlinde’s model is also connected with MOND. In fact Verlinde’s holographic model in an asymptotically de Sitter space leads to a new form of the second law of motion which is required by the MOND

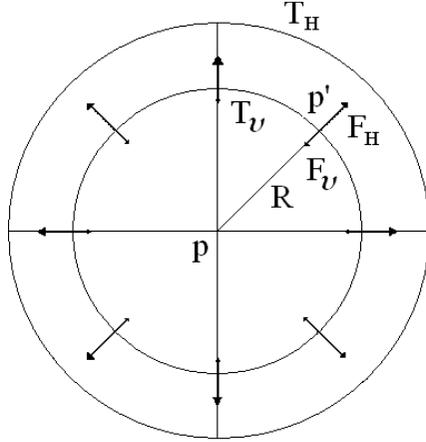


Fig. 1. Two screens influencing the test particle p' .

theory proposed by Milgrom, see Ref. 7. Therefore the phenomenological Milgrom formulation is supported by both models the holographic scenario and the modified inertia. In Ref. 7 it is demonstrated that, in a universe endowed by a positive cosmological constant Λ , the holographic model described by Verlinde leads naturally to a modification of the second Newton's law of the form

$$m[(a^2 + k^2)^{\frac{1}{2}} - k] = F, \tag{36}$$

where $k = \sqrt{\frac{\Lambda}{3}}$. In our case the positive cosmological constant Λ is given by the contribution of the boundary terms due to the existence of a screen in the comparable Hubble horizon. If we consider the Einstein field equations for general relativity with the cosmological constant Λ we have

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = \frac{8\pi G}{c^4}T_m^{\mu\nu} + \Lambda g^{\mu\nu}, \tag{37}$$

which give the modified acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{38}$$

Comparing with Eq. (23) we obtain that $\frac{\Lambda}{3} = H^2$. Moreover, Eq. (36) is identical to the specific formulation of MOND suggested by Milgrom in Ref. 23. In the limit $\frac{a}{k}$ arbitrarily large (36) becomes identical to the Newton second law and for $\frac{a}{k} \ll 1$, we have

$$m\frac{a^2}{2k} = F, \tag{39}$$

where $2k$ plays the role of the constant acceleration a_0 of the MOND theory. Consequently, the MOND observed in the dynamics of the astronomical objects can also be obtained in the holographic scenario and in the context of the modified inertia MiHsC obviating the dark matter. In Ref. 9 (see also Ref. 10) it is also showed

that the Pioneer anomaly can be explained in the context of the MOND theory and consequently in the context of the holographic scenario. Nevertheless, a simple derivation of MOND from both models does not save MOND from its inherent problems, see for instance Refs. 10, 13, 28, 29 and 34.

Verlinde's model and McCulloch model are based on the Unruh radiation resulting from the acceleration of the observer with respect to surrounding matter. This acceleration, if the observer describes the movement of a test particle in a point sufficiently far, is given by the Unruh³⁵ relationship

$$a_H = \frac{2\pi ck_B T_H}{\hbar}, \quad (40)$$

where T_H is the associated temperature. Therefore the total acceleration measured by this observer is

$$a = a_L + a_H, \quad (41)$$

where a_L is the local acceleration due to the local dynamics that suffers the particle. It is clear that only for very low local movements the acceleration a_H becomes important. We can assume that the local movement is the gravitational attraction of a central mass like the sun. Therefore we have

$$a - a_H = a_L = \frac{GM_\odot}{r^2}. \quad (42)$$

Equation (42) can be written into the form

$$a \left(1 - \frac{a_H}{|a|} \right) = \frac{GM_\odot}{r^2}. \quad (43)$$

Finally we obtain a modified inertia (in a similar way that in the MiHsC) given by

$$m_I = m_g \left(1 - \frac{a_H}{|a|} \right) = m_g \left(1 - \frac{2\pi ck_B T_H}{\hbar |a|} \right). \quad (44)$$

Obviously the T_H can be interpreted as the temperature of the horizon comparable to the Hubble horizon that in the context of the Verlinde's theory produce an entropic force.

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