Vacuum quantum fluctuations in curved space and the theory of gravitation

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In Einstein's theory of gravitation one postulates that the action of space-time depends on the curvature (R is the invariant of the Ricci tensor):

$$S(R) = -\frac{1}{16\pi G}\int (dx)\sqrt{-g}R. \quad (1)$$

The presence of the action (1) leads to a "metrical elasticity" of space, i.e., to generalized forces which oppose the curving of space.

Here we consider the hypothesis which identifies the action (1) with the change in the action of quantum fluctuations of the vacuum if space is curved. Thus, we consider the metrical elasticity of space as a sort of level displacement effect (cf. also Ref. 1).[1)]

In present-day quantum field theory it is assumed that the energy-momentum tensor of the quantum fluctuations of the vacuum $T^i_k(0)$ and the corresponding action $S(0)$, formally proportional to a divergent integral of the fourth power over the momenta of the virtual particles of the form $\int k^3 dk$, are actually equal to zero.

Recently Ya. B. Zel'dovich[3] suggested that gravitational interactions could lead to a "small" disturbance of this equilibrium and thus to a finite value of Einstein's cosmological constant, in agreement with the recent interpretation of the astrophysical data. Here we are interested in the dependence of the action of the quantum fluctuations on the curvature of space. Expanding the density of the Lagrange function in a series in powers of the curvature, we have (A and $B \sim 1$)

$$(R) = \mathcal{L}(0) + A\int k dk \cdot R + B\int\frac{dk}{k}R^2 + \dots \,. \quad (2)$$

The first term corresponds to Einstein's cosmological constant.

The second term, according to our hypothesis, corresponds to the action (1), i.e.,

$$G = -\frac{1}{16\pi A\int k dk}, \quad A \sim 1. \quad (3)$$

The third term in the expansion, written here in a provisional form, leads to corrections, nonlinear in R, to Einstein's equations.[2)]

The divergent integrals over the momenta of the virtual particles in (2) and (3) are constructed from dimensional considerations. Knowing the numerical value of the gravitational constant G, we find that the effective integration limit in (3) is

$$k_0 \sim 10^{28}\ \text{eV} \sim 10^{+33}\text{cm}^{-1}.$$

In a gravitational system of units, $G = \hbar = c = 1$. In this case $k_0 \sim 1$. According to the suggestion of M. A. Markov, the quantity k_0 determines the mass of the heaviest particles existing in nature, and which he calls "maximons." It is natural to suppose also that the quantity k_0 determines the limit of applicability of present-day notions of space and causality.

Consideration of the density of the vacuum Lagrange function in a simplified "model" of the theory for noninteracting free fields with particles $M \sim k_0$ shows that for fixed ratios of the masses of real particles and "ghost" particles (i.e., hypothetical particles which give an opposite contribution from that of the real particles to the R-dependent action), a finite change of action arises that is proportional to M^2R and which we identify with R/G. Thus, the magnitude of the gravitational interaction is determined by the masses and equations of motion of free particles, and also, probably, by the "momentum cutoff."

This approach to the theory of gravitation is analogous to the discussion of quantum electrodynamics in Refs. 4 to 6, where the possibility is mentioned of neglecting the Lagrangian of the free electromagnetic field for the calculation of the renormalization of the elementary electric charge. In the paper of L. D. Landau and I. Ya. Pomeranchuk the magnitude of the elementary charge is expressed in terms of the masses of the particles and the momentum cutoff. For a further development of these ideas see Ref. 7, in which the possibility is established of formulating the equations of quantum electrodynamics without the "bare" Lagrangian of the free electromagnetic field.

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1) Here the molecular attraction of condensed bodies is calculated as the result of changes in the spectrum of electromagnetic fluctuations. As was pointed out by the author, the particular case of the attraction of metallic bodies was studied earlier by Casimir.[2]

2) A more accurate form of this term is $\int(dk/k)(BR^2 + CR^{ik}R_{ik} + DR^{iklm}R_{iklm} + ER^{iklm}R_{iklm})$ where $A, B, C, D, E \sim 1$. According to Refs. 4 to 7, $\int dk/k \sim 137$, so that the third term is important for $R \gtrsim 1/137$ (in gravitational units), i.e., in the neighborhood of the singular point in Friedman's model of the universe.

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