

The Prospects for Magneto-Aerodynamics[†]

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SUMMARY

The equations describing the flow of an electrically conducting fluid in the presence of electric and magnetic fields are written down with the aid of certain simplifications appropriate to aeronautical applications. In order to estimate the probable significance of magneto-aerodynamic effects, some data on conductivity of pure and "seeded" air are first examined. Dimensionless quantities representing the ratios of forces and of currents are then formed and their values studied for conditions of flight in the atmosphere.

Some examples of magneto-hydrodynamic and magneto-gasdynamics effects in simple flows are given. These include two cases of Poiseuille flow of conducting liquids with applied magnetic fields and the case of quasi-one-dimensional gas flow with applied electrical and magnetic fields. In the last case, attractive possibilities are found for controlled acceleration or deceleration of gas at subsonic and supersonic speeds, even in constant-area channels. The behavior of the flow is characteristically different in different regimes of Mach Number and flow speed relative to certain "significant speeds" that are dependent on the ratio of electrical to magnetic field strengths. These are studied, and a chart is constructed to relate the length to the speed ratio of a maximum-acceleration constant-area channel.

It is concluded that the advantages that may accrue from magneto-aerodynamic methods are sufficiently attractive to justify the considerable research and engineering development that will be required. Among the unsolved engineering problems are the reduction of surface resistance of electrodes in contact with a conducting gas, development of techniques for seeding, and provision of the required magnetic fields in flight.

SYMBOLS

a	= half-width of channel in cases of Figs. 4 and 5
A	= $\sqrt{\gamma p/\rho}$ in compressible flow
A	= cross-section area of channel
c	= speed of light
c_p	= specific heat for constant pressure
c_v	= specific heat for constant volume
D	= channel cross-section dimension [Eq. (15)]
$\mathbf{E}$	= electric field vector (magnitude E)
$\mathbf{F}$	= body force
$\mathbf{j}$	= current density vector (magnitude j)
$\mathbf{H}$	= magnetic field vector (magnitude H)
Ha	= Hartmann Number
$\mathbf{h}$	= perturbation magnetic field vector (magnitude h)
h_0	= stagnation enthalpy
K_B	= breakdown-potential constant [Eq. (50)]
L	= typical body length
M	= Mach Number

m	= mass flow per unit area
P	= constant pressure gradient in Eq. (28)
p	= static pressure
$\mathbf{q}$	= fluid velocity vector (magnitude q)
Q	= heat added by dissipation and/or other effects excluding Joule heating
R	= resistance per unit length of channel (Fig. 5)
Re	= Reynolds Number
R_m	= magnetic Reynolds Number
t	= time
T	= temperature
u	= flow speed (average) in channel
u_1, u_2, u_3	= significant speeds in channel flow
V	= typical flow speed; speed of flight
$\mathcal{U}$	= electrical potential
x, y, z	= Cartesian coordinates
α	= fractional ionization
γ	= c_p/c_v
ξ	= $(1/m) \int_{x_0}^x \sigma H^2 dx$
μ	= viscosity coefficient
ρ	= mass density
σ	= conductivity

Subscripts

x, y, z	= components of vectors, along x, y, z axes
0	= constant, especially initial conditions in channel flow

INTRODUCTION

IF A FLUID is a conductor of electricity, the possibility arises that an *electric body force* may be produced in it that will affect the fluid flow pattern in a significant way. To be sure, we are not unaccustomed to dealing with fluid flows with body forces—e.g., the gravitational force and the inertial, centrifugal, and Coriolis forces in moving coordinate systems—but all of these are examples where the body force is more or less outside of our control. The attractive thing about the electric body force (which we will call the e.b.f.) is that it can be controlled, insofar as the current and the magnetic field can be controlled, and perhaps made to serve useful purposes such as acceleration or deceleration of flow, prevention of separation, and the like.

A body force in fluid mechanics is any force that is applied by an external agency directly to the fluid elements, as distinguished from the pressure, for example, which is a force between adjacent particles. The e.b.f. involved here is the counterpart of the force that drives an electric motor; it is given by the vector product of the current and the magnetic field strength and is therefore perpendicular to both of these vectors. In the case of a fluid, we expect electric-current flow through a volume rather than along a wire and are

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therefore concerned with the current-density vector $\mathbf{j}$, measured in amperes per unit area, for example. Similarly, the strength of the magnetic field $\mathbf{H}$ is measured in lines-of-force per square centimeter, usually called "gauss." Then the body force per unit volume is given by the vector equation

$$\mathbf{F} = \mathbf{j} \times \mathbf{H} \quad (1)$$

It is assumed here that the fluid is nonmagnetic—i.e., that it has the same permeability as empty space; this permeability has been taken equal to 1, since we propose to formulate our problem in electromagnetic units.

This force can be used to pump a conducting fluid, such as mercury, through pipes.¹ Recently it has been used in gasdynamics laboratories to help drive shock tubes;² here the conducting fluid is air or another gas heated to high temperature by discharge of a condenser through it. Astrophysicists also have a current interest in magneto-hydrodynamics because there are at least two different situations in their field where this body force apparently acts: (1) in the sun and stars where gases are highly ionized by the extreme temperatures, and (2) in nearly empty space where the temperatures of gas clouds are not high but the ionization is nevertheless sufficient, because of the low densities, to make them good conductors.

The possibility of these effects becoming important in aeronautics has naturally arisen, since the temperatures that occur in flight at high speeds are sufficient to ionize air appreciably. If, as predicted, these temperatures approximate the temperatures of electric arcs, it is clear that the air becomes a conductor and that the e.b.f. can be important aerodynamically.

The purpose of the present paper is to make an assessment of these aeronautical possibilities. How large are the effects likely to be, and under what conditions? What are some of the engineering problems posed?

Before proceeding to this task, however, we should notice another basic phenomenon of magneto-aerodynamics, equal in importance to the e.b.f. and even, in some cases, overshadowing it—namely, the Joule heating. This is one of the most familiar phenomena of electricity. We are used to it in metallic conductors, and it may be even more significant in magneto-aerodynamics, since the conductivities of gases are so much less. The electricians' familiar formula I^2R becomes, in this application,

$$\text{heat released per unit volume per unit time} = j^2/\sigma \quad (2)$$

where σ denotes the conductivity or the reciprocal of the specific resistance. The unit of conductance is the mho; here σ is a material property, the conductance of a unit cube. The conductance of such a sample is proportional to its cross-sectional area and inversely to its length. Thus σ has the units mho per unit length.

THE EQUATIONS OF MOTION

To summarize what has been written above, the presence of electrical currents in a fluid introduces essentially two new effects, the e.b.f., $\mathbf{j} \times \mathbf{H}$; and Joule heating, j^2/σ . The only change required in the equations of fluid motion is the introduction of these new terms in the equations of momentum and energy. The equation of continuity remains unaltered. We have then

Continuity:

$$(\partial\rho/\partial t) + \text{div}(\rho\mathbf{q}) = 0 \quad (3)$$

Momentum:

$$(D\mathbf{q}/Dt) + (1/\rho)\nabla p = (\mu/\rho)\{\nabla^2\mathbf{q} + (1/3)\nabla\nabla\cdot\mathbf{q}\} + (1/\rho)\mathbf{j} \times \mathbf{H} \quad (4)$$

Energy:

$$c_v(DT/Dt) + p(D\rho^{-1}/Dt) = (j^2/\rho\sigma) + Q \quad (5)$$

Here ρ , $\mathbf{q}$, p , μ , T , and σ are the density, velocity, pressure, viscosity, temperature, and electrical conductivity of the fluid; c_v is its specific heat at constant volume; and t is the time. The symbol Q denotes all heat added to the fluid except Joule heat, thus Q includes heat conduction, viscous dissipation, etc. One simplification has been made, for brevity, in writing Eq. (4)—namely, the viscous term has been written for the special case of constant μ —but this will not affect what is done below. It should be remembered however that μ is actually dependent upon T . The symbols $\mathbf{j}$ and $\mathbf{H}$ represent the current density and magnetic field strength defined above. The derivative D/Dt is the convective derivative—i.e., the rate of change for a fluid particle.

Eq. (5) is an expression of the fact that as heat is added to, or subtracted from, the fluid medium it appears either in internal energy or in mechanical work done. This is the simple expression of the First Law of Thermodynamics that applies when the thermodynamic and mechanical forms of energy are the only ones concerned. Eq. (5) does not attempt to include electrical forms of energy in the statement of the First Law. It is therefore useful only if there is no interchange of energy except what has been described above—i.e., Joule heating. In more complicated cases, for example, where the magnetic permeability of the fluid is a function of temperature or of the field strength, a more sophisticated formulation of the energy equation from the first law of thermodynamics is required.

Assuming that this is not the case in most aeronautical applications, the simple statement Eq. (5) is all that is needed. It takes on an interesting form if it is added to Eq. (4) after scalar multiplication of the latter with the velocity vector (thus changing Eq. (4) to a statement involving work done by forces). This form is, assuming the perfect-gas equation of state,

$$c_p(\partial T/\partial t) + (1/2)(\partial q^2/\partial t) + p(\partial \rho^{-1}/\partial t) + \mathbf{q} \cdot \nabla h_0 = \\ (1/\rho)\mathbf{q} \cdot \mathbf{j} \times \mathbf{H} + (j^2/\rho\sigma) + \mu\{\nabla^2 \mathbf{q} + (1/3)\nabla \nabla \cdot \mathbf{q}\} \cdot \mathbf{q} + Q \quad (6)$$

where h_0 denotes the stagnation enthalpy, $c_p T + q^2/2$. This form is useful for steady-flow problems, since the first three left-hand terms are then absent, and the left-hand side of Eq. (6) becomes the rate of increase of the stagnation enthalpy of a fluid particle. This rate involves two magneto-aerodynamic terms, one being the mechanical work done by the e.b.f., the other being the Joule heating. As we shall see below, it is possible for these two to oppose one another.

Ohm's Law

In Eqs. (3) to (5) we have introduced two new unknowns into the already complicated equations of gasdynamics—viz., $\mathbf{j}$ and $\mathbf{H}$ (assuming σ to be a known property of the fluid). We must provide additional equations concerning these. One of these is Ohm's Law:

$$\mathbf{j} = \sigma\{\mathbf{E} + \mathbf{q} \times \mathbf{H}\} \quad (7)$$

This should be thought of as an empirical result, describing in a macroscopic way how the electric current flows through a volume of conducting matter under the influence of an electric field $\mathbf{E}$ and the electromotive force $\mathbf{q} \times \mathbf{H}$ that is set up by motion relative to a magnetic field. The process of electrical conduction in gases in the presence of a magnetic field is really somewhat involved. Basically, a force acts on any charged particle that is moved across a magnetic field. In our case the charged particles are the electrons and ions of the gas, and, in relatively dense gases, they are constrained to move with the gas velocity $\mathbf{q}$. The electromotive force on them causes them to drift across the flow, their motion being impeded by frequent collisions with the other particles, and a net flux results that is described by the second term of Eq. (7). The first term represents a similar drift of charged particles, but here caused by the electric field $\mathbf{E}$.

The symbol $\mathbf{E}$ represents the total electric field strength (e.g., volts per unit length) due to external effects, such as the field set up by charged boundaries of the flow, and internal effects—i.e., separation of charges or *polarization*.³ $\mathbf{E}$ is measured in the frame relative to which $\mathbf{q}$ is measured. In our approximation ($q \ll c = \text{speed of light}$), $\mathbf{H}$ is not a function of $\mathbf{q}$. One can think of the term $\mathbf{q} \times \mathbf{H}$ as representing a tiny generator or source of e.m.f. at any point in the moving fluid; the effect of its e.m.f. is divided between driving a current through the gas and separating the positive and negative charges. If no current can flow, because of the presence of insulators, for example, the whole e.m.f. must be opposed by the field $\mathbf{E}$ —i.e., by charge separation. More generally, the current density $\mathbf{j}$ cannot always be distributed like $\mathbf{q} \times \mathbf{H}$, so that a net field $\mathbf{E}$ will generally result, in magneto-aerodynamics. A calculation shows that the distance

through which the positive and negative charges are moved in a gas to set up any reasonable field strength $\mathbf{E}$ is extremely small. It is a good approximation, therefore, to consider any infinitesimal sample of the gas to be neutral in charge, thus the field $\mathbf{E}$ appears to be produced by doublets—i.e., the gas is polarized. This is the approximation usually made for dielectrics, but it is also appropriate here. It does not imply that $\mathbf{E}$ is zero or negligibly small.

The importance of the quantity $\mathbf{E}$ is illustrated by combining Eqs. (6) and (7). The first two right-hand terms combine as follows:

$$\mathbf{q} \cdot \mathbf{j} \times \mathbf{H} + (j^2/\sigma) = \mathbf{q} \cdot \mathbf{j} \times \mathbf{H} + (\mathbf{E} + \mathbf{q} \times \mathbf{H}) \cdot \mathbf{j} \} \\ = \mathbf{E} \cdot \mathbf{j} \quad (8)$$

where the rule of signs of a scalar triple product has been used. Thus the net magneto-aerodynamic contribution to the energy is seen to involve directly the electric field $\mathbf{E}$.

Maxwell's Equations

The introduction of Ohm's Law has related $\mathbf{j}$ to $\mathbf{q}$ and $\mathbf{H}$, but only with the involvement of a new unknown, $\mathbf{E}$. The set of equations is rendered determinate by means of Maxwell's field equations:

$$4\pi\mathbf{j} = \text{curl } \mathbf{H} \quad (9)$$

$$\text{curl } \mathbf{E} = -(\partial \mathbf{H}/\partial t) \quad (10)$$

As before, we have assumed the fluid to have the same magnetic permeability as empty space and have used electromagnetic units. We have neglected "displacement currents" in writing Eq. (9).⁷

RELATIVE MAGNITUDES OF TERMS

Having written down the equations of motion, we are now able to investigate the relative magnitudes of the terms that appear, and in this way make some prediction of the importance of magneto-aerodynamic effects. There is an equally important reason for making this investigation—namely, that the set of equations is formidable and will require simplification. Aerodynamicists have had little success in trying to solve the equations of compressible viscous gas flow without drastic approximations, and it is even less likely that they can be handled with the added complexity of electro-magnetic terms. It will be necessary to approximate in the same rational way—i.e., to retain the most significant effects while discarding others in appropriate areas and types of flow. One may ask in this spirit, will there be any application of the magneto-aerodynamics of ideal fluids? Will the boundary-layer approximation still apply? Will the same approximations that the astrophysicists make in magneto-hydrodynamics (which will be described below) be permissible in aeronautics?

To begin this part of the study, we need, first of all, some estimate of the probable magnitudes of the conductivity σ and the magnetic field strength $\mathbf{H}$.

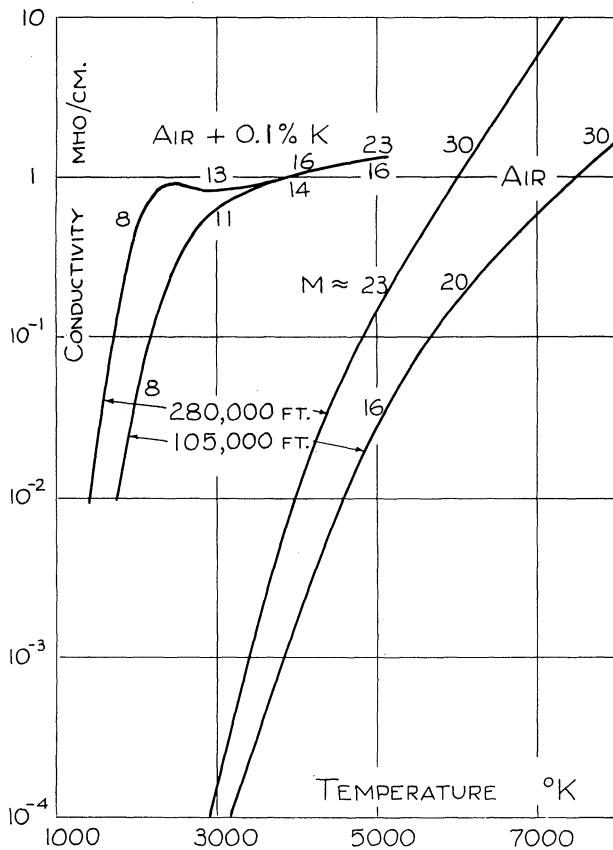


FIG. 1. Calculated conductivity of air and of air seeded with 0.1 per cent potassium (by weight) as functions of temperature for two altitudes. Corresponding Mach Numbers M giving approximately these stagnation temperatures are indicated. (From references 4 and 5.)

In the light of Ohm's Law, Eq. (7), we might also ask about the probable magnitude of $\mathbf{E}$, but we shall answer this question by specifically assuming that it has the same order of magnitude as $\mathbf{q} \times \mathbf{H}$. This assumption implies that the only electric fields present in the air are those induced by magneto-gasdynamic phenomena.

Conductivity

As has already been pointed out, electrical conduction in a gas is accomplished by a drift of charged particles. Conductivity is therefore increased by increasing the number of charged particles and reduced by increasing the probability of their collisions with impeding particles—i.e., atoms, molecules, and charged particles.

Let α denote the "fractional ionization" of the gas, defined as the number of original particles ionized. Thus α is nearly zero for air at room temperature, and $\alpha = 1$ represents a completely ionized gas, in which each original particle has given up one electron. Although α can exceed 1, the state of complete ionization represents a relatively large value for most applications, attained only at very high temperature. When α is small, so that charged particles in the main collide with neutral molecules, one arrives at the result

$$\sigma \sim \alpha T^{-1/2} \quad (11)$$

This regime is probably typical of aeronautical cases, but, in general, relation (11) is misleading in form since α itself is strongly dependent on the temperature T . Thus even in this regime, σ varies rapidly with T .

At the other extreme, when collisions between charged particles are dominant, the conductivity becomes independent of α , and relation (11) is replaced by

$$\sigma \sim T^{3/2} \quad (12)$$

There is an important case, however, in which the strong dependence of σ on T may be minimized, namely the case where conductivity is achieved by *seeding*. This means that an easily ionizable substance, such as potassium or sodium, is introduced into the air. The air temperature may be high enough to ionize completely this seeding substance, though it is insufficient to ionize the air itself appreciably. The result is then to remove the temperature dependence of α on T in Eq. (10), making σ relatively insensitive to temperature. The effects of seeding with various substances have been studied by R. Rosa in a Cornell University thesis.⁴

Some data on conductivity of air with and without seeding, from references 4 and 5, are presented in Fig. 1. The Mach Numbers spotted along the curves are only approximate and are intended to orient the reader as regards the relationship of stagnation temperature to flight speed. It is seen that σ for atmospheric air does not exceed about 0.1 mho/cm. even at the temperature corresponding to satellite speeds. On the other hand, one may obtain values near 1.0 mho/cm. by seeding, even at Mach Numbers contemplated for manned aircraft.

For purposes of orientation, it might be helpful to list the conductivities of other similar substances, in the same units. For example,

Copper	600,000 mho/cm.
Mercury	10,800 mho/cm.
Salt water (saturated at 25°C.)	0.25 mho/cm.
Pure water	0.0000002 mho/cm.

On the basis of Fig. 1, we shall adopt the value $\sigma = 1.0$ mho/cm. as a standard for certain numerical comparisons below. This value has the virtue that it is easy to change in any numerical result by appropriate multiplication.

Magnetic Field Strength

For the magnetic field strength H , on the other hand, there is no completely rational basis of estimation. Nevertheless, some milestones can be identified. A field strength of about 20,000 gauss saturates an iron core at room temperature. It is not certain that iron cores can be utilized in aeronautics in view of their weight; still, we can be sure that if they are used the saturation value will be the maximum field. This would seem to set only a very high upper limit even though in principle an air-core magnet could be made stronger.

Another indication of practical field strengths may be suggested by the value available in a permanent magnet. This runs to perhaps 1,500 or 2,000 gauss for good modern materials.

It seems probable that in aeronautical applications magnetic fields will be produced by air-core electromagnets whose currents are generated magneto-aerodynamically. The dimensions of such solenoids can be estimated in terms of vehicle dimensions, but since the number of turns will be determined by such matters as weight and resistance, one still does not have an estimate of H . In some simple computations carried out below we shall adopt rather arbitrarily the value $H = 1000$ gauss. Once again, its main virtue may be that it affords easy correction to other values for those who do not concur.

Dimensionless Ratios

In fluid mechanics it is useful to compare the magnitudes of dynamic (inertial) and viscous effects by forming their ratio, assuming that geometrically similar patterns of flow are being compared. The result, of course, is the Reynolds Number. It often has values of hundreds of thousands if the typical length is taken to be body diameter or wing chord, and one concludes that the viscosity can be ignored throughout most of the flow field. It is only a slight refinement of this procedure to introduce the boundary-layer thickness as the typical length (or, conversely, to inquire what typical length is required to make viscosity important) and to show thereby that inertial and viscous effects are comparable in magnitude in a boundary layer.

Let us undertake to apply this method to compare the e.b.f. with the dynamic force—i.e., the first and last terms of Eq. (4). In order of magnitude, the ratio is

$$\frac{\text{e.b.f.}}{\text{dynamic force}} = \frac{jH}{(1/2)\rho V^2/L} = \frac{\sigma VH^2L}{(1/2)\rho V^2} \quad (13)$$

where V is the typical speed, say the flight speed, and L the scale-determining length. Here we assume, as mentioned above, that the electric field $\mathbf{E}$ has the same order of magnitude as $\mathbf{q} \times \mathbf{H}$; in other words, that the current density j is of order σVH . If L is put equal to an aircraft or missile length (or diameter), one is comparing the total force per unit cross section of the flow, which can be applied over this length, to the dynamic pressure of flight. Some values are plotted in Fig. 2, where altitude and Mach Number are the variables and the assumptions are stated. It appears that the e.b.f. will be big enough to affect the flow appreciably at high altitudes and high speeds.

Naturally, if we form the ratio of body force to viscous force we obtain the product of this number and the Reynolds Number:

$$\frac{\text{e.b.f.}}{\text{viscous force}} = \frac{\text{e.b.f.}}{\text{dynamic force}} \times Re \quad (14)$$

where $Re = \rho VL/\mu$. Thus, this ratio is many times as large as the one plotted in Fig. 2, except in boundary layers, wakes, etc., where the significant Reynolds Number is much smaller. The immediate conclusion is that an inviscid theory of magneto-aerodynamics is generally useful but must be interpreted in the light of boundary-layer theory. In other words, boundary layers and similar viscous regions have their common significance, in magneto-aerodynamic flows as well.

The number calculated in Eq. (14) becomes twice the square of the Hartmann Number⁶ when the significant length is the diameter or breadth, D , of a pipe or channel:

$$2Ha^2 = [\sigma VH^2D/(1/2)\rho V^2] \times [\rho VD/\mu] = 2\sigma H^2D^2/\mu \quad (15)$$

A third interesting ratio is obtained by comparing electrical currents rather than forces. Let us compute the ratio of motion-induced current to the current necessary to produce the field $\mathbf{H}$. In doing so, we shall assume again that the current density j is of order σVH . The current to produce $\mathbf{H}$, on the other hand, depends on the dimension and the number of turns in the coils of the electromagnet. Dimensionally, this current $\approx H/L$, but a large factor, the number of turns, may be involved. Ignoring this omission, one has

$$\sigma VH/(H/L) = \sigma VL = R_m \quad (16)$$

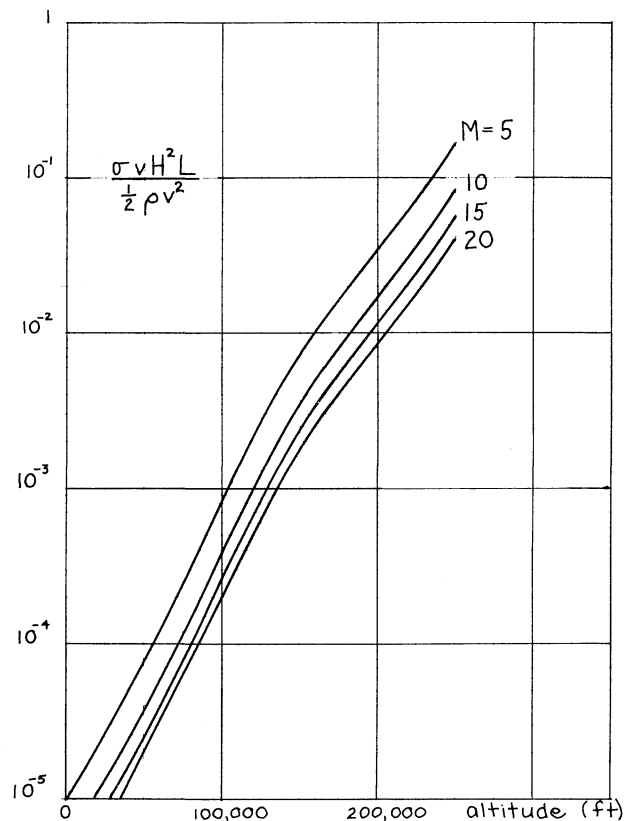


FIG. 2. Values of the ratio of typical electric body force to dynamic force as function of Mach Number and altitude, assuming $\sigma = 1$ mho/cm., $H = 1,000$ gauss, and $L = 1$ m.

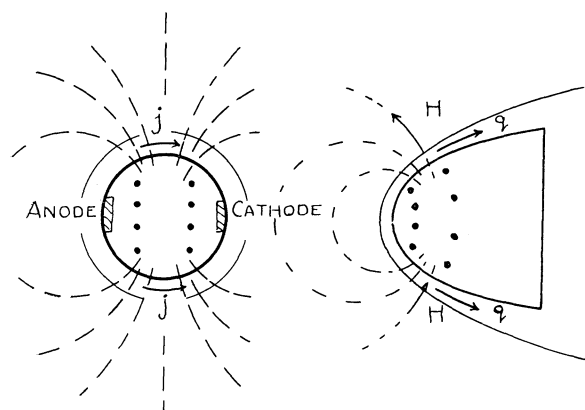


FIG. 3. Sketches of proposed device using magneto-aerodynamic effects to decelerate or control re-entry missiles.

Because of its resemblance to a Reynolds Number, this ratio has been called the "magnetic Reynolds Number."⁷ It can also be interpreted as the ratio of motion-induced magnetic field to applied magnetic field. In this sense, one recognizes that the currents generated involve their own magnetic fields, in accordance with Eq. (9). This can be thought of as a perturbation of an applied magnetic field. The magnitude of the perturbation would follow from Eq. (9) by a process of integration equivalent to the calculation of the velocity field of a vortex; the result would certainly be of magnitude σVHL . Thus the ratio of induced to applied magnetic field strengths is equal to the magnetic Reynolds Number. This may be a more satisfactory interpretation of R_m since it does not involve the number of coil-turns mentioned above.

In astrophysical and similar problems involving either high conductivities and/or very large dimensions, the magnetic Reynolds Number may be so large as to show that the motion-induced magnetic field is many times as large as the applied field. In such cases, the applied field may sometimes be neglected entirely. This leads to the concept of a fluid of infinite conductivity. For such a fluid the differential Eqs. (3) to (10) are much simplified. For example, Eq. (7) requires that $\mathbf{E} + \mathbf{q} \times \mathbf{H}$ be zero, since $\mathbf{j}$ must remain finite. It can be shown that under these conditions the magnetic field is convected with the fluid. Since $\mathbf{j}$ is finite, the Joule heating disappears. There is a considerable body of literature pertaining to this case, especially because of its astrophysical importance, as has been mentioned.

Returning to aeronautics, let us investigate the probable values of our magnetic Reynolds Numbers. Suppose that $\sigma = 1.0$ mho/cm. and $L = 1.0$ m. as before, then

$$R_m = \sigma VL = 10^{-5} V(\text{m./sec.}) \quad (17)$$

where the factor 10^{-5} has been introduced to relate the present units to electromagnetic units. Thus R_m is less than 1/10 even at satellite speeds, and the approximation of infinite conductivity seems not to

be appropriate for aeronautics. On the other hand, considering the missing factor related to coil-turns, the values of R_m in Eq. (17) may be large enough to make an aeronautical system self-powered—i.e., the currents induced by flow of seeded air may be sufficient to power the electromagnets that produce the field.

EXAMPLES

In this section we propose to present a few examples of magneto-aerodynamic flows, chosen to illustrate some of the effects discussed above and some of the difficulties of solving the equations.

Deceleration of Flow

Consider the body force $\mathbf{j} \times \mathbf{H}$ with the current density given by Ohm's Law (Eq. (7)):

$$\mathbf{j} \times \mathbf{H} = \sigma \{ \mathbf{E} \times \mathbf{H} + (\mathbf{q} \times \mathbf{H}) \times \mathbf{H} \} \quad (18)$$

The force $\sigma(\mathbf{q} \times \mathbf{H}) \times \mathbf{H}$ always has a component opposing the flow—i.e., tending to decelerate the flow.

A possible application of this effect occurs in missile re-entry where it is sometimes desirable to increase the missile drag. This might be accomplished as sketched in Fig. 3. A missile nose is shown enveloped by a layer of hot, ionized air. It is proposed that a magnetic field $\mathbf{H}$ be provided and that the current generated, $\mathbf{j}$, be made to flow through electrodes, as sketched. Inside the missile, the current generated would activate the electromagnet. In this situation, as is mentioned again below, the electric field strength due to polarization, $\mathbf{E}$, would be small if substantially two-dimensional conditions occur in the ionized layer. Thus the vectors representing the e.b.f. are shown in Fig. 3 to be opposite to the flow direction everywhere.

The result would be a considerable drag on the missile; the force would be applied to the structure of the electromagnets. From an energy viewpoint, one recognizes that kinetic energy is being converted to electrical form and then being dissipated in Joule heating of both the ionized air and the coils of the electromagnets.

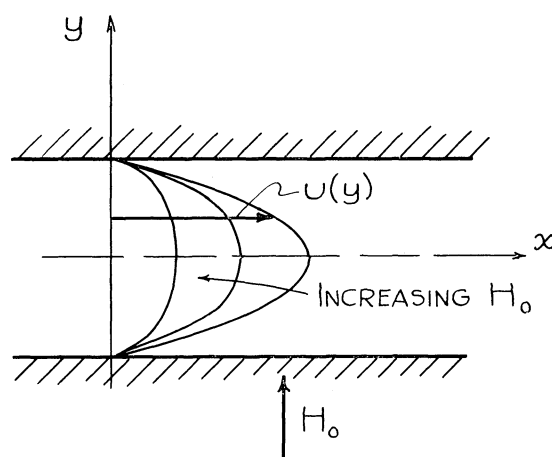


FIG. 4. Poiseuille flow of a conducting liquid, I. Magnetic field applied in plane of flow.

We have not made a detailed study of this device. A device based on similar principles was studied in detail by R. M. Patrick in a Cornell thesis.⁵ Patrick's objective was to produce lift rather than drag, so that he considered the device at angles of attack.

In applications of this kind it is clear that the distribution of the e.b.f. across the ionized layer is nonuniform because the distribution of $\mathbf{q}$ is nonuniform. In view of the usual boundary-layer profile, it appears that the decelerating force must increase in magnitude with distance from the body surface. The result must then be a tendency to alter the boundary-layer profile in a sense that would diminish both heat transfer and skin friction. These tendencies have been recognized by Rossow,⁸ whose work will be mentioned again in connection with the next example.

Two-Dimensional Channel Flow (I)

A classical case of magneto-hydrodynamic flow is the one sketched in Fig. 4, namely two-dimensional steady flow between infinite, parallel, insulating walls (Poiseuille flow) with a uniform magnetic field $\mathbf{H}_0$ applied normal to these walls. This case was treated by Hartmann¹ for an incompressible fluid. Under the hypothesis stated it can be assumed that all physical quantities are functions of y only (excepting, of course, the pressure p , which can vary linearly with x).

Eq. (10) implies $\text{div } \mathbf{H} = 0$, and, in the present problem, this means that the y -component of $\mathbf{H}$ is constant and equal to H_0 . Similarly, Eq. (9) states $\text{div } \mathbf{j} = 0$, and this means that the y -component of $\mathbf{j}$ is constant; it must therefore be equal to zero since the top and bottom walls are insulators. For incompressible flow, then, Eq. (4) becomes

$$P = \mu(d^2u/dy^2) - j_z H_0 \quad (19)$$

where P denotes the constant pressure gradient $\partial p/\partial x$ and j_z is the z -component of $\mathbf{j}$.

The equation for the z -component of momentum now states that no current flows in the x direction—i.e., $\mathbf{j}$ has only a z -component. The remaining equation of momentum then becomes

$$\partial p/\partial y = H_x j_z \quad (20)$$

where H_x is the x -component of $\mathbf{H}$. Further consequences of the single-component nature of $\mathbf{j}$ follow from Eq. (9)—namely, that H_z , the z -component of $\mathbf{H}$, must be constant and

$$4\pi j_z = -\partial H_x/\partial y \quad (21)$$

According to the hypotheses of the present problem, the constant H_z must be equal to zero, but in any case it does not affect the dynamics of the flow—i.e., the solution is the same if the applied magnetic field has a z -component.

An analysis of the electric field vector $\mathbf{E}$ is accomplished by means of Eq. (10), which states $\text{curl } \mathbf{E} = 0$ in this problem. This leads to the conclusion that both the x - and z -components of $\mathbf{E}$ are constants. But

Ohm's Law, Eq. (7), now states that the x -component is zero, that the y -component is equal to uH_z , and that j_z is given by

$$j_z = \sigma\{E_0 + uH_0\} \quad (22)$$

where E_0 is the constant z -component of the electric field, determined by boundary conditions at the extremities of the flow. Combining Eqs. (19) and (22), we have the differential equation for $u(y)$,

$$\mu(d^2u/dy^2) - \sigma H_0^2 u = P + \sigma E_0 H_0 \quad (23)$$

the solution of which is

$$u = -\frac{P + \sigma E_0 H_0}{\sigma H_0^2} \left\{ 1 - \frac{\cosh(\sqrt{\sigma/\mu} H_0 y)}{\cosh(\sqrt{\sigma/\mu} H_0 a)} \right\} \quad (24)$$

where a is the channel half-width as shown in Fig. 4. The familiar parabolic profile of Poiseuille flow is obtained from Eq. (24) by carrying out the limiting process $\sigma \rightarrow 0$.

It is interesting that in this case the velocity profile shape does not depend on E_0 , although the current density certainly does, as well as the Joule heating.

The profiles in Fig. 4 are sketched from Hartmann's paper. They show how the parabolic profile of Poiseuille flow is flattened by the magneto-hydrodynamic effect. The flow of fluid for given pressure gradient $\partial p/\partial x$ is reduced.

The solution is completed by integration of Eq. (21) for H_x , j_z being given by Eqs. (22) and (24). Also, one can combine Eqs. (20) and (21) and integrate, yielding

$$\begin{aligned} p &= -(1/8\pi)H_x^2 + \text{function of } x \\ &= -(1/8\pi)H_x^2 + Px + \text{constant} \end{aligned} \quad (25)$$

This type of result has led to the terminology "magnetic pressure" for terms like $H_x^2/8\pi$. It is clear that in the present problem the pressure gradients across the channel arise from the interaction of the induced current with the accompanying induced magnetic field.

The alteration of profiles by magneto-hydrodynamic effects in this problem suggests that a similar effect may occur in the analogous boundary-layer problem. This leads to the problem studied by Rossow⁸ (the case of two-dimensional incompressible flow over an infinite flat plate with a normal magnetic field). Rossow's results show changes of profile analogous to those of Hartmann, as predicted above. Rossow proceeds to calculate the alteration of temperature profiles, accounting for Joule heating, heat conduction, and viscous dissipation. He concludes that heat transfer to the wall may be appreciably reduced by magneto-hydrodynamic effects.

Rossow's paper involves two interesting extensions of this problem. One is a calculation of the analogous case with magnetic field fixed to the undisturbed fluid stream. This represents, for example, the case of a plate moving through the atmosphere and the earth's magnetic field. The second is an investigation of the

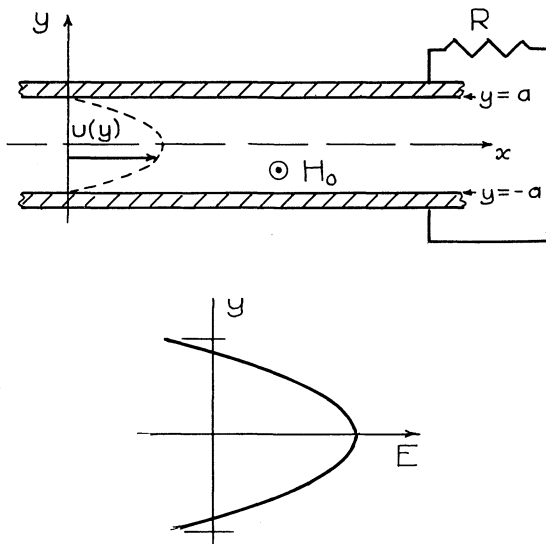


FIG. 5. Poiseuille flow of a conducting liquid, II. Magnetic field applied normal to plane of flow. Top and bottom walls of channel are conducting.

effects of variable σ . It has already been pointed out that σ varies rapidly with temperature; accordingly, Rossow writes

$$\sigma = \text{constant} [(u_\infty - u)/u_\infty] \quad (26)$$

where u_∞ denotes the stream speed. Although Eq. (26) is rather arbitrary, it should give an indication of the reduction of magneto-aerodynamic effects when conductivity drops off to a negligible value at the edge of a boundary layer. This is not quite the case sketched in Fig. 3, to be sure, for there the ionized layer is bounded by a shock wave, and the approximation of constant σ does not seem so objectionable.

Two-Dimensional Channel Flow (II)

A slightly different modification of Poiseuille flow is sketched in Fig. 5; this represents the same case as in Fig. 4, but with the uniform magnetic field directed along the z -axis—i.e., normal to the plane of the flow. This problem has the virtue of illustrating how an electric field $\mathbf{E}$ is induced by the motion; it also shows how relatively complicated a seemingly simple flow may become with magneto-hydrodynamic effects present. We shall treat it only in the approximation of first-order perturbations. Specifically, we assume that the magnetic field vector is made up of the uniform applied component $\mathbf{H}_0$ plus a much smaller perturbation $\mathbf{h}$.

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h}, \quad h \ll H_0 \quad (27)$$

The principal effect of the magnetic field is then the production of a current-density component j_y in the y direction, and the x -component of Eq. (4) becomes, approximately,

$$P = \partial p / \partial x = \mu(\partial^2 u / \partial y^2) + j_y H_0 \quad (28)$$

Now Eq. (9) requires $\text{div } \mathbf{j} = 0$. As in Hartmann's problem above, we postulate that conditions do not

vary with the streamwise coordinate x , but we cannot also require independence of z . For the existence of a component j_y implies, according to Eq. (9), that the induced magnetic field varies with z —i.e.,

$$j_y = \partial h_x / \partial z \neq 0 \quad (29)$$

Thus the flow can be only quasi-two-dimensional. Eq. (29) implies that a streamwise component h_x is induced and that it varies with z . This variation signifies that the magnetic lines of force are actually curved whereas in our approximation we consider them to lie straight across the flow in the z direction. The current-density component j_y is then approximately constant and the term $j_y H_0$ in Eq. (28) represents an additional pressure gradient. The solution of Eq. (28) is the parabolic profile of Poiseuille flow, only the flow quantity for given pressure gradient being changed.

The approximate evaluation of j_y follows from Eq. (7):

$$j_y \approx \text{constant} = \sigma \{E_y - uH_0\} \quad (30)$$

If the top and bottom channel walls are insulators, or if the circuit connecting them is open, the only solution is $j_y = 0$. Suppose, however, that the circuit is closed, as sketched, by a conductor of resistance R per unit length and breadth of channel. Integrating Eq. (30) with respect to y , one has, for constant σ ,

$$2\sigma j_y = -\sigma \Delta \mathcal{V} - \sigma H_0 \int_{-a}^a u dy \quad (31)$$

where the top and bottom walls are at $y = \pm a$ and $\Delta \mathcal{V}$ denotes the voltage difference between these walls—i.e.,

$$\int_{-a}^a E_y dy = -\mathcal{V}(a) + \mathcal{V}(-a) \equiv -\Delta \mathcal{V} \quad (32)$$

But the same current must flow through the exterior circuit—i.e.,

$$j_y = \Delta \mathcal{V} / R \quad (33)$$

Thus, combining Eqs. (31) and (32),

$$j_y = \frac{-H_0 \int_{-a}^a u dy}{R + (2a/\sigma)} \quad (34)$$

The distribution of electric field is given by

$$E_y = (j_y / \sigma) + uH_0 \quad (35)$$

as sketched in Fig. 5.

It is interesting to note that the current density is proportional to the quantity of flow through the channel. We are informed that flowmeters operating on this principle are available. It is also interesting to consider the energy balance in this case. For this purpose we use Eqs. (6) and (8). The only alteration of Eq. (6) for the case of an incompressible fluid is the deletion of the term $p \partial \rho^{-1} / \partial t$. (To be sure, it

is not common to use the concept of stagnation enthalpy for an incompressible fluid because the total head, $p + \rho q^2/2$, is equally convenient and there is ordinarily no coupling between temperature and velocity. In magneto-hydrodynamics, however, there is such coupling by virtue of Joule heating.)

Thus, the rate at which stagnation enthalpy is being increased is given by Eq. (8). This rate is $\mathbf{j} \cdot \mathbf{E}$, or in our present approximation, $j_y E_y$. By integration of this quantity from wall to wall, this rate is easily found to be

$$j_y \int_{-a}^a E_y dy = -j_y^2 R \quad (36)$$

In other words, energy is being extracted from the fluid at a rate equal to the Joule heating of the external resistance. Another check on the energy balance can also be made—namely, that the total work done by the pressure gradient $\partial p/\partial x$ appears in heating of the fluid and the external resistance.

The induced streamwise component h_x of the magnetic field is not negligible, as Eq. (29) indicates. Thus the body-force component $j_y h_x$ in the equation of z -momentum is also appreciable, and there must be a pressure gradient $\partial p/\partial z$ of the same magnitude. To complete this solution by calculating to a better approximation all of the components set up by the motion is a complicated process. This problem has been treated, though not by the perturbation process suggested here, by Shercliffe.⁹

Although this sample does illustrate a case where the electric field is not originally known and must be calculated, it is a particularly simple case in the approximation employed here because the directions of the vectors $\mathbf{j}$, $\mathbf{E}$, and $\mathbf{q} \times \mathbf{H}$ are determined by the hypotheses of the problem. In general, in two- and three-dimensional problems, this is not the case; in fact, these vectors may have three different directions. The technique then employed is to eliminate $\mathbf{j}$ from Eq. (4) means of Eq. (9), and to obtain an auxiliary partial differential equation by combination of Eqs. (7), (9), and (10)—viz.,

$$(1/4\pi) \text{curl}^2 \mathbf{H} = \sigma \{ -(\partial \mathbf{H}/\partial t) + \text{curl}(\mathbf{q} \times \mathbf{H}) \} \quad (37)$$

One-Dimensional Channel Flow—Acceleration of a Gas

Let us consider here a problem of more immediate aeronautical interest—namely, the acceleration of a gas flow in a channel such as a nozzle or tail-pipe. If the Reynolds Number is large and the channel cross-section A changes relatively slowly, one can neglect viscosity and treat the problem by the familiar methods of one-dimensional channel flow. In other words, one can consider the flow speed u to be a function of the coordinate x measured along the channel length only. Thus it is really an average speed. The same approximation can be made for the pressure p , density ρ , etc. The equations of motion, Eqs. (3), (4), and (8), become, for steady flow,

$$\text{Continuity:} \quad \rho u A = \text{constant} \quad (38)$$

$$\text{Momentum:} \quad \rho u u' + p' = \sigma \{ E - u H \} H \quad (39)$$

$$\text{Energy:} \quad \rho u c_p T' + u^2 u' = \sigma \{ E - u H \} E \quad (40)$$

where the prime denotes differentiation with respect to the coordinate x .

We recall that, in ordinary flow problems without magneto-aerodynamical effects, Eq. (40) becomes a statement of conservation of stagnation enthalpy. Eq. (38), together with this statement, leads to the familiar results of one-dimensional gasdynamics such as nozzle flow and choking. These appear as special cases in what follows.

Suppose now that not only $A(x)$ but also the field strengths $E(x)$ and $H(x)$ are given. Our point of view is therefore somewhat different from that adopted in our earlier examples, for here we do assume that an electric field is applied by a device exterior to the channel. We imagine, for example, that this field might be generated magneto-aerodynamically by motion of an aircraft and applied across an interior duct or tailpipe. Thus, within reasonable limits, we assume both E and H to be under control of the designer.

In general, for given $A(x)$, $E(x)$, and $H(x)$, Eqs. (38)–(40) cannot be integrated, but the acceleration u' and the derivative of the Mach Number, M' , can be calculated. (The Mach Number that appears here is the conventional ratio u/a , where a denotes $\sqrt{\gamma p/\rho}$, $\gamma = c_p/c_v$.) The results are

$$u' = \frac{1}{M^2 - 1} \left\{ \frac{u}{A} A' - \frac{\sigma H^2}{p} (u - u_3)(u - u_1) \right\} \quad (41)$$

and

$$M' = \frac{1 + [(\gamma - 1)/2] M^2}{M^2 - 1} \times \left\{ \frac{M}{A} A' - \frac{\sigma H^2}{a p} (u - u_3)(u - u_2) \right\} \quad (42)$$

where the significant velocities u_1 , u_2 , and u_3 are defined as follows:

$$u_1 = [(\gamma - 1)/\gamma](E/H) \quad (43)$$

$$u_2 = \{ (1 + \gamma M^2)/[2 + (\gamma - 1)M^2] \} u_1 \quad (44)$$

$$u_3 = E/H \quad (45)$$

The terms involving $A'(x)$ in Eqs. (41) and (42) are familiar to gasdynamicists; they state that the velocity and Mach Number decrease in an expanding channel at subsonic speeds and increase in an expanding channel at supersonic speeds.

The signs of the magneto-gasdynamic terms, on the other hand, depend on the magnitude of u relative to the significant velocities u_1 , u_2 , and u_3 . In particular, u' can be made positive even in constant-area channels at supersonic speeds provided only that u lies in the interval between u_1 and u_3 . Furthermore, M' can be made positive in a constant-area supersonic channel if $u_2 < u < u_3$. This is in sharp contrast

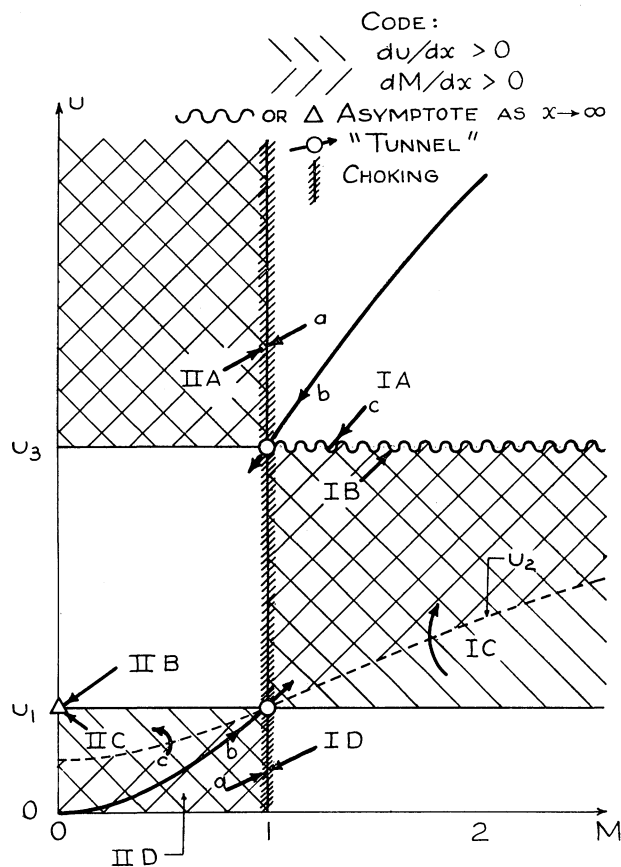


Fig. 6. Magneto-gasdynamic acceleration of one-dimensional flow in a constant-area channel. Regimes of positive acceleration and increasing Mach Number are designated by cross-hatching as shown. u_1 , u_2 , and u_3 are the "significant speeds" defined in this paper. Arrows denote typical behaviors of flows in the various regimes; arrowheads show direction of increasing x (downstream). IA, IB, etc. denote areas of different behaviors as indicated.

to the effects of pure heat addition, which always decelerates a supersonic constant-area flow and drives its Mach Number toward 1.

A summary of the various possibilities for constant-area flow is presented in Fig. 6, where the signs of u' and M' in a u, M plane are indicated. There are eight distinct regions in the u, M plane, in which different behaviors of $u(x)$ and $M(x)$ can be predicted from Eqs. (41) and (42). These are identified in Fig. 6 as follows:

- | | |
|---------------------|---------------------|
| (I) $M > 1$ | (II) $M < 1$ |
| (A) $u_3 < u$ | (A) $u_3 < u$ |
| (B) $u_2 < u < u_3$ | (B) $u_1 < u < u_3$ |
| (C) $u_1 < u < u_2$ | (C) $u_2 < u < u_1$ |
| (D) $u < u_1$ | (D) $u < u_2$ |

In general, a "barrier" exists at $M = 1$, where the phenomenon of choking occurs: the denominators of both Eqs. (41) and (42) disappear. That this represents choking is evident, from the fact that the curve of u versus x turns back upon itself and becomes double valued after reaching the barrier. Thus the sonic speed can never actually be attained in such a channel, and the phenomenon would have to occur, if at all, at the open end of a channel.

There are, however, two exceptional cases, denoted

"tunnels" in Fig. 6. These are two one-way tunnels through the barrier at sonic speed. For these special combinations of parameters (IAb and IIDb) the numerators and denominators of both Eqs. (41) and (42) disappear simultaneously, and, as can be verified, finite and continuous deceleration and acceleration, respectively, occur. The curves that lead to these tunnels have been plotted in Fig. 6, as they always separate choking from nonchoking channels and represent the special combinations that permit smooth passage through the sonic speed.

Maximum-Acceleration Constant-Area Duct, Supersonic Flow

According to Eq. (41), the acceleration of a supersonic stream in a constant-area channel can be made a maximum by choosing a certain value of E/H —i.e., the value that maximizes the product $-(u - u_3) \times (u - u_1)$. This value is

$$E/H = [(2\gamma - 1)/2(\gamma - 1)]u \quad (46)$$

The maximum acceleration is obtained by substituting this value in Eq. (41); the result is

$$u_{max}' = [1/(\gamma - 1)](\sigma H^2/4m)[M^2 u/(M^2 - 1)] \quad (47)$$

where m denotes the constant ρu (the mass flow per unit cross-section area).

As an indication of the possible velocities to be achieved magneto-aerodynamically, we have integrated Eq. (47) for constant γ . In other words, we assume that E/H is varied with x so as to maintain u' always at its maximum value. This calculation involves, first, integration of M as a function of u , which can be carried out using Eqs. (41), (42), and (47). The results are

$$u = \text{constant} \times \left| \frac{M^2}{[(2\gamma + 1)/\gamma] - M^2} \right|^{1/(2\gamma + 1)} \quad (48)$$

and finally,

$$\begin{aligned} (1/m) \int_{x_0}^x \sigma H^2 dx &\equiv \xi \\ &= \frac{4\gamma(\gamma - 1)}{2\gamma + 1} \left\{ \frac{\gamma + 1}{\gamma} \ln \frac{u}{u_0} - \frac{\gamma M_0^2 - (2\gamma + 1) \left[\left(\frac{u_0}{u} \right)^{2\gamma + 1} - 1 \right]}{(2\gamma + 1)\gamma M_0^2} \right\} \end{aligned} \quad (49)$$

Here the quantities with subscript 0 denote values at an initial station of the channel. If σH^2 is constant, Eq. (49) permits an easy calculation of the channel length $x - x_0$ for velocity ratio u/u_0 . Eq. (49) is plotted in Fig. 7 for several values of M_0 , for $\gamma = 1.4$. It is interesting to note that the curve of u/u_0 against ξ becomes practically independent of M_0 for $M_0 > 5$, making u/u_0 a function of ξ only.

The corresponding Mach Numbers are not plotted, but could be determined from Eq. (48). Since u always increases with x in this case, the form of Eq. (48) indicates that M tends toward the value

$\sqrt{(2\gamma + 1)/\gamma}$, or, for $\gamma = 1.4$, $M \rightarrow 1.65$. Numerical calculations show that this approach is very rapid; M is nearly equal to 1.65 when u/u_0 reaches 3 for any initial Mach Number greater than 1. It is clear from this result that there is considerable heating of the gas in a typical case, and that appreciable further increase of flow speed might be achieved by expanding in a supersonic nozzle.

The present results (Eq. (49) and Fig. 7) must not be used when $E - uH$ is greater than the "breakdown" potential of the gas. This critical potential (where arcing occurs and therefore the assumptions of the present studies are violated) varies directly with density in the range of densities anticipated here; i.e., the restriction on our results may be written as

$$E - uH < K_B \rho \quad (50)$$

where K_B has the value of about 2×10^7 volt cm.²/gm. for air.* Thus the restriction can easily be expressed by combining Eq. (46) and inequality (50) to give a maximum permissible value for u , above which Eq. (46) will lead to a violation of (50). It is

$$u < \sqrt{2(\gamma - 1)10^8 K_B m/H} \quad (51)$$

where H is measured in gauss, m in gm. cm.⁻² sec.⁻¹, and u in cm./sec. This does not mean that speeds greater than this cannot be achieved, but only that they cannot be achieved under the maximum-acceleration conditions of Eqs. (46)–(49). For greater velocities, acceleration might have to be accomplished under conditions of constant potential $E - uH$, for example. We are studying further the consequences of this limitation in practical cases.

CONCLUSIONS

The studies reported here suggest that electromagnetic effects may become important in aerodynamics, especially if seeding can be used to increase the conductivity of the air. Here we understand "aerodynamics" to mean the realm of flight where the continuum approximation is relatively useful. There are, for example, possibilities of decelerating re-entry missiles, controlling the skin friction and heat transfer in boundary layers, and accelerating or decelerating channel flows magneto-aerodynamically.

The last mentioned of these possibilities appears especially interesting to the present authors, since it permits some significant departures from the restrictions of conventional gasdynamics, such as acceleration of supersonic flows in straight tubes.

There are, however, some formidable practical obstacles in the path of engineering applications of the principles mentioned here. One of these may be the complexity of the equations of motion, since this will hamper the theoretical studies that might lead to significant and efficient applications. More serious,

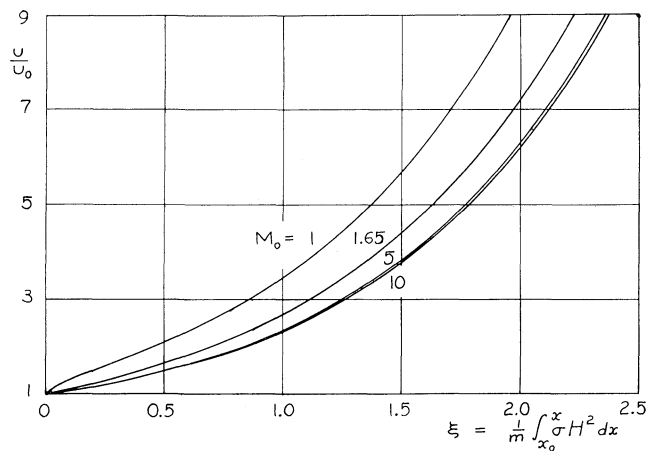


FIG. 7. Magneto-gasdynamic channel flow. Flow speed ratio as function of the channel-length variable ξ for a maximum-acceleration channel of constant area for several initial Mach Numbers.

however, is the problem of the development of magnets powerful enough to provide the required fields and yet light enough for use in flight. Techniques for rendering gases conducting by means of seeding must also be made practical.

Another engineering problem of particular significance seems to be posed by the need for electrodes in all the magneto-aerodynamic devices suggested in the present paper. Hartmann¹ mentioned the high surface resistance of electrodes in contact with mercury in his early experiments. Brogan¹⁰ found a similar phenomenon in a shock tube, where the overall resistance of a circuit including cold copper electrodes in contact with ionized argon was many times the calculated resistance of the gas alone. The solution of this rather basic problem is certainly not obvious at present.

Thus, successful achievement of magneto-aerodynamic effects in flight must be preceded by considerable research and engineering development. The possible benefits to be obtained seem to justify such efforts.

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* We can expect the same value to apply for seeded air.

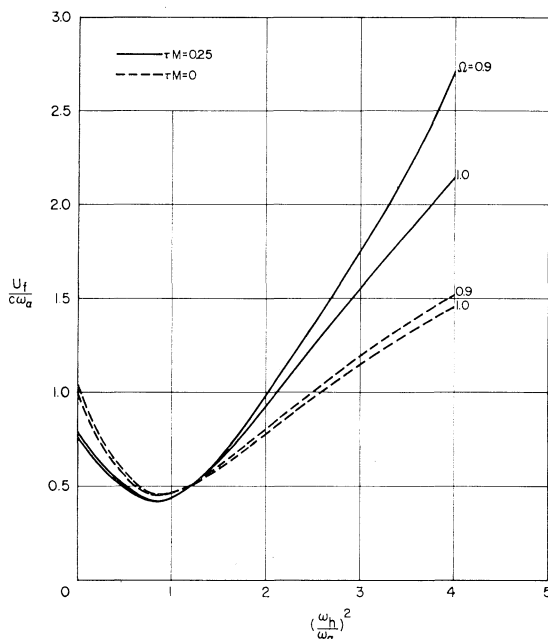


FIG. 15. Effect of mode shape on flutter speed of double wedge profile at zero angle of attack. $\xi_0 = 0.5$, $\xi_\alpha = 0.1$, $r\alpha^2 = 1/3$, $\mu M = 250$.

For $0 < (\omega_h/\omega_\alpha) < 1.0$, a wing should be stiffened torsionally and, for $(\omega_h/\omega_\alpha) > 1.0$, it should be stiffened flexurally in order to raise its flutter speed.

At high Mach Number, the frequency at flutter is independent of altitude.

With changes in altitude, the flutter Mach Number and the dynamic pressure at flutter speed are inversely proportional to the pressure ratio, and the flutter speed is inversely proportional to the product of density ratio and speed of sound ratio, $\sigma(a/a_0)$.

Airfoil thickness lowers the flutter speed for $0 < (\omega_h/\omega_\alpha) < 1.0$. The effect increases with a forward shift in the axis of pitch. For $(\omega_h/\omega_\alpha) > 1$ these effects are reversed. These characteristics depend upon airfoil profile also.

Small angles of attack and spanwise mode shapes produce almost equal and opposite effects on flutter

speed over a wide range of frequency ratio. Neglecting them in preliminary calculations should not result in a great loss of accuracy.

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