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Cosmological Consequences of the Holographic Scenario

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Abstract In this paper we discuss some consequences of Verlinde's holographic gravity model. Among other things, it yields the observed acceleration of the universe and the inflationary period at early universe obviating the Dark energy. From the Verlinde's theory of gravity the first phenomenological Modified Newtonian dynamics obviating the Dark matter can be deduced. Moreover through the connection with the Modification of inertia resulting from a Hubble-scale Casimir effect (MiHsC) of McCulloch the model gives a promising possible explanation to the Pioneer anomaly, the flyby anomalies, the Tajmar effect and the minimum mass observed in the disc galaxies.

Keywords Modified Newtonian Dynamics · Modified inertia · Cosmology · Dark energy · Dark matter · Pioneer anomaly · Flyby anomalies · Tajmar effect

1 Verlinde Holographic Scenario and MOND

Verlinde proposed a model where the second Newton law and Newton's law of gravitation arise from basic thermodynamic mechanisms, see [90]. The model is based in the Unruh effect [88]. In the context of Verline's holographic model, the response of a body to the force may be understood in terms of the first law of thermodynamics, see [90]. Moreover the entropic origin of gravity is claimed because the acceleration is related with an entropy gradient. More precisely, gravity is explained as an entropic force caused by changes in the information associated with the positions of material bodies. The consequences of this general theory are currently being analyzed and discussed, see for instance [17, 18, 23, 31]. For example, the cosmological acceleration can be explained using the entropic force, see [17]. However in [17] the general relativity is still considered a fundamental theory but including a boundary term (contrary to what is claimed in the Verlinde's holographic model of gravity where the Einstein equations of the general relativity are deduced from the model, see [90]). In [17] the entropic force arises from the contribution of this boundary term. The

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model leads to the current acceleration of the universe and the inflationary period at early universe, see also [18, 23]. Other important consequence related to Verlinde's model is that the Verlinde's holographic model in an asymptotically de Sitter space leads to a new form of the second law of motion which is the required by the Modified Newtonian dynamics theory (MOND) proposed by Milgrom, see [24, 48, 49].

The MOND was introduced by Milgrom to solve the galaxies rotation curves problem as an alternative to the dark matter. The MOND can be implemented by a modification of the Newton's second law or the Newton's law of gravity. In the formulation where the Newton's second law is modified, Milgrom allowed for an inertia term not to be proportional to the acceleration of the object but rather to be a more general function of it. More precisely, it has the form

$$m_i \ \mu(a/a_0)\ \mathbf{a} = \mathbf{F},$$

where $\mu(x \gg 1) \approx 1$, and $\mu(x \ll 1) \approx x$ and $a = |\mathbf{a}|$, replacing the classical form $m_i\ \mathbf{a} = \mathbf{F}$. Here m_i is also the inertial mass of a body moving in an arbitrary static force field $\mathbf{F}$ with acceleration $\mathbf{a}$, see [61]. For accelerations much larger than the acceleration constant a_0, we have $\mu \approx 1$, and Newtonian dynamics is restored. However for small accelerations $a \ll a_0$ we have that $\mu = a/a_0$. In [27] (see also [25]) is found a new form of $\mu(x)$ for the Milgrom formula given by

$$m_i \left(\frac{|\mathbf{a}|}{|\mathbf{a} + \mathbf{a}_e|} \right) \mathbf{a} = \mathbf{F}, \tag{1}$$

where $\mathbf{a}_e$ is an effective acceleration given by $\mathbf{a}_e = \mathbf{a}_0(1 - R_{obs}/R_U)\hat{\mathbf{R}}_U$, R_{obs} is the distance to the object and R_U is the radius of the causal connected universe. For local objects we have $a_e \sim a_0$ and for far away objects we get $a_e \sim 0$. The form of $\mu(x)$ presented in [27] is a modification of the inertia following the ideas developed by Milgrom in [63, 65] and using the classical Newtonian dynamics assuming the accelerated expansion of the universe. Nevertheless, is a phenomenological formulation that requires a rigorous and comprehensive theory, see [11] to see the first attempts.

However, as we have said, the phenomenological Milgrom formulation is supported by Verlinde's theory. In [24] it is demonstrate that, in a universe endowed by a positive cosmological constant Λ, the holographic model described by Verlinde leads naturally to a modification of the second Newton's law of the form

$$m[(a^2 + k^2)^{1/2} - k] = F, \tag{2}$$

where $k = \sqrt{\Lambda/3}$. Moreover Eq. (2) is identical to the specific formulation of MOND suggested by Milgrom in [63]. In the limit a/k arbitrarily large (2) becomes identical to the Newton second law and for $a/k \ll 1$ we have

$$m \frac{a^2}{2k} = F,$$

where $2k$ plays the role of the constant acceleration a_0. In fact, if we assume that the present evolution of the universe is dominated by the cosmological constant Λ, as corroborated by observation [84], we can set $cH_0 \sim \Lambda^{1/2}$ which implies that $k \sim a_0$ in orders of magnitude. The relation between a_0 and the cosmological constant it is also discussed in [29, 30] in the context of the scaling laws that suggest a quantum and fractal universe. Nevertheless the relation between the Verlinde's theory and MOND is widely regarded in [51] where a modified entropic force is connected with modified Newtonian dynamics. Therefore the MOND can be derived from the Verlinde's theory and it must be considered as a consequence of it.

2 Other Consequences of the Holographic Scenario

There are several unexplained anomalies connected with astrometric data of the solar system, see [7]. The Pioneer anomaly is one of them (see [5, 6, 19, 86]) and consists of unexpected, almost constant and uniform acceleration directed approximately towards the Sun $8.74 \pm 1.33 \times 10^{-10}\ \mathrm{m\,s^{-2}}$ first detected in the analyzed data of the Pioneer probes after they passed the threshold of 20 Astronomical units. Milgrom realized that MOND could explain the Pioneer anomaly, see [64]. The modified-inertia approaches to solve the Pioneer anomaly have been also considered under Unruh radiation by McCulloch, see [52]. In particular [52] it is found that the acceleration of the Pioneer craft is given by

$$a = \frac{GM_\odot}{r^2} + \frac{\beta\pi^2 c^2}{\Theta},$$

where $M_\odot$ is the sun mass, β appear in the Wien's constant and has the value $\beta = 0.2$, and Θ is the Hubble diameter $\Theta = 2c/H_0 = 2R_U$. The second term can be rearranged to give

$$a = \frac{GM_\odot}{r^2} + \frac{1}{2}\beta\pi^2 cH_0 \sim \frac{GM_\odot}{r^2} + 0.99 \times cH_0. \tag{3}$$

In [28] it is obtained Eq. (3) in the context of phenomenological formulation of MOND. In fact, in the limit case $a_0/a \ll 1$ i.e., $a_0 \ll a$, the expression obtained is

$$a = \frac{GM_\odot}{r^2} + a_0, \tag{4}$$

where a_0 is the acceleration constant of MOND. In [25] and [26] it is justified by different arguments that $a_0 \sim cH_0$, where H_0 is the actual value of the Hubble constant, see also [27]. Therefore we have found Eq. (3) but in the context of the MOND theory. The arguments to obtain $a_0 \sim cH_0$ are the following. In [25] using the equivalence principle, which implies the equality between inertial mass m_i and gravitational mass m_g, it is obtained the relation $GM_U = c^2 R_U$ where M_U and R_U is the mass and the radius of the universe respectively. The Mach's principle following the implementation of Sciama [79] affirms that the inertial mass is caused by the rest of the matter in the universe. Then in the case without local movements (or low acceleration $a \ll a_0$) the main contribution to the acceleration is a_0 and we have

$$m_i a_0 = G\frac{m_g M_U}{R_U^2}. \tag{5}$$

Assuming $m_i = m_g$ and substituting $GM_U = c^2 R_U$ in (5) the result follows. In [26] the relation $a_0 \sim cH_0$ is obtained through the scale factor of the universe $R(t)$ and the Hubble law of expansion of the universe. In fact $cH_0 = 6.9 \times 10^{-10}\ \mathrm{m/s^2}$ and the constant a_0 empirically derived from the galaxy data is typically $2 \times 10^{-10}\ \mathrm{m/s^2}$. However in [58] it is also established the connection between the MiHsC of McCulloch and the MOND and it is proved that the MiHsC predicts a Tully-Fisher relation [85] (in a similar form of MOND $v^4 = GMa_0$) of the form

$$v^4 = GM\frac{2c^2}{\Theta} = GMcH_0, \tag{6}$$

which is in agreement with the observed data, given the errors.

Nevertheless a simple modification of the $\mu(x)$ function does not save MOND from its inherent problems, see for instance [27, 78, 86]. It is still open to find the form of $\mu(x)$

consistent with the observational data which establish differences between the unbounded orbits (like the Pioneer craft) and the bounded orbits (like the planets). A first attempt is the use of the External Field Effect (EFE), see for instance [14, 36, 37, 39, 41, 44, 61, 62, 66]. We recall that the form of $\mu(x)$ presented in [27] was obtained assuming the accelerated expansion of the universe, hence it must be applied to systems that follow the cosmological expansion. In [73] Price shows that a bound system does not follow the cosmological expansion, whereas an unbound system does.

In principle, any gravitational explanation must face the issue of why the motions of the major bodies of the solar system do not show the Pioneer Anomaly themselves. Suitable references about this key issue are [20, 21, 33–35, 40, 45, 46, 68–70, 81–83, 91]. A gravitational mechanism able to accommodate the Pioneer Anomaly without destroying the agreement with planetary observations can be found in [89]. Moreover in recent times several studies appeared proposing conventional, non-gravitational explanations for the Pioneer Anomaly, see for instance [12, 13, 22, 74–77, 87].

Anyway, the correct version of MOND must be derived from the correct cosmology. There is nothing yet to prove experimentally that the Verlinde holographic scenario or the MiHsC of McCulloch is correct. In a series of works McCulloch apply with success the modified-inertia by a Hubble scale Casimir effect (MiHsC) to several problems and anomalies. However a similar modified-inertia developed by McCulloch can also be derived from the MOND theory and consequently from the holographic scenario. In order to see this we consider a local object in (1) i.e., $a_e \sim a_0$. Then we consider that we are in a strong Newtonian regime i.e., $a_0 \ll a$ and we get

$$m_i \left(1 - \frac{a_0}{|\mathbf{a}|}\right) \mathbf{a} = \mathbf{F}. \tag{7}$$

It is clear that we can obtain (4) from Eq. (7) (taking the modulus) and choosing $\mathbf{F}$ the gravitational force of a central mass $M_\odot$. Consequently we can define the modified-inertia of the form

$$m_{im} = m_i \left(1 - \frac{a_0}{|\mathbf{a}|}\right) = m_g \left(1 - \frac{a_0}{|\mathbf{a}|}\right), \tag{8}$$

taking into account the equivalence principle that implies $m_i = m_g$. The reduction of the inertial mass for low accelerations was first suggested by Milgrom [63]. The problem with this approach is that it violates the equivalence principle. However, as noted by McGaugh [59] this principle has not been tested at very low accelerations, which is difficult to attain on Earth.

In the following we apply this definition to different scenarios and we obtain the same results that those obtained by McCulloch.

Other large-scale dynamical anomaly that remain unexplained are the flyby anomalies described in [3, 4, 8]. As before this problem can also be interpreted as unexpected increases in gravitational interaction. To analyze the trajectories of the flyby craft (which are not bound to the Earth) it is assumed conservation of momentum.

$$m_{1e} v_{1e} + m_1 v_1 = m_{2e} v_{2e} + m_2 v_2.$$

Replacing the inertial masses m_1 and m_2 of the unbound craft by the modified inertial mass (8) we have

$$m_{1e} v_{1e} + m_g \left(1 - \frac{a_0}{|\mathbf{a}_1|}\right) v_1 = m_{2e} v_{2e} + m_g \left(1 - \frac{a_0}{|\mathbf{a}_2|}\right) v_2.$$

This gives an additional term in the velocity change that gives the correct corrections (with some exceptions) to the unexplained velocity anomalies of the observed Earth flybys, see [53, 54].

The Tajmar effect is an unexplained acceleration observed by gyroscopes close to rotating supercooled rings. In [55, 57] the model of modified inertia is also applied to this problem. The model assumes that the inertia of the gyroscope is caused by Unruh radiation that appears as the ring and the fixed stars accelerate relative to it, and that this radiation is subject to a Hubble-scale Casimir effect. Assuming the conservation of the momentum parallel to the ring's edge we have

$$m_{g1} v_{gr1} = m_{g2} v_{gr2},$$

where v_{gr} is the velocity of the gyroscope with respect to the ring. Replacing the inertial masses with the modified inertial masses we get

$$\left(1 - \frac{a_0}{|\mathbf{a}_{g1}|}\right) v_{gr1} = \left(1 - \frac{a_0}{|\mathbf{a}_{g2}|}\right) v_{gr2},$$

where a_{g1} and a_{g2} are the initial and final accelerations of the gyroscope with respect to all the surrounding masses. Working with this last equation, taking into account the spinning of the Earth and the acceleration of the Tajmar ring it is possible to derive the acceleration change observed in the Tajmar effect, see [55, 57].

In [60] it has recently been observed that there are no disc galaxies with masses less than $10^9 M_\odot$ (within the central 500 parsecs) and this cutoff has not been explained. In [56] it is shown that this minimum mass can be predicted using MiHsC. In the words of the author "The model predicts that as the acceleration of an object decreases, its inertial mass eventually decreases even faster stabilising the acceleration at a minimum value, which is close to the observed cosmic acceleration". In the present paper we see that in fact the cosmic acceleration has a key role and that this minimum value can be deduced in the context of the holographic scenario. Equation (4) for a disc galaxy is

$$a \sim \frac{GM}{r^2} + cH_0 = \frac{GM}{r^2} + \frac{c^2}{R_U}. \tag{9}$$

Therefore considering smaller and smaller galaxies, the mass M in equation (9) will decrease and the extra acceleration due to the new second term will become ever more important. The second term in (9) c^2/R_U is attributed to dark mass M_{dark} (in fact misinterpreted) so that

$$a = \frac{GM_{dark}}{r^2} = \frac{c^2}{R_U}. \tag{10}$$

Therefore this apparent dark mass must be

$$M_{dark} = \frac{c^2 r^2}{GR_U}, \tag{11}$$

which within a radius of $r = 500$ parsecs gives the value of $M_{dark} = 2.3 \times 10^{39}$ kg $= 1.1 \times 10^9 M_\odot$ and we obtain a minimum apparent mass close to the observed minimum mass.

At large-scale equation (11) can be applied to the observable universe (which has radius R_U) so that all apparent dark mass in the universe must be

$$M_{U_{dark}} = \frac{c^2 R_U}{G} = 2 \times 10^{53} \text{ kg}. \tag{12}$$

In fact Eqs. (11) and (12) are based on those given in [56]. In [25] was obtained the identity

$$\frac{GM_U}{c^2 R_U} = 1, \tag{13}$$

applying the Mach's principle and taking the equivalence principle as nature principle which implies the equality between the inertial mass and the gravitational mass $m_i = m_g$. An expression like (13) was obtained by Whitrow (1946) [92], Whitrow and Randall (1951) [93], Sciama (1953) [79], Brans and Dicke (1961) [15, 16] and also by Assis (1989) [9, 10] in different contexts. From Eqs. (12) and (13) we see that the mass of the universe M_U and the apparent dark mass $M_{U_{dark}}$ coincide. This implies that most important contribution to the mass of the universe is from the apparent dark mass and also gives an explanation of the flatness problem, see also [56].

3 Concluding Remarks

This paper attempts to connect two new potential cosmologies, the Verlinde model and the MiHsC of McCulloch. From both can be derived a version of Modified Newtonian Dynamics MOND and the equations obtained are similar. In a series of papers, McCulloch has tested MiHsC quite successfully against some observational data. In this paper we conclude that therefore the Verlinde's model also gives a promising possible explanation to the Pioneer anomaly, the flyby anomalies, the Tajmar effect and the minimum mass observed in the disc galaxies.

The two gravitational mechanisms presented in this work, Verlinde's theory of gravity and the MiHsC of McCulloch must be put to the test on as many different and independent phenomena as possible, in order to discriminate between them. There are other more or less established astrometric anomalies in the solar system which may serve as benchmark. For instance the anomalous secular increase of the eccentricity of the orbit of the Moon, see [42, 43, 94–96], the anomalous perihelion advance of Saturn, see [21, 38, 71], the mass variation of the solar mass [72] and the secular increase of the Astronomical unit, see [1, 2, 32, 47, 50, 67, 80]. Moreover the links between both models through the origin of both which is the Unruh radiation should be studied in future.

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