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Bose Condensation of Phonons in Biological Systems

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Abstract

Using the Bogolyubov's rate equation from the theory of superfluidity the possibility of Bose condensation of phonons in biological systems and the validity of Fröhlich's hypothesis has been proved. We took into account both the third and the fourth anharmonism in the rate equation. All the processes with active phonons (from one to four) of biological active modes have been investigated. Taking into account these processes the expression for the chemical potential is shown to be changed, but the conditions for Bose condensation of phonons still exist. For the first time we point out the possibility of soliton wave packet propagation in the coherent systems of phonons and photons. The possibility of Bose condensation of excitons in biological systems is also discussed.

1. Introduction

In a series of papers [1–13] the possibility of the phenomenon of Bose–Einstein condensation of phonons in biological systems has been investigated. According to Fröhlich [1–6] in some biological systems the modes of active phonons may be excited, which through interaction among themselves are split into a narrow band of frequencies $\{\omega_i\}$, spread over a range ω_0 to ω_N. The rest of the total number of phonons of the biological system, i.e., a set of independent phonons of frequencies $\{\Omega_\mu\}$, forms a heat bath. The active phonons are the longitudinal electric dipolar oscillations of regulated units of quasilinear macromolecules. The active phonons interact both among themselves and with the phonons of the heat bath. If the energy is supplied to the active modes at the rate s beyond a certain critical amount s_0, the lowest mode is very strongly excited, i.e., the phenomenon of Bose condensation of phonons occurs. In experimental papers [14–19] the influence of 6–7 mm coherent electromagnetic radiation on the metabolism of cells and on their other properties, on the intensity of cell division of different culture's yeast, on the marrow of mice, and on the structural components of a cell has been studied. The experiments have demonstrated that these biological systems possess a branch of phonon modes in the 10^{11}–10^{12} $\sec^{-1}$ frequency region. The section of the cell membrane or H-bonds of the giant molecules (protein, ATP, etc.) are oscillating units. Some effects described in Refs. 14–18 may be explained by means of Bose condensation of phonons. For example, the observed effects have both frequency resonance and amplitude threshold character. In Ref. 15 a number of cells of the marrow of mice remaining nondestroyed under the action of x-irradiation have been studied. It was found that the number of nondestroyed cells essentially increases if the cells were preliminary irradiated by mm electromagnetic irradiation with the power greater than the critical amount. The threshold character of this irradiation correlates

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with the threshold character of Bose condensation of phonons. For the first time this fact was indicated by Fröhlich [1]. The Bose condensation of excitons, phonons, and other elementary excitations in solids have been studied in Refs. 21 and 22 and discussed in Refs. 23 and 24. From this point of view the problem of Bose condensation of phonons is not new. Unexpected is the fact that it is actual in biological systems. There is a relation between the phenomenon of Bose condensation of phonons and the problem of cancer [25].

From the above it is clear that we have to investigate the collective phenomena not only of the system of phonons, but of other elementary excitations in biological systems. For the first it is worthwhile to investigate Bose condensation of phonons in Fröhlich's model, which is now being discussed in the literature. The validity of the model from the microscopic viewpoint has been discussed in Refs. 9–11. Nevertheless, the papers do not give the answer to Lifshitz's criticism [7, 8]. Lifshitz, without having enough basis, has added the two-quantum terms in the Fröhlich's rate equation. These terms describe the simultaneous creation and annihilation of two active phonons. He assumed that the probability of these processes is equal to the probability of scattering of two active phonons. He concluded that Bose condensation of phonons on the lowest mode does not occur. We shall investigate the nature of two-quantum process terms and discuss the probability of their appartition in the rate equation from the microscopic point of view. It will be shown that Bose condensation of phonons taking into account the two-quantum terms under some conditions may exist. We shall use the Bogolyubov's rate equations [26], which were generalized in Ref. 27 for the two-component system of phonons and high density excitons. The equations have been obtained using the third-order terms in the Hamiltonian of the system. We shall add also the terms [28] which appear in the rate equation after taking into account in the Hamiltonian the fourth-order operator term. It will allow us to take into account correctly not only two-quantum terms, but even three- and four-quantum terms. It will be shown that three- and four-quantum terms also do not destroy the examined coherent state. Our results are in agreement with the conclusion of a recent paper [13] in which two-quantum terms are correctly included but the higher-order terms are not considered.

2. Rate Equation

We start from the rate equation [27], to which is added the fourth-order terms [28]. It determines the time evolution of the average number of elementary excitations n_f:

$$\begin{aligned}\frac{\partial n_f}{\partial t} = &\sum_{xy} A_{fxy}\delta(E_f+E_x-E_y)[(1+n_f)(1+n_x)n_y-(1+n_y)n_f n_x] \\ &+\sum_{xy} B_{fxy}\delta(E_x+E_y-E_f)[(1+n_f)n_x n_y-(1+n_x)(1+n_y)n_f] \\ &+\sum_{xyz} C_{fxyz}\delta(E_f+E_x-E_y-E_z)[(1+n_f)(1+n_x)n_y n_z \\ &\qquad -(1+n_y)(1+n_z)n_f n_x] \qquad (1)\end{aligned}$$

where A, B, and C are the interaction constants of elementary excitations. If the system is in the nondegenerate state, the coefficients A and B are determined by the constant of the triple anharmonism of phonons, but C is determined by the fourth one. They are discussed in a series of papers [29–31] on the phonon theory of molecular crystals. In this case we may trace the system approaching the coherent state at constant temperature while the excited power and the whole number of quasiparticles are increasing. These rate equations, but with other coefficients, are also applicable in the case of Bose condensation. We designate the active modes by i, j, k, and l and the passive modes by μ, ν, and ξ. As we investigate the state of the active mode, we take $f = i$. The right-hand side of Eq. (1), when x, y, and z are substituted by the Greek letters, gives one-quantum terms (according to the number of active modes). If one of x, y, or z is substituted by the Roman letter, and the rest of the subscripts by the Greek ones, Eq. (1) gives two-quantum terms. If one of the subscripts x, y, z is substituted by the Greek letters, we obtain three-quantum terms, and finally if all the subscripts are substituted by the Roman letters, Eq. (1) gives four-quantum terms. We suppose that phonons of the heat bath are thermalized, i.e., $n_\mu = [\exp(\beta\hbar\Omega_\mu) - 1]^{-1}$. Since the energy of heat bath quanta is small $\hbar\Omega_\mu \ll k_0 T$ at room temperature, the approximation $n_\mu \simeq k_0 T/\hbar\Omega_\mu$ is correct. If to Eq. (1) is added the term which describes the same pump of energy s in all active modes, we obtain

$$
\begin{aligned}
\frac{dn_i}{dt} = s &+ \Phi_i[(1+n_i) - n_i e^{\beta\hbar\omega_i}] \\
&+ \sum_j \Gamma_{ij}[(1+n_i)(1+n_j) - n_i n_j e^{\beta\hbar(\omega_i+\omega_j)}] \\
&+ \sum_j \Lambda_{ij}[(1+n_i)n_j - (1+n_j)n_i e^{\beta\hbar(\omega_i-\omega_j)}] \\
&+ \sum_{jk} \{\Psi_1[(1+n_i)(1+n_j)n_k - (1+n_k)n_i n_j] \\
&\quad + \Psi_2[(1+n_i)n_j n_k - (1+n_j)(1+n_k)n_i] \\
&\quad + \Psi_3[(1+n_j)(1+n_i)n_k - (1+n_k)n_i n_j e^{\beta\hbar(\omega_i+\omega_j-\omega_k)}] \\
&\quad + \Psi_4[(1+n_i)n_j n_k - (1+n_j)(1+n_k)n_i e^{\beta\hbar(\omega_i-\omega_j-\omega_k)}]\} \\
&+ \sum_{jkl} \Theta_{ijkl}[(1+n_i)(1+n_j)n_k n_l - (1+n_k)(1+n_l)n_i n_j]
\end{aligned} \tag{2}
$$

where

$$
\begin{aligned}
\Phi_i = &\sum_{\mu\nu} A_{i\mu\nu}\delta(\omega_i + \Omega_\mu - \Omega_\nu) n_\mu n_\nu e^{\beta\hbar\Omega_\mu} + \sum_{\mu\nu} B_{i\mu\nu}\delta(\Omega_\mu + \Omega_\nu - \omega_i) n_\mu n_\nu \\
&+ \sum_{\mu\nu\xi} C_{i\mu\nu\xi}\delta(\omega_i + \Omega_\mu - \Omega_\nu - \Omega_\xi) n_\mu n_\nu n_\xi e^{\beta\hbar\Omega_\mu}
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_{ij} = &\sum_\mu A_{ij\mu} \frac{k_0 T}{\hbar(\omega_i + \omega_j)} \delta(\omega_i + \omega_j - \Omega_\mu) \\
&+ \sum_{\mu\nu} C_{ij\mu\nu} \frac{k_0 T}{\hbar(\omega_i + \omega_j)} \delta(\omega_i + \omega_j - \Omega_\nu - \Omega_\mu)
\end{aligned}
$$

$$\Lambda_{ij}=\sum_{\mu}(B_{ij\mu}+B_{i\mu j})\frac{k_0T}{\hbar(\omega_i-\omega_j)}\delta(\omega_j+\Omega_\mu-\omega_i)$$
$$+\sum_{\mu\nu}(C_{i\mu j\nu}+C_{i\mu\nu j})\frac{k_0T}{\hbar(\Omega_\mu+\omega_i-\omega_j)}\delta(\omega_i+\Omega_\mu-\omega_j-\Omega_\nu)$$
$$+\sum_{\mu}A_{i\mu j}\delta(\omega_i+\Omega_\mu-\omega_j)$$
$$\Psi_1=A_{ijk}\delta(\omega_i+\omega_j-\omega_k) \tag{3}$$
$$\Psi_2=B_{ijk}\delta(\omega_i+\omega_k-\omega_j)$$
$$\Psi_3=\sum_{\mu}(C_{ijk\mu}+C_{ij\mu k})\frac{k_0T}{\hbar(\omega_i+\omega_j-\omega_k)}\delta(\omega_i+\omega_j-\omega_k-\Omega_\mu)$$
$$\Psi_4=\sum_{\mu}C_{i\mu jk}\delta(\omega_i+\Omega_\mu-\omega_j-\omega_k)$$
$$\Theta_{ijkl}=C_{ijkl}\delta(\omega_i+\omega_j-\omega_k-\omega_l)$$

Let us assume that the frequencies ω_i of the active phonons are spread in a narrow band $\Delta\omega$, i.e., $\Delta\omega\ll\omega_i$. In addition it is reasonable to take A and B as being of the same order. Then, as follows from Eq. (3), $\Gamma_{ij}\ll\Lambda_{ij}$. In the stationary state $dn/dt=0$ and neglecting both three- and four-quantum terms we obtain Fröhlich's equation [1]:

$$s+\phi_i[(1+n_i)-n_i\,e^{\beta\hbar\omega_i}]+\chi[(1+n_i)n_j-(1+n_j)n_i\,e^{\beta\hbar(\omega_i-\omega_j)}]=0$$

In order to obtain Lifshitz's result we have to admit that $\Gamma_{ij}=\Lambda_{ij}e^{-\beta\hbar\omega_i}=b_{ij}$ and neglect the three- and four-quantum terms. In this case Bose condensation of phonons does not occur as the chemical potential $\mu=0$. However, as we see from (3) this equality is not fulfilled when $\Delta\omega\ll\omega_i$ and $A\sim B$. Under this condition Lifshitz's conclusion is not true. Neglecting the three- and four-quantum terms in rate equation (2) we obtain the result of Wu and Austin [12, 13].

In the general case under the stationary condition we get from (2) the expressions for the distribution functions n_i and chemical potential μ:

$$n_i=\Big(s+\Phi_i+\sum_j\Gamma_{ij}(1+n_j)+\sum_j\Lambda_{ij}n_j$$
$$+\sum_{jk}[\Psi_1(1+n_j)n_k+\Psi_2n_jn_k$$
$$+\Psi_3(1+n_j)n_k+\Psi_4n_jn_k]+\sum_{ikl}\Theta_{ijkl}(1+n_j)n_kn_l\Big)$$
$$\times\Big(\Phi_i+\sum_j\Gamma_{ij}(1+n_j)+\sum_j\Lambda_{ij}n_j$$
$$+\sum_{jk}[\Psi_1(n_j-n_k)-\Psi_2(1+n_k+n_j)+\Psi_3n_k(1+n_j)+\Psi_4n_jn_k]$$
$$+\sum_{jkl}\Theta_{ijkl}[n_kn_l-(1+n_l+n_k)n_j]\Big)^{-1}\{e^{\beta\hbar(\omega_i-\mu)}-1\}^{-1} \tag{4}$$

where

$$e^{-\beta\mu\hbar}=\Big(\Phi_i+\sum_j\Gamma_{ij}n_j\,e^{\beta\hbar\omega_i}+\sum_j\Lambda_{ij}(1+n_j)\,e^{-\beta\hbar\omega_i}$$
$$+\sum_{jk}\big[\Psi_3(1+n_k)n_j\,e^{\beta\hbar(\omega_j-\omega_k)}+\Psi_4(1+n_j)(1+n_k)\,e^{-\beta\hbar(\omega_j-\omega_k)}\big]\Big)$$
$$\times\Big(\Phi_i+\sum_j\Gamma_{ij}(1+n_j)+\sum_j\Lambda_{ij}n_j$$
$$+\sum_{jk}\big[\Psi_1(n_j-n_k)-\Psi_2(1+n_k+n_j)+\Psi_3n_k(1+n_j)+\Psi_4n_jn_k\big]$$
$$+\sum_{jkl}\Theta_{ijkl}[n_kn_l-(1+n_l+n_k)n_j]\Big)^{-1} \tag{5}$$

The occupation numbers from (4) are positive if $0\leq\mu<\hbar\omega_0$. Bose condensation of phonons on the lowest active mode ω_0 occurs when the chemical potential μ approaches ω_0. As follows from Eqs. (4) and (5) taking into account three- and four-quantum terms together with two-quantum terms does not destroy the coherent state but leads only to the renormalization of the chemical potential μ. Thus Fröhlich's conclusion about the possibility of Bose condensation of phonons in biological system is true.

3. Solitons in a System of Coherent Phonons and Photons

Above we discussed the phenomenon of Bose–Einstein condensation of phonons when the rate of the energy supplied s is the same in all the biological active modes. Nevertheless, the selective excitation of a single mode by the coherent laser irradiation is possible too. For this it is necessary for the examined mode to be dipole active. The interaction of coherent phonons and photons leads to the formation of coherent phonon polaritons. In this case Bose condensation of phonons is analogous to the Bose condensation of excitons in a strong electromagnetic field [32] or to Bose condensation of excitons and photons in a resonator [33]. Such Bose condensation of phonons induced by the external field in a biological system has not yet been discussed. Coherent phonons in biological systems can also appear in the regime of ultrashort impulses under the action of resonance maser irradiation or at the Raman scattering of laser irradiation on phonons. Such experiments have been made for a number of crystals and liquids [34]. The coherent phonons and photons arising in the regime of ultrashort pulses or Bose condensation can propagate in the medium under some conditions in the form of steady-state soliton wave packets [35, 36]. If the pulse width is smaller than the relaxation time of phonons, the propagation of solitons in biological systems without the dissipation of energy will lead to the phenomenon of self-induced transparency discovered in gases by McCall and Hann [37]. It is necessary to note that solitons in biological objects have already been discussed by Davydov and Kislucha [38].

However, in this paper the interaction of an electron excitation with the atoms' displacements in a linear molecular chain was considered and the propagation of autolocalized states of a polaron type was investigated.

In our case the polariton consists of a phonon and a photon. It may be called a polariton soliton. Introducing the phonon and photon quasi-wave number k in medium we can speak about the polariton dispersion law and its translational mass. The changes in the dispersion law are stronger in the crossing of the dispersion branches of noninteracting phonons and photons. In that region where $k \sim k_0$ the steepness of the polaritons' dispersion branches is maximal and the polariton mass can become equal to or less than the electromagnetic photon mass $\hbar\omega/c^2$.

Let us introduce the envelope of the coherent wave packet of phonons and photons as $A/\cosh\xi$ where $A^2/2$ is the phonons' linear density in the linear chain. ξ is the running variable $\xi=(t-x/v)/\tau$, and v is the soliton velocity. Taking into consideration only the fourth-order anharmonism with the constant g the time width τ of the wave packet is defined by

$$\tau^2=\frac{8\hbar(k/v-\omega/c^2)}{gA^2(k^2-\omega^2/c^2)}$$

Here ω and k are the frequency and the wave number of the carrier wave and lay on one of the polariton branch for which $g(k^2-\omega^2/c^2)>0$. In the actual region $k\sim k_0$, $\omega \simeq ck_0 \pm (ck_0\Omega_0/2)^{1/2}$, where Ω_0 is the splitting between the frequencies of longitudinal and transversal phonons at $k=0$. Let us take $k=k_0$, $v=c/2$, $\Omega_0=10^{-3}ck_0$, $ck_0\approx 10^{11}\ \mathrm{sec}^{-1}$, $g\approx 10^{-23}$ erg cm, and $A^2/2\sim 10^6\ \mathrm{cm}^{-6}$. Then for ω lying on the lower polariton branch we obtain

$$\tau^2=\frac{4\hbar}{gA^2(ck_0\Omega_0/2)^{1/2}}\approx 2\times 10^{-19}\ \mathrm{sec}^2$$

i.e., $\tau\approx 4\times 10^{-10}$ sec. For the relaxation time of the order 10^{-9} sec in a linear biological chain the ultrashort pulses with the given τ will propagate without energy losses, and the phenomenon of self-induced transparency will realize.

4. Coherent Excitons

In the conclusion we will state some remarks relative to the collective properties of excitons in biological structures. Excitons are of great importance in the migration of energy in the process of photosynthesis. Their existence has been experimentally detected under the investigation of the energy migration in DNA [39] and other systems [40]. The high density excitons can essentially change their own life times due to the interaction between themselves and lead to the biexciton formation. Both these possibilities have been discussed in Borisov's report [41]. We would like to pay attention to the possibility of the creation of high density coherent excitons in biological systems. They may be created in different ways: One of them is the thermalization of excitons and the establishment of quasi-equilibrium in the band, if the excitons' life time is considerably greater than their relaxation time and the translational mass is not too large. Otherwise too large a concentration of excitons is needed. In this case the excitons destroy one another. The second way is Bose condensation of excitons in the field of an external electromagnetic wave. And finally coherent excitons may be created in the regime of ultrashort pulses and they may exist only during times less then the relaxation time. These problems are well known and have been discussed for a comparatively long time for semiconductors and other crystals [23]. It is possible that the idea of Bose condensation of excitons will be helpful in the understanding of some consequences of the laser action on biological structures and will open the new perspective in this branch of science.

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